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The mathematical model of inverter

Streszczenie. *Zapropono model matematyczny nowej generacji jednofazowego inwertora. Transformator jest opisany przez równania potencjału wektorowego kwazi-stacjonarnego pola elektromagnetycznego. Operacje sterowania tyrystorów opisane są przez wielkości logiczne, które odzwierciedlają rzeczywisty proces fizyczny. Kondensator jest opisany przez zwyczajne równania różniczkowe. Przedstawiono rezultaty symulacji komputerowej przebiegów przejściowych. (**Model matematyczny przekształtnika**)*

Abstract. *The new generation mathematical model of single-phase inverter is proposed. The iron-clad transformer is described by equation of quasi stationary electromagnetic field vector potential. The adapter operation of thyristors is described by logical values which reflect the real physical process. Capacitor is described by ordinary differential equation. The coil resistance is changed for balance resistor. There are shown the results of computer simulation of model device transient processes.*

Slowa kluczowe: model matematyczny, konwertor mostkowy, elektromagnetyczne pole, cieplne pole.
Keywords: single-phase inverter, electromagnetic field, iron-clad transformer, logical variable, *C*-filter, *RL*-load.

Introduction

Modern mathematical theory of high precision electronic systems indispensable run into the problem of the most modern mathematical models elaboration of devices of converter technics. The analysis of the problem show that such mathematical models must be make with employment of electromagnetic field theory methods only because the electromagnetic circuit theory methods reach the limit of their resources completely. In the paper is proposed the field mathematical model of single-phase semi-conductor thyristor for the first time. The inverter is the most widespread electric device of electronic systems.

The mathematical model of inverter

The main elements of inverter are transformer, semi-conductor thyristor bridge, filter of course, electric voltage source and load. Its scheme is shown on fig.1 The choking coil is changed for balance resistor R_b. The electromagnetic process in iron-clad transformer is described by quasi stationary electromagnetic field equations. The semi-conductor thyristor bridge is circumscribed by logical values which reflect the real physical process. Condenser is circumscribed by ordinary electric circuit differential equation.

The main idea of this articles is concluded in order to device duty with condition of thyristor ideal commutation to circumscribe by special control of transformer electromagnetic field so as to recreate real device duty. In practice we have inverse situation: the time changed non-symmetry that is created by thyristor duty generates strong determine behavior of electromagnetic filed in time. Therefore we go to backdirection. The field control we realize by use of control logical variable. As will be shown later proposed solution gives perfect results: we control the electromagnetic filed - recreate the perfect device duty.

The perfection of mathematical model of device on the whole depends on perfection of transformer mathematical model. Magnet conductor and electric windings are the base of transformer construction. The cross-section of quarter of transformer body are shown on fig.2.

The equation of electric windings we write as

$$\oint_{l_k} \frac{\partial \mathrm{A}}{\partial t} d\mathrm{l} = u_k - r_k i_k, \qquad k = 1, ..., n \tag{1}$$

where $\mathbf{A}$ is vector potential of electromagnetic field; u, i, r are voltages, current and resistance of winding; l is length of winding wire; t is time. The integral in left part of equation (1) is time derivative of full linkage of k-th winding.

The equation of quasi stationary electromagnetic field we use in the following form [1, 2]

$$\frac{\partial \mathrm{A}}{\partial t} = -\Gamma^{-1}\left(\nabla \times \left(\mathrm{N}\nabla \times \mathrm{A}\right) - \delta\right) \tag{2}$$

where δ is vector of current density; N, Γ are matrixes of reluctivities and conductivities; ∇ is the Hamilton's operator.

The module of current density vector we calculate as

$$\delta_k = i_k / \mathrm{S}_k, \qquad k = 1, 2 \tag{3}$$

where S_k is square of cross-section of k-th winding conductor.

