Sliding Mode Control Based on Adaptive Backstepping Approach for a Quadrotor Unmanned Aerial Vehicle

Abstract. A sliding mode control based on adaptive backstepping approach is developed respectively in order to synthesise tracking errors and to ensure Lyapunov stability, handling of all system nonlinearities and desired tracking trajectories. Under ground effects and wind disturbances, the body inertia becomes badly known, and non parametric uncertainties are considered in the system model. Dynamic modelling of quadrotor takes into account the high-order nonholonomic constraints, that are considered here in order to test this new control scheme on a model that takes into account the various physical phenomenas, which can influence the dynamics of a flying structure. Finally simulation results are provided to illustrate the performances of the proposed controller.

Słowa kluczowe: Tuning function, Nonholonomic constraints, Adaptive backstepping, Sliding mode.

Introduction

Unmanned aerial vehicles are characterized by high level of complexity, high dimension of the state space, multiple inputs and outputs, parametric uncertainty, unmodeled dynamics. Its inertia will slowly vary as a result of wind disturbances and ground effects. We need a control law that adapts itself to such changing conditions. Affected by aerodynamic forces, the Quadrotor dynamics is nonlinear, multivariable, and is subject to parameter uncertainties and external disturbances. In turn, controlling of the Quadrotor is required i) to meet the stability, robustness and desired dynamic properties. ii) to be able to handle nonlinearity. iii) to be adaptive to changing parameters and environmental disturbances.

The adaptive control deals with system which the parameters are slowly time-varying or uncertain. The adaptive backstepping control has been widely studied in the last two decade [3], [13], [9], [11], [10], [12], and [14]. In the latter newly developed strategies are presented. The designed controllers are shown to guarantee all signals bounded in the system and yield good transient and tracking performance. Based on backstepping approach, this method use the tuning functions for updating the unknown parameter to be estimated and overcome the overparametrisation problem. An adaptive controller is designed by combining a parameter estimator, which provides the unknown parameter estimation, with a control law. The parameters of the controller are adjusted during the operation of the plant. Affected by wind aerodynamic force and wind aerodynamic torque, the control scheme is reinforced by sliding mode control, based on adaptive backstepping approach. This control is developed by taking error of virtual controls as sliding surface. The same strategy used into [15] is followed, except the backstepping approach is replaced by adaptive backstepping approach, while taking into account the high-order nonholonomic constraints. This method presents the following major advantages: i) It ensures Lyapunov stability. ii) It ensures the robustness and all properties of the desired dynamics. iii) It handles the overparametrisation problem.

In this paper, the dynamic modelling of system [4] with the nonholonomic constraints taken into account [15] and neglecting a Coriolis torque and a gyroscopic torque [1] are presented in section II. Theoretical background of adaptive backstepping approach and his application to quadrotor system are developed in section III. In section IV, sliding mode control, based on adaptive backstepping approach of the quadrotor system is developed. Simulation results are presented in Section V and a conclusion is drawn in Section VI.

Dynamical Model of X4-Flyer

The coordinate system and X4-flyer configuration are shown in (Fig. 1).

Let \( E = \{E_x, E_y, E_z\} \) denote a right-hand inertial frame such that \( E_x \) denotes the vertical direction downwards into the earth. Let the vector \( \zeta = [x \ y \ z]^T \) denote the position of the centre of mass of the airframe in the frame \( E \) relative to a fixed origin \( O \in E \). Let \( c \) be a (right-hand) body fixed frame for the airframe. When defining the rotational angles \( \eta = [\phi \ \theta \ \psi]^T \) around X-, Y-, and Z-axis in the frame \( c \), the orientation of the rigid body is given by a rotation \( R : c \to E \), where \( R \in \mathbb{R}^{3 \times 3} \) is an orthogonal rotation matrix given by

\[
(1) \quad R = \begin{pmatrix}
\cos \psi \cos \phi \cos \theta - \sin \phi \sin \psi & \cos \psi \sin \phi \cos \theta + \cos \phi \sin \psi & 
-\sin \psi \cos \phi \\
\cos \phi \sin \psi + \cos \psi \cos \phi \sin \theta & 
\cos \psi \cos \phi \cos \theta - \sin \phi \sin \psi & 
\cos \phi \sin \psi \cos \theta + \cos \psi \cos \phi \\
-sin \phi & 
\cos \psi \cos \phi & 
\cos \psi \sin \phi
\end{pmatrix}
\]

