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Application of probabilistic three-phase load flow for electrical distribution systems with photovoltaic generators

Abstract*. This paper shows how to solve a probabilistic three phase load flow in radial distribution networks with photovoltaic distributed generation (PDG). Voltage regulation is one of the principal problems to be addressed. This research study applies a three phase power flow combined with the Monte Carlo method to solve this problem. Load and PDG are modeled as random variables. A case study is presented. The results obtained show the decrease of the unbalance factor due to the presence of PDG.*

Streszczenie. *W artykule pokazano jak rozwiązać problem trójfazowego przypadkowego obciążenia w sieci dystrybucyjnej ze źródłami fotowoltaicznymi. Do rozwiązania problemu zastosowano metodę Monte Carlo. (****Zastosowanie metod probabilistycznych do przypadku trójfazowej sieci dystrybucyjnej ze źródłami fotowoltaicznymi****)*

Keywords: Monte Carlo method; Probabilistic load flow; Three-phase load flow; Photovoltaic systems.
Słowa kluczowe: metoda Monte Carlo, sieci zasilające, ogniwa fotowoltaiczne

Introduction

The electric distribution systems have unbalanced lines supplying three-phase loads. These systems present voltage unbalances. The unbalance is a state of a three-phase system in which the RMS values of the line voltages, and/or the phase angles between successive line voltages, are not all equal and/or 120° displaced.

Distributed Generation (DG) is electricity generation sited close to the load it serves, typically in the same building or complex. DG creates a variety of well documented impacts on distribution network operation and implies significant changes to planning and design practices. Research has suggested that the benefits of distributed resources could be substantial. However, these distributed advantages are site specific [2,3]. A Photovoltaic Grid-Connected System (PVGCS) is chosen for DG.

To assess the unbalanced voltages of the electric distribution systems, the unavoidable uncertainties that are relevant to the information of entry of the model must be considered. The uncertainties normally are due to variations in the time of the demands of phase load, generation and topology of the system. In the last years, the new and significant causes of the uncertainties in the distribution systems are due to the DG. For example, the production of energy by renewable sources - as the photovoltaic one - is powerfully associated to the uncertainties that concern the available energy of the sun.

Hence, the right valuation of the impact of renewable sources on current systems of distribution must be faced by means of a probabilistic approach [3] that also considers the unbalance of the electrical systems.

In addition, probabilistic approaches seem to be particularly useful to study in depth the influence of voltage unbalance on the performance of induction machines in stationary condition, with the objective to set up recommendations for their functioning [4]. This work studies the unbalanced three-phase systems with PVGCS. These PVGCS will be considered as negative charges, because they have a very low performance and it is advisable to work normally at a power factor close to the unit [5].

In this work, the model of PVGCS is incorporated into the three-phase load flow equations and the Monte-Carlo simulation method is used to consider the random inputs, not only the active and reactive loads, but also the distributed generation.

Using deterministic load flow analysis, it is not possible to measure with objectivity how often and where overvoltages or undervoltages happen in the network during a period of time. This can be accomplished by employing probabilistic techniques like the probabilistic load flow or the Monte Carlo simulation. Probabilistic load flow demands modeling of loads and power productions as probability density functions and supplies the complete range of all probable values of the node voltages and load flows in the study with their respective probabilities assuming generation and load uncertainties and correlations and topological changes. The probabilistic load flow was enunciated in [6, 7] and further developed at a greater extent in [8, 9].

In [10] the probabilistic power flow was extended to the three-phase field to evaluate the uncertainties which affect the steady-state operating conditions of an unbalanced power system. Both Monte Carlo procedure and a linearized form were proposed.

Various distribution system load flow algorithms, based on the forward/backward sweeps, were reviewed, and their convergence ability was quantitatively evaluated for different loading conditions in [11, 12].

In [13] was presented a three-phase power flow solution method for real-time analysis of primary distribution systems. This method is a direct extension of the compensation-based power flow method for weakly meshed distribution systems from single phase to three-phase. For asymmetrical three-phase load-flow study, two methods based on symmetrical component theory, the bus admittance method and the decoupling compensation method were proposed in [14].