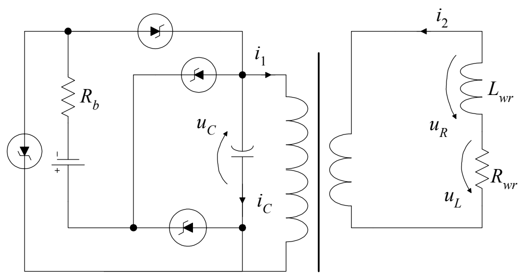

Fig.1. The scheme of single-phase bridge rectifier

From the system of equations (1)-(3) we are received such computation equation too [2]

$$\Gamma \frac{\partial \mathrm{A}}{\partial t} + \mathrm{n}_0 \frac{\gamma}{l} \oint \frac{\partial \mathrm{A}}{\partial t} d\mathrm{l} = -\nabla \times \left(\mathrm{N}\nabla \times \mathrm{A}\right) + \mathrm{n}_0 \frac{\gamma}{l} u\left(t\right) \tag{4}$$

where $\mathbf{n}_0$ is normal vector. This equation is used for given voltage regime.

The equations (4) circumscribes magnetization winding zones.

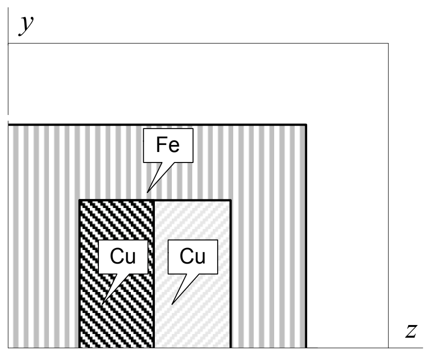

Fig.2. The calculation zone of quarter cross-section iron-clad transformer

The equation (2) covers following zones of spatial integration: laminated magnet conductor and air. It changes in each zones. Having the conditions

$$\mathbf{A} = \mathbf{x}_0 A; \;\; \delta = \mathbf{x}_0 \delta \tag{5}$$

in the zone of laminated magnet conductor section the equation (2) transforms into time algebraic equation

(6) $$\frac{\partial}{\partial z}\left(\nu_y \frac{\partial A}{\partial z}\right)+\frac{\partial}{\partial y}\left(\nu_z \frac{\partial A}{\partial y}\right)=0$$

where ν_y, ν_z are y- and z-direction reluctivities of medium.

Having taken the conditions (5) in the zone of windings equation (4) assumes new form

(8) $$\frac{\partial A}{\partial t}+\frac{c}{l_k}\oint_{l_k}\mathrm{x}_0\frac{\partial A}{\partial t}d\mathrm{l}=\frac{\nu}{\gamma_x}\left(\frac{\partial^2 A}{\partial y^2}+\frac{\partial^2 A}{\partial z^2}\right)+\frac{c}{l_k}u_k$$

where γ_x is equivalent conductivity of k-th winding medium; ν_0 is reluctivity of dielectric medium; c is constant coefficient. The integral in the left part of (5) is taken over the trajectory on the surface of winding conductors. For c =1 the equation (6) describes electromagnetic process in massive conductor. Eddy current need depends if winding are constructed from thin conductors. This can make easily by adaption $c \to \infty$.

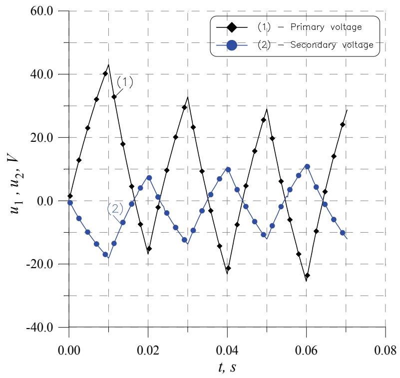

Fig.3 The time function of primary (1) and secondary (2) voltages of transformer

In the practical spatial discretization (8) we substitude the integral for the following sum

(9) $$\int_l \mathrm{x}_0\frac{\partial A}{\partial t}d\mathrm{l} = 4d\xi_2\sum_{m=1}^{q_2}\frac{\partial A_m}{\partial t}$$

where d is thickness of magnet conductor.