Let \( J \) denote the inertia matrix around the center of mass (expressed in the body fixed frame \( c \)), where the moment of inertia around each axis is given by \( I_x, I_y, I_z \). Moreover, let \( f_i \) denote the thrust generated by the rotor \( i \) in free air (expressed in \( c \)), \( m \) is the mass of the airframe, \( g \) denotes the acceleration due to gravity, and \( l \) denotes the distance from the rotors to the center of gravity of the airframe. We present a dynamical model of the X4-flyer using a Lagrangian approach. The translational kinetic energy of the X4-flyer will be

\[
(2) \quad T_{trans} = \frac{1}{2} m \dot{\zeta}^T \dot{\zeta}
\]

The rotational kinetic energy is

\[
(3) \quad T_{rot} = \frac{1}{2} \dot{\eta}^T J \dot{\eta}
\]

with \( J = diag \{I_x, I_y, I_z\} \). And the only potential energy which needs to be considered is the standard gravitational potential energy
The Lagrangian is given by

\[ L = T_{\text{trans}} + T_{\text{rot}} - U \]

\[ = \frac{1}{2} m \dot{\zeta}^T \zeta + \frac{1}{2} \eta^T J \eta + m g z \]

\[ F = [F_\zeta + F_\tau]^{T}, \text{Where } F_\zeta \text{ is the translational force applied to the X4-flyer due to the control inputs, } F_\tau \text{ is the resultant of the drag forces along } (X, Y, Z) \text{ axis and } \tau = [\tau_\phi \ \tau_\theta \ \tau_\psi]^T \text{ is the generalized moments around the airframe.} \]

\[ u_1 = f_1 + f_2 + f_3 + f_4 \]

And the unit vector of $Z$-direction in the airframe be $e_3 = [0 \ 0 \ 1]^T$, it yields that

\[ F_\zeta = - R e_3 u_1 \]

\[ = - u_1 \begin{bmatrix} c \phi s \theta c \psi + s \phi s \psi \\ c \phi \theta s \psi - s \phi c \psi \\ c \phi e \theta \end{bmatrix} \]

\[ F_\tau = \begin{bmatrix} -K_{ftx} & 0 & 0 \\ 0 & -K_{fty} & 0 \\ 0 & 0 & -K_{ftz} \end{bmatrix} \zeta \]

Such as $K_{ftx}$, $K_{fty}$ and $K_{ftz}$ are the translation drag coefficients, and $\tau$ becomes

\[ \tau_\phi = (f_2 - f_4) l \]

\[ \tau_\theta = (f_1 - f_3) l \]

\[ \tau_\psi = \sum_{i=1}^{4} \tau_{Mi} \]

\[ J \dot{\omega} = \omega \times J \omega + \sum_{i=1}^{4} J_r \omega e_3 \omega_i + \tau \]

\[ \omega = \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix}^{T}, \text{J}_r \text{ denotes the moment of inertia for the rotor, } \omega_i \text{ is the rotational speed of the rotor } i. \]

Neglecting a term consisting of a Coriolis torque and a gyroscopic torque, the final dynamics of X4 flyer can be reduced to

\[ m \ddot{x} = (\cos \phi \sin \theta \cos \psi + \sin \phi \sin \psi) \ u_1 - K_{ftx} x \]

\[ m \ddot{y} = (\cos \phi \sin \theta \sin \psi - \sin \phi \cos \psi) \ u_1 - K_{fty} y \]

\[ m \ddot{z} = (\cos \phi \cos \theta) \ u_1 - m g - K_{ftz} z \]

Assuming that the electric rotors are velocity controlled then $(u_1, u_2, u_3$ and $u_4)$ may be considered directly as control inputs.