Probabilistic PV system model

Solar irradiation on a horizontal surface inside the atmosphere cannot be predicted exactly; it depends on the irregular presence of clouds. The randomness introduced by clouds on terrestrial radiation is characterized with two random variables [15, 16]: the daily clearness index K_T and the hourly diffuse fraction k_d. The statistical properties of the components of solar radiation allow constructing a probability model, using probability density functions (PDFs) and cumulative distribution functions (CDFs). These properties provide the probability for K_T [15] and k_d [16].

If the random variables K_T and k_d are known, then it is possible to determine the total irradiance on a surface sloped, $G_{t,\beta}$, as a linear combination of K_T and k_d. At this point, the Cumulant Method [17] allows a statistical information mapping of predefined random variables K_T and k_d with the new random variable $G_{t,\beta}$. If the variable $G_{t,\beta}$ is known, the power output is obtained from a linear combination of $G_{t,\beta}$ once the correlations used between both variables are linearized.

Probability density function for global irradiance

Most works involving the analysis of the statistical properties of global irradiation use set of daily solar irradiation data. Holland and Huget suggested the following PDF [15]

(1) $$P_K(K_{T,}\ \overline{K}_T)\ = C_1\left(1-\frac{K_T}{K_{Tu}}\right)\exp\left(\lambda K_T\right)$$

where C_1 and λ are functions of K_{Tu} and $\overline{K}_T$. K_{Tu} is upper limit of K_T.

Probability density function for diffuse irradiance

Using the expected value approach of probability theory, Hollands [16] gave a general-purpose expression containing the PDF for k_d

(2) $$P_k(k_{d,}\ k_t)\ = C_3\left(k_d-k_{dl}\right)\left(1-k_d\right)\exp\left(\lambda k_d\right)$$

where C_3, λ *and* k_{dl} are functions of $\overline{k}_d$.

Probability density function for PV electrical power

The PV electrical power (DC side) P_{pv} is calculated as linear function of the total irradiance on PV surface ($G_{t,\beta}$) [18, 19]:

(3) $$P_{pv}\ = \eta_c A G_{t,\beta}$$

where A is generator surface area and η_c is electrical efficiency.

Probabilistic load model

The electric load of a power system has deterministic and stochastic components. The two main deterministic factors that affect the load are: time (multiple seasonal patterns: yearly, weekly and intra-daily) and weather conditions. However, there is also a random component in load which cannot be explained, resulting from the random behaviour of energy customers. Customers are classified by electric utilities into different subjective classes [20].

Typical load patterns of customer classes can be obtained from statistical analysis of historical data. Thus, reference [20] defined for each customer class typical daily profiles (TDPs) which contained information about daily load profile after extracting all exogenous information, i.e. seasonal (yearly and weekly cycles) and weather information. Additionally, Jardini et al. [20] applied statistical analysis methods to the TDPs, allowing to construct a probability model capable of giving the probability that a value of observed load will be within specified limits. Its approach treated the TDPs of each *j*th customer class $L_j(m,h)$ as a random variable, normally distributed, which changed month to month and hour to hour. This means that its hourly mean value $\mu_j(m,h)$ and the corresponding standard deviation $\sigma_j(m,h)$ changed throughout the 12 months and 24 hour period.

However, seasonal information when TDPs are built is only partially extracted. Thus, for the *j*th customer class are built two TDPs per month, one for weekend days and another one for working days. The one-year TDPs are arranged into two-dimensional layout with 12 columns representing 12 month of a year and 2 rows representing weekend or working days.

Known the TDPs of customer classes it is possible to determine the random variable hourly active power consumed by the *kth* node of a feeder at *m*th month and *h*th hour adding the random variables individual consumptions $L_j(m,h)$ of all customer classes. For example for working days

$$P_{L\ k}^{wo}\left(m,h\right)=\sum_{j=1}^{j=ncc} cn_{j,k}L_j^{wo}\left(m,h\right)$$

(4)

Assuming a deterministic power factor the relevant reactive power consumed is

(5) $$Q_{L\ k}\left(m,h\right)=P_{l\ k}\left(m,h\right)\tan\ \varphi$$

Probabilistic radial three phase load flow (PRTPLF)

In this paper, the proposed three-phase probabilistic load flow for radial networks is a combination of the three-phase load flow purposed in [21] with the Monte Carlo simulation method [22].