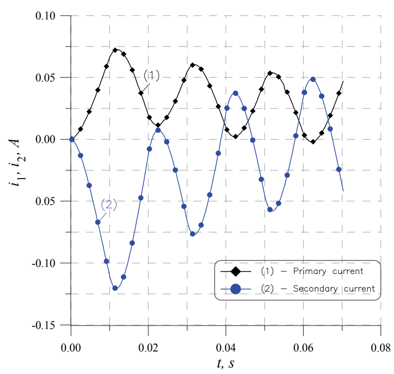

Fig.4. The time function of primary (1) and secondary (2) transformer currents

The laminated magnet conductor has been changed by continuous anizotropic ferromagnetic medium. The magnetization characteristic of this medium **H** = **H**(**B**) we receive from recalculation of the ferromagnetic characteristic $H_f = H_f(B_f)$ by axes x and y [1]

(10) $$B=\frac{B_f}{\chi};\qquad \chi=\frac{d_f+d_0}{d_f+(d_0\nu_f(B))/\nu_0}$$

where d_0, d_f are thicknesses of ferromagnetic sheet and air interval; $\nu_f(B_f)$ is static reluctivity of ferromagnetic that is calculated from ferromagnetic magnetization characteristic $H_f = H_f(B_f)$

(11) $$\nu_f=\frac{H_f\left(B_f\right)}{B_f}$$

In the air zone the equation (2) will be more simple

(12) $$\nu_0\left(\frac{\partial^2 A}{\partial z^2}+\frac{\partial^2 A}{\partial y^2}\right)=0$$

where ν is reluctivity of medium.

Providing the condition (1) and (9) primary current equation we write as

(13) $$i_1=\frac{u_C-\oint_{l_1}\mathrm{x}_0\frac{\partial A}{\partial t}d\mathrm{l}}{r_1}$$

where u_C is capacitor voltage.

According to positive directions of current and voltage capacitor current equation will be

(14) $$i_C=\frac{\varepsilon u-u_C}{R_{wr}}-i_1$$

where u is input voltage; i_C is capacitor current; R_b is balance resistance; ε is logical variable which can be calculated from

(15) $$\varepsilon=\begin{cases}1, & \text{if } \; hT\le t<hT+T/2;\\ -1, & \text{if } \; hT+T/2\le t<(h+1)T,\end{cases}$$

where T - period; $h = 1, 2,...$.

The capacitor ordinary differential equation assumes form

(16) $$\frac{du_C}{dt}=\frac{1}{C}\left(\frac{\varepsilon u-u_C}{R_{wr}}-i_1\right)$$

where C is condensance of condeser.

According to the Kirkhof's law we receive the differential equation of inductance coil

(17) $$L_{wr}\frac{di_2}{dt}+(R_{wr}+r_2)i_2=\oint_{l_2}\mathbf{x}_0\frac{\partial A}{\partial t}d\mathbf{l}$$

where R_{wr} is resistance; $L_{wr} = L_{wr}(i_2)$ is differential inductance

The analysis of transient processes of single-phase inverter is interconnected with integration of differential equations (6), (8), (12) of transformer, (17) of inductance coil and capacitor (16).

The spatial discretization of partial derivatives has been realized by the finite difference method

(18) $$\frac{dy}{dx}=\frac{y(x+\Delta x)-y(x-\Delta x)}{2\Delta x}$$

$$\frac{d^2y}{dx^2}=\frac{y(x-\Delta x)-2y(x)+y(x+\Delta x)}{(\Delta x)^2}$$

where y is function; Δx is spatial step of discretization.

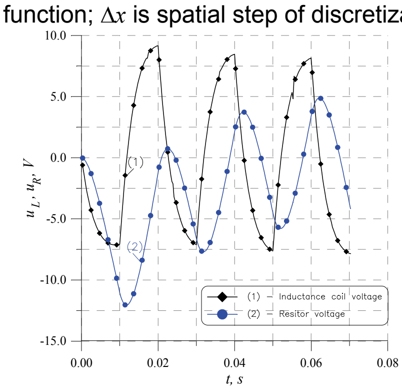

Fig.5. The time function of inductance coil (1) and resistor (2) voltages

The time discretization has been realized by the implicit method

(19) $$y(t+\Delta t) = y(t) + \Delta t\, f(y,\, t+\Delta t)$$

where $f(y, t)$ is rigth part of space discretizated differential equations.

The simultaneous algebraic equation system

(20) $$(B + C)x = b$$

where B is bottom triangular matrix with diagonal elements; C is top triangular matrix, is solved by the top relaxation method.