**Nonholonomic Constraints**

Taking into account nonholonomic constraints which define the coupling between various states of the system. From the equations (10), (11), and (12) we can extract the expressions of the high-order nonholonomic constraints:

\[ \tan \theta = \left( \frac{z + K_{ftx} x \sin \psi + (y + K_{fty} y) \sin \psi}{z + g + \frac{K_{ftz}}{\sqrt{x^2}}} \right) \]

\[ \sin \phi = \left( \frac{(z + K_{ftx} x) \sin \psi + (y + K_{fty} y) \cos \psi}{\sqrt{(z + K_{ftx} x)^2 + (y + K_{fty} y)^2 + (z + g + \frac{K_{ftz}}{\sqrt{x^2}})^2}} \right) \]

**Adaptive Backstepping**

**Theoretical background**

In this section, we will consider unknown parameters which appear linearly in system equations[14]. An adaptive controller is designed by combining a parameter estimator, which provides estimates of unknown parameters, with a control law. The parameters of the controller are adjusted during the operation of the plant. In the presence of such parametric uncertainties, the adaptive controller is able to ensure the boundedness of the closed-loop states and asymptotic tracking.

The adaptive backstepping control with tuning function is considered here. Let the following sytem:

\[ x_{n-1} = x_n + \phi^{T}_{n-1} x_n + \psi_n \]

\[ x_n = b u + \phi^{T}_n x \theta + \psi_n \]

where $\phi$ and $\psi$ are the parameter estimates of unknown parameters.
Where \( \mathbf{x} = [x_1, \ldots, x_{n-1}]^T \in \mathbb{R}^n \), the vector \( \theta \in \mathbb{R}^r \) is constant and unknown, \( \phi_i \in \mathbb{R}^r, \psi_i \in \mathbb{R}, i = 1, \ldots, n \) are known nonlinear functions and \( b \) is unknown constant gain. The control objective is to force the output \( x_1 \) to asymptotically track the reference signal \( x_r \), with the following assumptions.

**Assumption 1**: The sign of \( b \) is known.

**Assumption 2**: The reference signal \( x_r \) and its \( n \) order derivatives are piecewise continuous and bounded. For system \((22)\), the number of design steps required is equal to \( n \). At each step, an error variable \( z_i \), a stabilizing function \( \alpha_i \), and a tuning function \( \tau_i \) are designed.

Step 1:

\[
\begin{align*}
\dot{z}_1 &= x_1 - x_r \\
\dot{z}_1 &= x_1 - x_r - z_2 + \alpha_1 + \phi_i^T \hat{\theta} + \psi_1 \\
\alpha_1 &= -c_1 z_1 - \phi_i^T \hat{\theta} - \psi_1 \\
\tau_1 &= \phi_1 z_1 \\
V_1 &= \frac{1}{2} z_1^2
\end{align*}
\]

Step \( i \): \( i = 2, \ldots, n \)

\[
\dot{z}_i = z_{i+1} + \alpha_i + \psi_i - \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_j} (x_{j+1} + \psi_j) + \theta^T \phi_i - \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \theta} \Gamma \tau_i + \sum_{j=2}^{i-1} z_j \frac{\partial \alpha_{i-1}}{\partial \theta} \Gamma \phi_i - \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_j} \phi_j + \sum_{j=i+1}^{\infty} \frac{\partial \alpha_{i-1}}{\partial x_j} x_j^{(j)}
\]

We select the stabilizing function \( \alpha_i \)

\[
\begin{align*}
\alpha_i &= -c_i z_i - z_{i-1} - \psi_i + \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_j} (x_{j+1} + \psi_j) - \theta^T \phi_i - \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \theta} \Gamma \tau_i + \sum_{j=2}^{i-1} z_j \frac{\partial \alpha_{i-1}}{\partial \theta} \Gamma \phi_i - \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_j} \phi_j + \sum_{j=i+1}^{\infty} \frac{\partial \alpha_{i-1}}{\partial x_j} x_j^{(j)}
\end{align*}
\]

And tuning function

\[
\tau_i = \tau_{i-1} + \left( \phi_i - \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_j} \phi_j \right) z_i
\]

In the last step \( n \), the actual control input \( u \) appears and is at our disposal. We derive the \( z_n \) dynamics

\[
\begin{align*}
\dot{z}_n &= bu + \psi_n - \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_j} (x_{j+1} + \psi_j) + \theta^T \phi_n - \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial \theta} \hat{\theta} - \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_j} x_j^{(n)} + x_{r}^{(n)}
\end{align*}
\]

We are finally in this position to design control \( u \) and update laws \( \hat{\theta} \) and \( \hat{p} \) as

\[
\begin{align*}
u &= \hat{p} \hat{n} \\
\hat{\theta} &= \Gamma \tau_n \\
\hat{p} &= -\gamma \text{sign}(b) \hat{\pi} z_n
\end{align*}
\]

