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Support Region Determination of the Quasilogarithmic Quantizer for LAPLACIAN source

***Abstract**. This paper proposes the method for the support region determination of the quasilogarithmic quantizer designed for the Laplacian source and an arbitrary variance. The method is based on the minimal distortion criteria and it implies some approximations yielding the asymptotic formula for the optimal support region threshold of the quasilogarithmic quantizer. This formula shows which parameters influence the support region threshold of the quasilogarithmic quantizer which is of great importance for practical implementation of the considered quantizer in the high-quality quantization of signals, which, as well as speech signals, have statistics modeled by the Laplacian probability density function.*

***Streszczenie**. W artykule zaproponowano metodę określania region pomocniczego (support region) w quasi logarytmicznym kwantyzerze zaprojektowanym do źródeł typu Laplasian o dowolnej wariancji. Kwantyzer może być zastosowany między innymi do kodowania mowy w systemach komunikacyjnych. (**Określenie parametru SR (suport region) quasi-logarytmicznego kwantyzera**)*

Keywords: quasilogarithmic quantizer, support region threshold, scaling coefficient
Słowa kluczowe: kwantyzer, SR - support region

Introduction

The worldwide growth in communication networks has spurred a renewed interest in the area of speech coding. The challenge to meet the G.711 standard specifications [1] has driven much of the research in this area and several speech and audio coding algorithms have been developed and eventually adopted in international standards [2, 3]. However, although a great number of quantizers have been developed to provide an additional enhancement of the speech signal quality, especially for the VoIP applications where identical PCM format is used, there is still a need to continue the research in this field. Quantizers designed for the particular variance are not suitable for quantization of non-stationary signals that, as well as speech signals, have changing variance characteristics [4 - 9]. In such situations the variance mismatch between the input signal's variance and the variance for which the quantizer is designed may occur. Since for a wide variance range of input speech signal it is necessary to provide a near constant quality of the quantized signal, in this paper the robust logarithmic quantization is considered. A widely accepted solution to the scalar logarithmic quantizer is defined by the G.711 standard [1]. In fact, the two modified logarithmic compressor characteristics obtained by piecewise linear approximation to the *A*-law and the *μ*-law characteristics have become widely used as a design guideline for nonuniform quantization of speech signals in digital telephony.

The quality of a quantized signal, usually measured by distortion or signal to quantization noise ratio (SQNR), is generally influenced by the width of a quantizer's support region and the number of quantization levels [4, 5]. Particularly, the region of an input signal is divided by the scalar quantization procedure into a granular region and an overload region, which are separated by the support region thresholds $-x_{\max}$ and $x_{\max}$. These thresholds define a quantizer's support region $[-x_{\max}, x_{\max}]$. The support region width has a large impact on the total distortion, because the granular distortion and overload distortion, which compose the total distortion, behave opposite in relation to the support region threshold $x_{\max}$. Particularly, the shrinkage of the quantizer's support region, for a fixed number of quantization levels, causes a reduction of the granular distortion, while at the same time possibly resulting in an unwanted increase of the overload distortion [4]. For the same number of quantization levels, with the increase of the support region width, the overload distortion is reduced at the expense of the granular distortion increase. For that reason, the main trade off in scalar quantizer design is making the support region width large enough to accommodate the signal's amplitude dynamic while keeping this support region width small enough to minimize the quantizer distortion. For the model of optimal companding quantizer, it has been shown that the analytical determining of the optimal support region meets some difficulties [10 - 12], so the approximate formulas for its determining have been proposed. The optimal support region is defined with the optimal support region threshold, which is determined so that the quantizer designed for the particular variance provides, for the same variance, the minimum of distortion or equally the maximum of SQNR. In this paper we observe the robust quasilogarithmic quantization which gives an almost constant SQNR in a wide range of variances, hence, it is preferable for use when the input signal's variance changes with time in a wide range. The motivation of this paper is based on the fact that, so far, an approximate formula for determining the optimal support region of quasilogarithmic quantizers has not been proposed.