(21) $$\left(B-D\right)x^{n+1} + D\left(\frac{x^{n+1}+px^{n}}{1+p}\right) + Cx^{n} = b$$

where D is diagonal matrix; p is relaxation parameter. If $0 < p < 1$ than we receive the top relaxation method; if $p = 0$ than we receive Seidel's method.

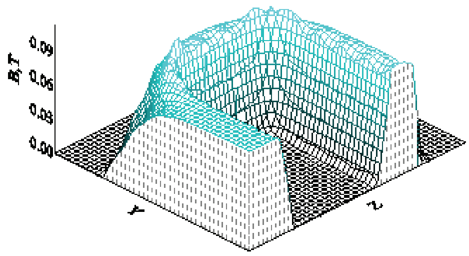

Fig.6. The spatial distribution of magnetic induction in fixed time of transient process (fig.1)

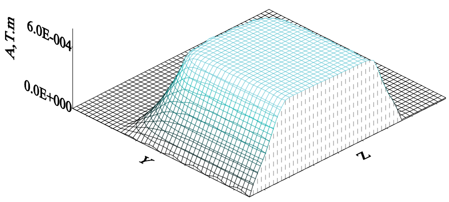

Fig.7. The spatial distribution of vector potential in fixed time of transient process (fig.1)

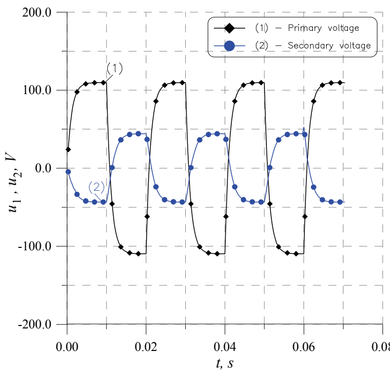

Fig.8. The analogue of fig.3 for another parameters of electric scheme

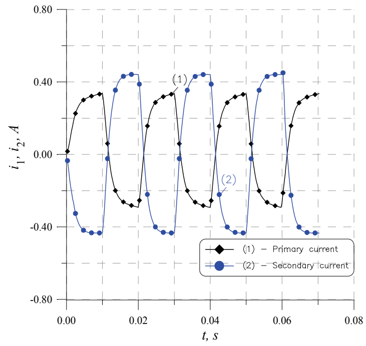

Fig.9. The analogue of fig.4 for another parameters of electric scheme

The results of computer simulation

The results of computer simulation of transient processes of single-phase inverter in the regime of sudden switching-in voltage source $u(t) = 110$ V are presented. The regime is calculated for the following parameters: $C = 0.02$ F; $L_{wr} = 0.5$ H. The time function of primary (1) and secondary (2) voltages are shown in fig.3 and currents shown in fig.4. The time function of inductance coil (1) and resistor (2) voltages are shown in fig.5. The spatial distribution in fixed time of magnetic induction and vector potential of electromagnetic field on cross-section of 2-windings iron-clad transformer body quarter is shown in fig.6-7. As an example there is computated the device regime for another parameters: $C = 0.001$F; $L_{wr} = 0.01$ H. The results (analogues of fig.3-4) are presented on fig.8-9

Conclusion

1. There is constructed mathematical model of calculation and research of electromagnetic processes of a single-phase frequency divider on the basis of computation of an electromagnetic field.
2. The constructed mathematical model allows, if necessary, to realize calculation of operating duties of a single-phase frequency divider and at the robot with inexact corners of discovery of thyristors (in this case enough to exchange the scheme of the description of operation of semiconducting valves). Nevertheless practical implementation of calculation in such case requires considerable computing costs and HIGH-POWER computer. At the same time the obtained information on distribution of an electromagnetic field enables to realize calculation of dynamic parameters of the device in view of commutation of valves.
3. The information on character of electromagnetic processes, which flow past in the researched device, enables to develop mathematical algorithm of calculation of design data of devices of different power and for different values of power with usage of optimization.
4. On the basis of the offered technique there can be constructed mathematical models of other devices, in which basic elements are the pole cores and windings.

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