Where \( \gamma \) is a positive constant and \( \hat{p} \) is an estimate of \( p = 1/b \). Note that

\[
\begin{align*}
bu &= b \hat{p} \hat{n} = \hat{n} - b \hat{p} \hat{n}
\end{align*}
\]

Where \( \hat{p} = p - \hat{p} \). We choose the Lyapunov function

\[
\begin{align*}
V_n &= V_{n-1} + \frac{|b|}{2 \gamma} \hat{p}^2 = \sum_{i=1}^{n} \frac{1}{2} z_{i}^2 + \frac{1}{2} \hat{\gamma} \Gamma^{-1} \hat{\theta} + \frac{|b|}{2 \gamma} \hat{p}^2
\end{align*}
\]

Where \( \gamma \) is a positive design parameter. Then its derivative is given by

\[
\begin{align*}
\dot{V}_n &= -\sum_{i=1}^{n} c_i z_i^2 + \left( \sum_{j=2}^{n} z_j \frac{\partial \alpha_{j-1}}{\partial \theta} \right) \left( \Gamma \tau_n - \hat{\theta} \right) + \hat{\theta}^T \left( \tau_n - \Gamma^{-1} \hat{\theta} \right) - \frac{|b|}{\gamma} \hat{p} \left( \hat{\gamma} \text{sign}(b) \hat{\pi} z_n \right)
= -\sum_{i=1}^{n} c_i z_i^2 \leq 0
\end{align*}
\]

The controller designed in this section achieves the goals of stabilization and tracking. The proof of these properties is a direct consequence of the recursive procedure, because a Lyapunov function is constructed for the entire system including the parameter estimates. The overparametrization problem is overcome by using tuning functions. The number of parameter estimates are equal to the number of unknown parameters.

**Adaptive backstepping control of the quadrotor**

The model \((10)\) to \((15)\) developed in the previous section can be rewritten in the state-space form \((22)\) with six subsystems and wind disturbances are considered. Let

\[
X = [x; \dot{x}; y; \dot{y}; z; \dot{z}; \phi; \dot{\phi}; \theta; \dot{\theta}; \psi; \dot{\psi}]^T
\]

\[
X = [x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}, x_{12}]^T
\]

\[
u = [u_1, u_2, u_3, u_4]^T
\]

\[
\begin{align*}
x_1 &= x_2 \\
x_2 &= U_\phi \frac{u_m}{m} - a_1 x_2 + \frac{A_x}{m} \\
x_3 &= x_4 \\
x_4 &= U_\phi \frac{u_m}{m} - a_2 x_4 + \frac{A_u}{m} \\
x_5 &= x_6 \\
x_6 &= \cos \left( x_7 x_9 \right) \frac{u_5}{m} - g a_3 x_6 + \frac{A_y}{m} \\
x_7 &= x_8 \\
x_8 &= \theta_1 x_{10} x_{12} + b_1 u_2 + \frac{A_{x}}{x} \\
x_9 &= x_{10} \\
x_{10} &= \theta_2 x_8 x_{12} + b_2 u_3 + \frac{A_{x}}{x} \\
x_{11} &= x_{12} \\
x_{12} &= \theta_3 x_8 x_{10} + b_3 u_4 + \frac{A_{x}}{x}
\end{align*}
\]
With
\[
\begin{align*}
a_1 &= \frac{K_{tx}}{m}, & a_2 &= \frac{K_{ty}}{m}, & a_3 &= \frac{K_{tz}}{m} \\
b_1 &= \frac{b_1}{I_{tx}}, & b_2 &= \frac{b_2}{I_{ty}}, & b_3 &= \frac{b_3}{I_{tz}} \\
\theta_1 &= \frac{(I_{tx} - I_{ty})}{I_{ty}}, & \theta_2 &= \frac{(I_{ty} - I_{tz})}{I_{tz}}, & \theta_3 &= \frac{(I_{tz} - I_{tx})}{I_{tx}}
\end{align*}
\]
\[
U_x = (\cos x_7 \sin x_9 \cos x_{11} + \sin x_7 \sin x_{11} + \sin x_7 \cos x_{11})
\]
\[
U_y = (\cos x_7 \sin x_9 \sin x_{11} - \sin x_7 \cos x_{11})
\]

