Jinlin GONG, Ramzi BEN-AYED, Frédéric GILLON, Stéphane BRISSET, Pascal BROCHET

University Lille Nord of France, ECLille, L2EP

Three-Level Adapted Output Space Mapping Technique for Two-Objective Optimization

Abstract. *Multi-objective optimization with 3D finite element model is a complex and time consuming process. Output space-mapping techniques allow having an affordable computation cost with a minimum number of computationally expensive FEM evaluations. In this paper, a three-level adapted output space-mapping technique is proposed to assist two-objective optimization problems. Three models having different granularity and describing the same device are used jointly in the optimization process. The results for the safety isolating transformer show that the proposed algorithm allows saving a substantial amount of computation time compared to the classical two-level output space-mapping technique.*

Streszczenie. *Przedmiotem artykułu jest dwukryterialna optymalizacja, trójpoziomowa, wykorzystująca metody odwzorowania zewnętrznej przestrzeni. Przypadek techniczny (transformator bezpieczeństwa) przeanalizowano trzema metodami, różniącymi się od siebie ziarnistością. Wyniki analizy wskazały na istotną redukcję czasu obliczeń w porównaniu do podejścia klasycznego dwupoziomowego. (**Trójpoziomowa metoda adaptująca odwzorowanie zewnętrznej przestrzeni dla dwukryterialnej optymalizacji**)*

Keywords: *Multi-objective, FEM, Three-level, Output space mapping*
Słowa kluczowe: wielokryterialność. MES, trójpoziomowość, odwzorowanie przestrzeni zewnętrznej

Introduction

Optimal design of electrical machines using high fidelity models, such as FEM runs into the wall of the computation time and of the number of parameters. Indeed, FEM is sensitive to numerical noise, its results depend on the mesh quality [1] and one evaluation of such model is computationally expensive. Therefore an analytical model is preferred within an optimization process for time-saving reason and the FEM is used as a virtual prototype in the late stage in order to validate the design.

The space-mapping optimization technique appears as a promising trade-off. It allows benefiting both from the rapidity of the analytical model (coarse model) and the accuracy of the FEM (fine model) by aligning them. Several variants of space-mapping technique [2]-[3] have recently being used in solving optimization problems of electromagnetic devices [4]-[6]. Among them, the output space-mapping (OSM) is the most recent and effective method. The OSM technique avoids numerical noise and has a faster convergence by reducing the use of the FEM during the optimization process.

Realistic design systems need to consider a multi-objective problem. The aim of multi-objective algorithms is to find compromise solutions rather than a single optimal one. Solving these problems requires building the Pareto front, which helps the designer to find a satisfying solution [7]. Many methods allow solving multi-objective problems, such as the well-known weighted sum (WS), ε-constraint methods [8] and non-dominated sorting genetic algorithm II (NSGA-II) [9]. Unfortunately, all these algorithms require a very high number of model evaluations. This is not compatible with the time consuming FEM. In order to have an affordable computation cost for two-objective optimization problems, the adapted OSM technique is proposed in [5]. In the adapted OSM, the scalar corrective coefficients of the model outputs are replaced by a set of corrective spline cubic functions. An extended and accurate Pareto optimal set can be obtained in an acceptable limit.

However the computation time of the conventional OSM (2L-OSM) is still rather high when one FEM evaluation is time-consuming such as 3D FEM, and the number of evaluations is important. To overcome this problem, a 3-level OSM (3L-OSM) algorithm is proposed in [11] using a medium model in terms of accuracy and computation time. This means that three models of the device are used. The added medium model is used to reduce the evaluations of the fin model, following the decrease of optimization time. The aim of this paper is to adapt the 3L-OSM for two-objective optimization problems.

This paper is structured in three main parts. Firstly, the principle of the conventional OSM technique is presented. Secondly, the proposed 3L-OSM strategy for two-objective problems is proposed. Thirdly, the proposed algorithm is tested on a safety isolating transformer. The solutions and computation time of the 2L-OSM are compared to 3L-OSM. To conclude, the advantages of using the proposed algorithm are highlighted.