Simulation method

This technique is basically to select values of input variables randomly from their distribution functions, and with these values to solve a deterministic radial three phase load flow. After a certain number of simulations, the probabilistic solution of the problem is reconstructed from deterministic data obtained for each simulation.

The number of simulations that has been estimated as adequate for this problem and has also been used in several articles to solve probabilistic load flow is 10000 [23, 24].

Simulation results

For the study of the method proposed, the IEEE-13 node test feeder system has been modified [25], as shown in the Fig. 1:

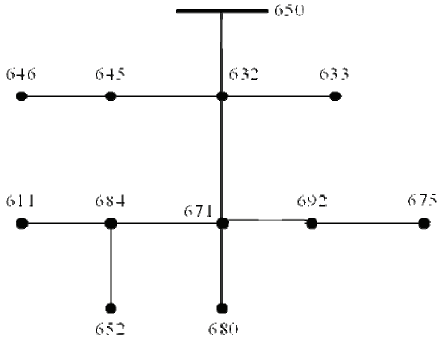

Fig.1. System used in case study.

Data of this the system are those that appear in [25], with the following caveats: 1) All loads have been considered constant PQ; 2) The load distributed between node 632 and node 671 has been eliminated; 3) Line 633-634 has been eliminated. 4) The voltage regulator in the node 650 has been deleted; 5) Line 671-692 is 100 feet length. System loads, modeled with normal random variables, are shown in Table 1.

Table 1. Loads of the system.

	Phase a		Phase b		Phase c		
Node	kW	KVAr	kW	KVAr	kW	kVAr	σ
646			230	132			0.07
645			170	125			0.06
671	385	220	385	220	385	220	0.10
611					170	80	0.04
692					170	151	0.07
675	485	190	68	60	290	212	0.06
652	128	86					0.04

First, the distribution system has been simulated without distributed generation. The distribution function of the voltage at each node has been determined, as well as the index of unbalance of voltage at each node, as defined in [21]:

$$V_{unbalance}(\%) = \frac{|\text{Maximum deviation from average}|}{|V_{average}|} \cdot 100 \quad (6)$$

Later distributed generation has been connected in some nodes, and the results have been compared with the previous case.

Results

Table 2 shows the mean value of the voltages of the three phases at some nodes of the system, without distributed generation.

Table 2. Voltages at nodes of the system without DG.

Node/Phase	**Voltage (p.u.)**	**Deviation**	**Angle (º)**
V650a	1.0199	6.36e-07	7.07e-05
V650b	1.0199	1.00e-06	-120.001
V650c	1.0199	2.21e-06	120.001
V632a	0.9932	0.0028	-2.17723
V632b	1.0073	0.0022	-121.404
V632c	0.9754	0.0031	117.859
V645a	0.9932	0.0028	-2.17726
V645b	0.9927	0.0025	-121.590
V645c	0.9790	0.0031	117.755
V671a	0.9718	0.0057	-5.42511
V671b	1.0219	0.0041	-121.571
V671c	0.9202	0.0064	115.8162
V684a	0.9698	0.0057	-5.4539
V684b	1.0219	0.0041	-121.57
V684c	0.9178	0.0065	115.70
V675a	0.9650	0.0061	-5.8028
V675b	1.0255	0.0040	-121.72
V675c	0.9162	0.0065	115.802

As shown in this table, voltages at node 650 form a balanced system. However, in the rest of the system, the voltages are unbalanced. In addition, unacceptably low voltages can be observed.

Table 3 shows the mean in per cent and standard deviation of the unbalance of voltage in each node.

Table 3. Unbalanced between phases without DG.

Node	**Mean value (%)**	**Deviation**
650	0.0039	0.0001
632	3.2148	0.5525
646	1.3203	0.5606
645	1.4425	0.5477
633	3.2151	0.5525
671	10.469	1.090
611	10.987	1.100
684	10.733	1.095
692	10.762	1.098
675	11.281	1.116
652	10.755	1.097
680	10.469	1.090

From node 633, the voltage unbalances are inadmissible for the proper functioning of the system. Fig. 2 shows the profile of the mean of the voltages of the three phases at each node without distributed generation.