Where assuming, values of inertias, unknown and constant or slowly varying in time, then: \( \theta_1, \theta_2 \) and \( \theta_3 \) are the unknown parameters but constant. \( b_1, b_2 \) and \( b_3 \) are constant gains unknown with a known sign. 
\[
(A_x \quad A_y \quad A_z)^T \quad \text{and} \quad (A_p \quad A_q \quad A_r)^T
\]
are resulting aerodynamic forces and moments acting on the UAV, and are computed from the aerodynamic coefficients \( C_i \) as
\[
A_i = (1/2) \rho_{air} C_i W^2
\]
Where \( \rho_{air} \) is the air density and \( W \) is the velocity of the UAV with respect to air.
Using the adaptive backstepping approach as a recursive algorithm for the control-laws synthesis, the following control and adaptation laws are obtained.
\[
\begin{align*}
U_x &= \frac{m}{u_1} (z_1 - c_2 x_2 + a_1 x_2 - c_1 (x_2 - x_{1d}) + x_{1d}) \\
U_y &= \frac{m}{u_1} (z_3 - c_4 x_4 + a_2 x_4 - c_3 (x_4 - x_{3d}) + x_{3d})
\end{align*}
\]
\[
\text{With} \quad u_1 \neq 0
\]
\[
\begin{align*}
u_1 &= \frac{m}{\theta} (-z_5 - c_6 x_6 + a_3 x_6 + g - c_5 (x_6 - x_{5d}) + x_{5d}) \\
\nu_2 &= (-z_7 - c_8 x_8 - \theta_1 x_{10} x_{12} - c_7 (x_8 - x_{7d}) + x_{7d}) \\
\nu_3 &= (-z_9 - c_10 x_{10} - \theta_2 x_{8} x_{12} - c_9 (x_{10} - x_{9d}) + x_{9d}) \\
u_4 &= (-z_{11} - c_{12} x_{12} - \theta_3 x_{10} x_{11} - c_{11} (x_{12} - x_{11d}) + x_{11d})
\end{align*}
\]
\[
\text{With}
\]
\[
D = c_3 x_{7} c_{10} x_{9} \quad \text{and} \quad c_i > 0 \quad \text{for} \quad i = 1, ..., 12.
\]
\( U_x \) and \( U_y \) are the effort of the control input \( u_1 \) along the x-axis and along the y-axis respectively.
The desired trajectories \( x_{7d} \) of \( \phi \) angle and \( x_{9d} \) of \( \theta \) angle are extracted from the equations (21) of nonholonomic constraints.
Adaptation laws:
\[
\begin{align*}
\hat{\theta}_1 &= \Gamma_1 \tau_8, & \hat{\theta}_2 &= \Gamma_2 \tau_{10}, & \hat{\theta}_3 &= \Gamma_3 \tau_{12},
\end{align*}
\]
Where \( \Gamma_1, \Gamma_2 \) and \( \Gamma_3 \) are the adaptation gain and with the tunning functions:
\[
\begin{align*}
\tau_1 &= 0 \quad i = 7, 9, 11 \\
\tau_8 &= \phi_8 \phi_8 = x_{10} x_{12}, \phi_8 \\
\tau_{10} &= \phi_{10} \phi_{10} = x_{8} x_{10} x_{12} \\
\tau_{12} &= \phi_{12} \phi_{12} = x_{8} x_{10} x_{12}
\end{align*}
\]
\[
\begin{align*}
\hat{\theta}_1 &= -\gamma_1 \text{sign}(b_1) \nu_2, \nu_8 \\
\hat{\theta}_2 &= -\gamma_2 \text{sign}(b_2) \nu_5, \nu_{10} \\
\hat{\theta}_3 &= -\gamma_3 \text{sign}(b_3) \nu_3, \nu_{12}
\end{align*}
\]
For robustness opposite the non parametric uncertainties (wind disturbances) the sliding mode control is developed in next section.