The rest of this paper is organized as follows. Sections 2 and 3 provide a detailed description of the proposed simple solution to the problem of determining the optimal support region of the quasilogarithmic quantizer designed for the Laplacian source of unit variance and also for an arbitrary variance. The achieved numerical results are the topic addressed in Section 4. Finally, Section 5 is devoted to the conclusions which summarize the contribution achieved in the paper.

Support region of the quasilogarithmic quantizer designed for the Laplacian PDF of unit variance

An N-level scalar quantizer Q is defined by mapping $Q: R \rightarrow Y$ [4, 5], where R is a set of real numbers, and $Y \equiv (y_1, y_2, y_3, ..., y_N) \subset R$ is a set of representation levels that makes the code book of size $|Y| = N$. Every N-level scalar quantizer partitions the set of real numbers into N cells $R_i = (t_{i-1}, t_i]$, $i = 1, \ldots, N$, where t_i, $i = 0, 1, \ldots, N$ are decision thresholds and where it holds that $Q(x) = y_i$, $x \in R_i$. For a quasilogarithmic quantizer Q_μ, compression is done using the μ-law compressor function $c_\mu(x)$: $[-x_{\max}, x_{\max}] \rightarrow [-x_{\max}, x_{\max}]$ [4, 5]:

$$c_\mu(x) = \frac{x_{\max}}{\ln(1+\mu)} \ln\left(1 + \mu \frac{|x|}{x_{\max}}\right) \operatorname{sgn}(x), \quad |x| \le x_{\max}, \tag{1}$$

where the parameter μ is the compression factor and $x_{\max}$ is the quasilogarithmic quantizer's support region threshold.

We assume Laplacian probability density function (PDF) which is commonly accepted as a good approximation to the actual distribution of speech samples [4]

$$p(x)=\frac{1}{\sqrt{2}\sigma}\exp\left(-\frac{|x|\sqrt{2}}{\sigma}\right). \tag{2}$$

For the assumed Laplacian PDF the expression for the total distortion of the quasilogarithmic quantizer is given by [13]:

$$D(Q_\mu)=C\sigma^2\left[\frac{1}{\mu^2}\frac{x_{\max}^2}{\sigma^2}+\frac{x_{\max}}{\sigma}\frac{\sqrt{2}}{\mu}+1\right]+\sigma^2\exp\left(-\frac{\sqrt{2}x_{\max}}{\sigma}\right), \tag{3}$$

where $C=\ln^2(\mu+1)/(3N^2)$ is a constant. Let the quasilogarithmic quantizer be designed for the unit variance, $\sigma^2=1$. Then, the expression for the total distortion becomes:

$$D(Q_\mu)=C\left[\frac{x_{\max}^2}{\mu^2}+\frac{\sqrt{2}x_{\max}}{\mu}+1\right]+\exp\left(-\sqrt{2}x_{\max}\right). \tag{4}$$

By setting the first derivate of the so obtained distortion to zero with respect to $x_{\max}$

$$\frac{\partial D(Q_\mu)}{\partial x_{\max}}=\frac{2Cx_{\max}}{\mu^2}+\frac{C\sqrt{2}}{\mu}-\sqrt{2}\exp\left(-\sqrt{2}x_{\max}\right)=0, \tag{5}$$

we derive the following expression for the optimal support region threshold of the considered quasilogarithmic quantizer:

$$x_{\max}=\frac{1}{\sqrt{2}}\ln\left[\frac{1}{C\left(\frac{\sqrt{2}x_{\max}}{\mu^2}+\frac{1}{\mu}\right)}\right]. \tag{6}$$

Obviously, in order to determine the optimal support region threshold the application of iterative numerical methods is required. However, if we assume that the value of the parameter μ is a large enough (for instance $\mu=255$ corresponds to the quantizer defined by G.711 standard), we obtain the following approximate formula:

$$x_{\max}^a=\frac{1}{\sqrt{2}}\ln\left(\frac{\mu}{C}\right)=\frac{1}{\sqrt{2}}\ln\left(\frac{3\mu N^2}{\ln^2(\mu+1)}\right). \tag{7}$$