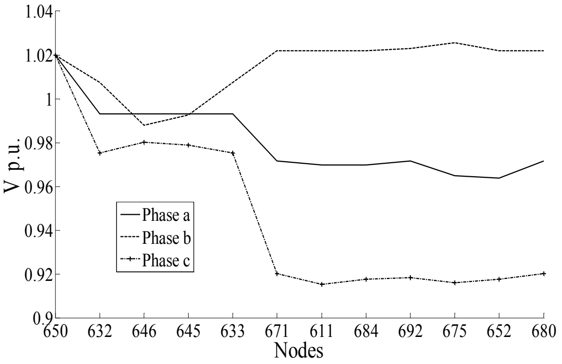

Fig.2. Voltage profiles without distributed generation.

Fig. 3 indicates the cumulative distribution function (CDF) of the voltages at the node 684.

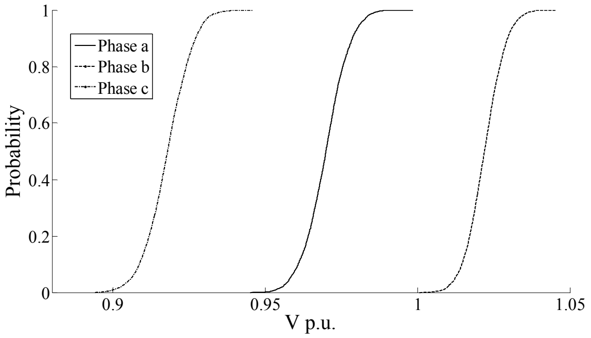

Fig.3. CDF of the voltages at node 684 without DG.

Next, several photovoltaic generators will be connected to the nodes. Table 4 shows the average power of each generator.

Table 4. Average power of each generator.

Node	Phase a (kW)	Phase b (kW)	Phase c (kW)
646		543.35	108.67
645		163.01	
671	434.68		163.01
611			163.01
692			163.01
675	489.02	54.33	271.68
652	108.67		

The first seven cumulants of the power for a summer day are presented in table 5. Fig. 4 shows the PDF corresponding to a generator.

Table 5. Cumulants of the generators.

k_1	54.3352985696210	108.670597100000	271.676492848105
k_2	169.966222748626	1529.69600473763	4249.15556871564
k_3	-2511.4709135059	-67809.714664660	-313933.86418824
k_4	36023.1884143184	2917878.26155979	22514492.7589490
k_5	106758.145880823	25942229.4490400	333619205.877572
k_6	-48422460.980892	-35299974055.070	-756600952826.43
k_7	2718128757.49367	5944547592638.66	212353809179193
k_1	434.682388556968	489.017687126589	543.352985696210
k_2	10877.8382559121	13767.2640426387	16996.6222748626
k_3	-1285873.1077150	-1830862.29594583	-2511470.9135059
k_4	147550979.745048	236348139.186343	360231884.143184
k_5	3498250924.22281	6303961756.11672	10675814588.0823
k_6	-12693657611375	-25733681086146.3	-48422460980892
k_7	5.7003291600e+15	1.30007255851e+16	2.718128757e+16

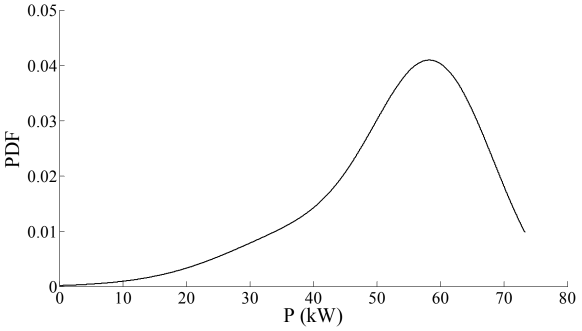

Fig.4. PDF of a generator.

It can be seen how the shape of this distribution function moves away from the normal distribution. Table 6 presents the results for the system with distributed generation.

Table 6. Voltages at nodes of the system with DG.