Sliding mode control of the quadrotor
The choice of the sliding surfaces is based upon the synthesized tracking errors which permitted the synthesis of stabilizing control laws [15], so from (23) we define:
\[
\begin{align*}
S_x &= x_2 - x_{1d} - c_1 x_2 - a_1 x_2 \\
S_y &= x_4 - x_{3d} - c_3 x_4 - a_2 x_4 \\
S_z &= x_6 - x_{5d} - c_5 x_6 - a_3 x_6 \\
S_{\phi} &= x_8 - x_{7d} - c_7 x_8 - a_4 x_8 \\
S_{\theta} &= x_{10} - x_{9d} - c_9 x_{10} - a_5 x_{10} \\
S_{\psi} &= x_{12} - x_{11d} - c_{11} x_{12}
\end{align*}
\]
Such as \( S_x, S_y, S_z, S_{\phi}, S_{\theta} \) and \( S_{\psi} \) are the dynamic sliding surfaces. To synthesize a stabilizing control law by sliding mode, the necessary sliding condition \( (S \dot{S} < 0) \) must be verified, so the synthesized stabilizing control laws are as follows:
\[
\begin{align*}
U_x &= \frac{m}{u_1} (-k_1 \text{sign}(S_x) - c_2 S_x - a_1 x_2) \\
U_y &= \frac{m}{u_1} (-k_2 \text{sign}(S_y) - c_3 S_y - a_2 x_4)
\end{align*}
\]
\[
\text{With} \quad u_1 \neq 0
\]
\[
\begin{align*}
u_1 &= \frac{m}{\theta} (-k_3 \text{sign}(S_z) - c_6 S_z - a_3 x_6 + g) \\
\nu_2 &= (-k_4 \text{sign}(S_{\phi}) - c_7 S_{\phi} - \hat{\theta}_1 x_{10} x_{12}) \\
u_3 &= (-k_5 \text{sign}(S_{\theta}) - c_9 S_{\theta} - \hat{\theta}_2 x_{10} x_{12}) \\
u_4 &= (-k_6 \text{sign}(S_{\psi}) - c_{11} S_{\psi} - \hat{\theta}_3 x_{10} x_{11})
\end{align*}
\]
\[
\text{With} \quad k_i > 0 \quad \text{for} \quad i = 1, ..., 6.
\]
\textbf{Proof} Apply to first subsystem (34), we know a priori from (32) and (39)
\[
\begin{align*}
V_2 &= \frac{1}{2} \dot{x}_2 + \frac{1}{2} S_x^2 \\
\dot{S}_x &= z_2 - x_{1d} - c_1 z_1
\end{align*}
\]
With equation (26) apply to $\dot{z}_2$ we have
\begin{equation}
\begin{cases}
V_2 = z_1 z_i + S_x \dot{S}_x \\
V_2 = z_1 z_i + S_x \left( a_1 x_2 + \frac{u_i}{m} U_x + c_1 \left( x_2 - x_{id} \right) - x_{id} \right)
\end{cases}
\end{equation}

The chosen law for the attractive surface is the time derivative of (41) satisfying ($S < 0$):
\begin{equation}
\begin{aligned}
\dot{S}_x &= -k_1 \text{sign} \left( S_x \right) - c_2 S_x \\
&= \dot{x}_3 - x_{id} + c_1 z_1 \\
&= a_1 x_2 + \frac{u_i}{m} U_x - x_{id} + c_1 \left( x_2 - x_{id} \right)
\end{aligned}
\end{equation}

From (44), the control input $U_x$ is extracted:
\begin{equation}
U_x = \frac{m}{u_i} \left( -k_1 \text{sign} \left( S_x \right) - c_2 S_x - c_1 \left( x_2 - x_{id} \right) \right)
\end{equation}

The same steps are followed to extract $U_u$ and $u_1$.

For $u_2$, we know a priori from (32) and (39)
\begin{equation}
\begin{cases}
V_8 = \frac{1}{2} \dot{z}_7^2 + \frac{1}{2} \dot{z}_8^2 + \frac{1}{2} \dot{\theta}_1^T \Gamma_1^{-1} \dot{\theta}_1 + \frac{b_1}{2 \gamma_1} \dot{\theta}_1^2 \\
S_\phi = \gamma_8 = x_8 - \dot{x}_{id} - \dot{c}_T \gamma_7 
\end{cases}
\end{equation}