It allows for simple calculation of the support region threshold of the quasilogarithmic quantizer designed for the Laplacian PDF of unit variance. In the following sections we provide an analysis of performances for the proposed quasilogarithmic quantizer, where along with distortion SQNR is used [4, 5]:

$$\mathrm{SQNR}(Q_\mu)=10\log\left(\frac{\sigma^2}{D(Q_\mu)}\right)[dB]. \tag{8}$$

Support region of the quasilogarithmic quantizer designed for the Laplacian PDF of an arbitrary variance

In the previos section, the optimal support region threshold of the quasilogarithmic quantizer has been determined so that the quantizer designed for the Laplacian PDF of unit variance provides the minimum of distortion or equally the maximum of SQNR. Let us consider designing the quasilogarithmic quantizer for the Laplacian PDF of an arbitrary variance σ_p^2. Assume that the support region of this quantizer is $[-x_{\max}^{\sigma_p}, x_{\max}^{\sigma_p}]$. By introducing the scaling coefficient k as:

$$k=\frac{\sigma_p}{\sigma_{\mathrm{ref}}}, \tag{9}$$

where σ_{ref}^2 is the reference unit variance, the total distortion of the considered quasilogarithmic quantizer can be expressed by:

$$D(Q_k)=Ck^2\left[\frac{1}{\mu^2}\frac{{x_{\max}^{\sigma_p}}^2}{k^2}+\frac{x_{\max}^{\sigma_p}}{k}\frac{\sqrt{2}}{\mu}+1\right]+k^2\exp\left(-\frac{\sqrt{2}x_{\max}^{\sigma_p}}{k}\right). \tag{10}$$

According to the minimal distortion criteria, the optimal support region threshold $x_{\max}^{\sigma_p}$ is derived as:

$$x_{\max}^{\sigma_p}=\frac{2k}{\sqrt{2}}\ln(k)+\frac{k}{\sqrt{2}}\ln\left(\frac{\mu}{C}\right)\approx\frac{k}{\sqrt{2}}\ln\left(\frac{\mu}{C}\right). \tag{11}$$

Further, by identifying $x_{\max}^a=x_{\max}^{\sigma_{\mathrm{ref}}}$, according to (7), one can come to the conclusion that $x_{\max}^{\sigma_p}$ can be presented by multiplying coefficient k by $x_{\max}^a$:

$$x_{\max}^{\sigma_p}=k\cdot x_{\max}^{\sigma_{ref}}=k\cdot x_{\max}^a. \tag{12}$$

In other words, designing the quasilogarithmic quantizer for an arbitrary variance σ_p^2 is based on the designing the same quantizer for the reference unit variance along with the scaling with the coefficient k.

Numerical Results

The performances that we have ascertained by applying the considered quasilogarithmic quantizer in quantization of signals having Laplacian PDF and a wide variance range are presented in this section. First of all, the parametres for analysis of the accuracy of the derived approximate formula for the support region threshold of the quasilogarithmic quantizer are listed in Table 1. Particularly, the numerical values for the support region threshold of the quasilogarithmic quantizer designed for the Laplacian PDF of unit variance have been calculated by using the proposed approximate formula $x_{\max}^a$ (7), and by numerically optimizing the support region threshold. The numerical optimization of the support region threshold has been performed from the condition that for $x_{\max}^{opt}$ the total distortion is minimum. The procedure is as follows: for $x_{\max}$ with changes in sufficiently small steps, total distortion has been calculated. The iterations have been interrupted when the distortion becomes higher than the distortion from the previous step. Table 1 also contains the values of the relative error of estimating the support region threshold calculated for $\mu=255$ and a different number of quantization levels N ($N=16$, $N=32$, $N=64$, $N=128$ i $N=256$) according to:

$$\delta[\%]=\left|\frac{x_{\max}^{opt}-x_{\max}^a}{x_{\max}^{opt}}\right|\times 100. \tag{13}$$

Since δ amounts approximately to 0.38 %, we have confirmed the correctness of the introduced approximation and the proposed expression for the support region threshold determining.