Node/Phase	Voltage (p.u.)	Deviation	Angle (º)
V650a	1.0199	2.65e-06	-4.3e-06
V650b	1.0199	3.01e-06	-120.00
V650c	1.0199	4.59e-06	120.00
V632a	1.0129	0.0126	0.513
V632b	1.0002	0.0060	-120.00
V632c	1.0023	0.0099	119.67
V645a	1.0129	0.0126	0.513
V645b	1.0015	0.0099	-119.33
V645c	1.0024	0.0098	119.67
V671a	0.9954	0.0285	1.155
V671b	0.9905	0.0071	-121.39
V671c	0.9893	0.0219	119.39
V684a	0.9940	0.0289	1.210
V684b	0.9905	0.0071	-121.39
V684c	0.9897	0.0227	119.39
V675a	0.9947	0.0303	1.145
V675b	0.9920	0.0076	-121.49
V675c	0.9889	0.0227	119.43

The voltages are acceptable at each node. Table 7 depicts the unbalances between phases at the nodes of the system with distributed generation.

Table 7. Unbalanced between phases with DG.

Node	Mean value (%)	Deviation
650	0.0038	0.0003
632	1.2614	0.5239
646	1.0411	0.4489
645	1.1346	0.4540
633	1.2614	0.5239
671	0.6152	1.5714
611	0.4215	1.6549
684	0.4338	1.6189
692	0.6676	1.6229
675	0.5848	1.7092
652	0.2431	1.6311
680	0.6153	1.5714

The unbalances at the nodes have decreased considerably after the connection of the distributed generation. Also, Fig. 5 shows the new voltage profiles with distributed generation.

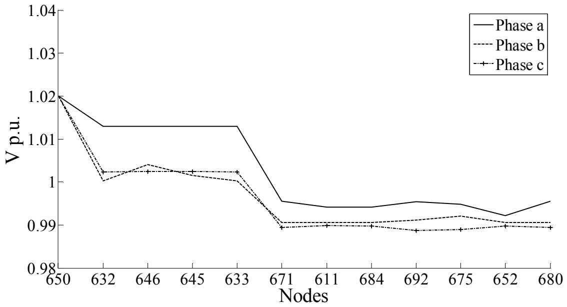

Fig.5. Voltage profiles with distributed generation.

Fig. 6 indicates the new cumulative distribution function (CDF) of the voltages at the node 684.

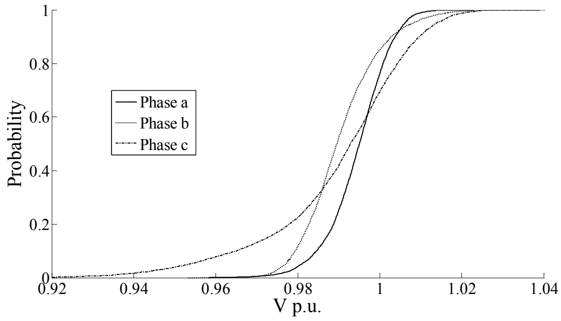

Fig.6. CDF of the voltages at node 684 with distributed generation.

Using the results obtained for the unbalances, it is possible to reconstruct a distribution function for the unbalanced in each node. Then, it can be determined the probability that the value of unbalance does not exceed a specified limit. The table 8 shows the probability that the unbalance of each node is less than 1.5 per cent.

Table 7. Unbalanced between phases with DG.

Node	Probability (%)	Node	Probability (%)
650	99.99	611	95.37
632	70.37	684	96.09
646	85.45	692	94.51
645	81.59	675	94.85
633	70.55	652	97.17
671	94.65	680	94.53

Conclusions

In this paper, loads and distributed generation production are modeled as random variables. Results have proved that the proposed method can be applied for the keeping of voltages within desired limits at all load nodes of a PVGCS.

In addition, the probability of unbalance at the nodes has been decreased considerably.

To evaluate the performance of photovoltaic system this paper has used a probabilistic model that takes into account the random nature of solar irradiance and load.

A new method utilizing three-phase probabilistic radial load flow has been introduced. The impact of the DG connections has been examined applying this method.

The probabilistic analysis in stationary condition of an unbalanced distribution system with PVGCS has been effectuated by means of a probabilistic three-phase load flow. The probabilistic analysis has permitted the consideration of the uncertainties associated with the active and reactive loads and the solar radiation.

The Monte-Carlo simulation method has been regarded in order to implement the three-phase probabilistic radial load flow. Numerical applications have been showed and discoursed with reference to the IEEE-13 node test feeder system including PVGCS at several nodes. The results obtained show the decrease of the unbalance factor due to the distributed generation.

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