And from (33) after applying (36) and (38), with equation (28) apply to $z_8$ we have
\begin{equation}
\begin{cases}
\dot{V}_8 = \dot{z}_7 \dot{z}_7 + S_\phi \dot{S}_\phi + \sum_{i=1}^{8} \frac{\partial \phi_i}{\partial \theta_i} \left( \Gamma_1^{-1} \dot{\theta}_1 - \dot{\theta}_1 \right) \\
\dot{\phi}_i = \left( \Gamma_1^{-1} \dot{\theta}_1 - \dot{\theta}_1 \right) \Theta_i + c_7 \left( x_{id} - x_7 \right) \\
V_8 = z_7 \dot{z}_7 + S_\phi \left( \dot{\theta}_1 x_{10} x_{12} + \frac{m^2}{p_i} + c_7 \left( x_8 - x_7 \right) \right)
\end{cases}
\end{equation}

The chosen law for the attractive surface is the time derivative of second equation of (46) satisfying ($S < 0$):
\begin{equation}
\begin{aligned}
\dot{S}_\phi &= -k_2 \text{sign} \left( S_\phi \right) - c_6 S_\phi \\
&= \dot{\theta}_1 x_{10} x_{12} + \frac{m^2}{p_i} + c_7 \left( x_8 - x_7 \right)
\end{aligned}
\end{equation}

From (48), the control input $u_2$ is extracted:
\begin{equation}
\begin{aligned}
\frac{\gamma_7}{u_i} &= -k_3 \left( \frac{\gamma_8}{\gamma_7} - \dot{\theta}_1 x_{10} x_{12} - c_7 \left( x_8 - x_7 \right) \right) \\
u_2 &= \dot{\theta}_1 \frac{u_i}{u_i}
\end{aligned}
\end{equation}

The same steps are followed to extract $u_3$ and $u_4$.

**SIMULATION RESULTS**

The simulation results are obtained with the following parameters:

\[ m = 1.5 \text{ kg}; \quad l = 0.35 \text{ m}; \quad I_x = I_y = 1.8 \times 10^{-1} \text{ N.rad/s}^2 \]
\[ I_z = 9.1 \times 10^{-2} \text{ N.rad/s}^2; \quad g = 9.81 \text{ m/s}^2 \]
\[ K_{ftx} = K_{fty} = K_{ftz} = 5.567 \times 10^{-4} \text{ N.rad/s} \]
\[ A_x = A_y = 3 \text{ N.m} \quad A_z = 5 \text{ N.m} \]
\[ A_p = 1.5 \text{ N.m} \quad A_q = A_r = 0.8 \text{ N.m} \]

The vector of parameters $\theta^T$ is initiated to zero, unknown constant gains $b_1$, $b_2$ and $b_2$ are initiated to any values except at zero.

**Quality of the results**

(Fig. 3) shows the good tracking of desired trajectory along the axis $x$, $y$, $z$ and along yaw angle $\psi$. (Fig. 4) shows the evolution of the quadrotor in space and its stabilization. The response time of the system is about 4s. (Fig. 5) shows the good tracking of desired trajectory along the angle $\phi$ and $\theta$, except in the beginning a little discrepancy for the angle $\theta$ is present. (Fig. 6) represents the errors made on the desired trajectory tracking. (Fig. 7) shows the convergence of estimated parameters to their true values and (Fig. 8) shows the convergence of estimated unknown constant gains, their convergence is slower then the estimated parameters.
Fig. 5. $\theta$ and $\phi$ angle tracking desired trajectories

Fig. 6. Tracking errors according ($x$, $y$, $z$) and yaw angle $\psi$ respectively

**Conclusion**

In this paper, stabilizing control law synthesis by sliding mode based on adaptive backstepping approach is presented. The dynamic model of the quadrotor taking into account the different physical phenomena which can influence the evolution of the system in the space is considered, with neglecting the phenomena less pertinent, as a Coriolis torque, a gyroscopic torque and the development of the high order nonholonomic constraints imposed to the system motions. The adaptive backstepping control is developed to overcome the parametric uncertainties caused by changes in the values of inertia under ground effects or sudden change of climatic conditions. The rapid convergence settings to their real value makes a good trajectory tracking and ensures the stability of the closed loop. The sliding mode control enhances the stability of the global system against disturbances experienced by the external system such as the wind and keeps the system performance obtained by The adaptive backstepping control. For all outputs, the tracking error reaches zero after a few seconds.

**REFERENCES**


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