Table 1. Parametres for analysis of the accuracy of the approximate formula for the support region threshold of the quasilogarithmic quantizer

N	$x_{\max}^{a}$	$x_{\max}^{opt}$	δ [%]
256	10.1147	10.0763	0.3813
128	9.1345	9.1	0.3787
64	8.1542	8.123	0.3841
32	7.1739	7.146	0.3911
16	6.1937	6.17	0.3839

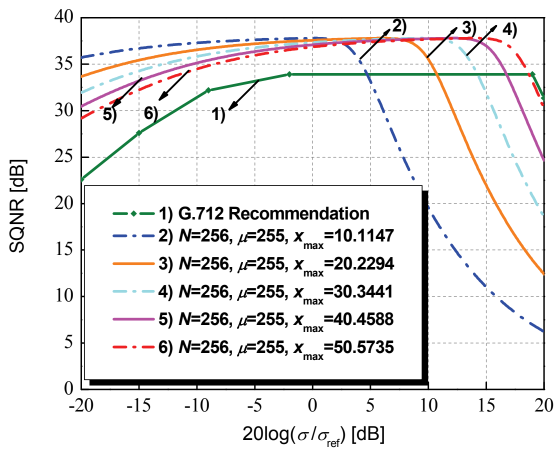

Fig. 1. SQNR characteristics in the wide variance range for N = 256, μ = 255 and a different $x_{\max}$

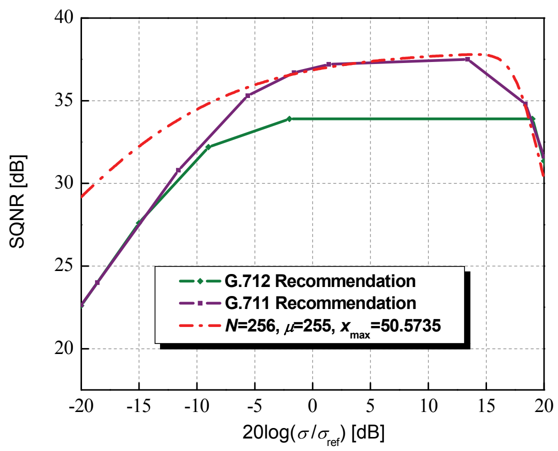

Fig. 2 Comparision of the SQNR characteristics of the G.712 and G.711 standards with SQNR characteristic of the quasilogarithmic quantizer obtained for N = 256, μ = 255 and $x_{\max}$ = 50.5735 (k = 5)

For the considered quasilogarithmic quantizer which is optimized for a different variances, we have calculated the performances in the wide variance range of 40 dB. In such a way we have ascertained the robustness of SQNR of our quasilogarithmic quantizer. From the Fig. 1 one can conclude that our quantizer, when designed for the unit variance and $x_{\max} = x_{\max}^{a}$ = 10.1147, N = 256, μ = 255, is only suitable for small values of the signal variance. Particularly, in the considered variance range, it can not completely overreach SQNR characteristic defined by G.712 recommendation [14], which defines the bottom level of SQNR that needs to be satisfied in order to provide high-quality quantization. This is not an issue in case of $x_{\max}$ = 50.5735, i.e. when the scaling coefficient k has the value k =5. As shown in Fig. 2, when the quasilogarithmic quantizer is designed for $x_{\max}$ = 50.5735 it not only satisfies the requirement for high-quality quantization, but in the whole variance range it also exceeds SQNR characteristic of the widespread used quantizer defined by G.711 standard. In addition, since for $x_{\max}$ = 50.5735 SQNR characteristic is significantly flattened, we believe that it is a good choice for the support region threshold of the considered quasilogarithmic quantizer.

Conclusion

In this paper, we have derived an approximate closed-form formula for the support region threshold of the quasilogarithmic quantizer designed for the Laplacian source of an arbitrary variance. Since the accuracy of the derived approximate formula of $\delta \approx 0.38$ % is ascertained, the proposed approximate formula can be used for simple and fast support region determination. It has been shown that designing the quasilogarithmic quantizer for an arbitrary variance is based on designing the same quantizer for the unit variance along with the multiplying by the scaling coefficient. Finally, since we have ascertained that the proposed quantizer can satisfy the G.712 recommendation in the considered variance range, one can believe that it will find practical implementation in the high-quality quantization of signals, which, as well as speech signals, have statistics modeled by the Laplacian PDF.

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