2-Level adapted Output Space Mapping

OSM technique is a common approach for the optimization of devices represented by accurate, but time consuming models. It has been recently used for solving optimization problems of electromagnetic converters. This technique requires two models: a coarse model and a fine one of the device to be optimally sized. The coarse model is faster but less accurate. This coarse model could be empirical, analytical or interpolated. The optimization is carried out with the coarse model and the final results are computed with the fine model. The OSM aligns iteratively the outputs of the coarse model and the fine one by adding correctors to the coarse model. For the design of electromagnetic devices, the coarse model is an analytical model and the fine one is a FEM as usually. The design space exploration is done by the optimization with the analytical model, avoiding thus the possible numerical noise and mesh problems [1]. By aligning the both models, the optimization results are given by the FEM evaluation, and the precision is assured. Therefore the OSM technique benefits both the rapidity of the analytical model and the accuracy of the FEM.

In order to adapt two-objective problems, the OSM technique combined with ε-constraint method is applied. Compared to mono-objective problems, a set of corrective functions instead of the scalar corrective coefficients is used to adjust coarse model outputs, and the Pareto optimal set obtained with the coarse model extends and gets close to the real one iteratively. The corrective functions can be obtained using interpolation functions. Within the OSM technique, the role of the fine model is to adjust coarse model outputs, in order to better fulfill the constraints. The adapted OSM provides a practical way not only to build an accurate Pareto optimal set but also to maintain a reasonable computation time.

A. Two-objective optimization using ε-constraint

The ε-constraint method is a useful approach to build a Pareto optimal set including the nonconvex region. It consists to transform a multi-objective problem into a single-objective problem [8]. One of the objectives is chosen and the others are transformed into inequality constraints:

$$x^* = \min_{x \in \mathbf{X}} f_i(x) \;\; s.t. \;\; f_{j \neq i}(x) \leq \varepsilon_j \; and \; g_f(x) \leq 0 \qquad (1)$$

where $f_i(x)$ - the objective kept, $f_{j\neq i}(x)$ - the other objectives, $g_f(x)$ - the ordinary constraints, and ε_j - the threshold values. By varying ε_j between ε_j^{min} and ε_j^{max}, the Pareto front can be found. In the case of two-objective optimization problem, ε_j^{min} can be found by minimizing $f_{j\neq i}(x)$ and ε_j^{max} can be found by minimizing $f_i(x)$.

B. 2 level adapted OSM Technique

Both the evaluation numbers of fine models and the computational time of optimization are decreased using space-mapping technique compared to the single-level optimization.

In general, the coarse computationally cheaper model is denoted by $c(x) \in \mathbf{R}^m$ with $x \in \mathbf{X} \subset \mathbf{R}^n$, and the fine computationally expensive model is denoted by $f(x) \in \mathbf{R}^m$ with $x \in \mathbf{X} \subset \mathbf{R}^n$. The nonlinear constraints of the coarse and fine models are $g_c(x)$ and $g_f(x)$, respectively.

This two-objective optimization problem given in (1) is hard to solve and is replaced by (2):

$$\begin{aligned} x_j &= \arg\min_{x \in X} \left\| c_1(x, \theta_j(\varepsilon)) - y \right\| \;\; s.t. \; c_2(x, \theta_j(\varepsilon)) \leq \varepsilon, g_c(x, \theta_j(\varepsilon)) \leq 0 \\ & and \;\; \varepsilon \in [\varepsilon_{\min}, \varepsilon_{\max}] \end{aligned} \qquad (2)$$

where $y \in \mathbf{R}^m$ - a vector of design specification that can be zeros in the case of minimization. $\theta_j(\varepsilon) \in \Theta \subset \mathbf{R}^p$ - the corrective functions, p - the number of responses (objectives and constraints). In conventional OSM, θ is a set of p scalar coefficients expressed by (3).

$$\theta_{j+1} = \begin{bmatrix} f(x_j)/c(x_j) \\ g_f(x_j)/g_c(x_j) \end{bmatrix} \qquad (3)$$

The p scalar coefficients must be changed for each solution from the Pareto front, i.e. for each value of ε. The 2L adapted OSM [5] replaces the scalar corrective coefficient θ by the corrective functions expressed by (4):

$$\theta_{j+1}(\varepsilon) = \begin{bmatrix} S_j \\ \vdots \end{bmatrix} \qquad (4)$$

where S_j - spline cubic interpolation functions which can avoid the oscillations compared to other polynomial interpolation approaches when the order is high. These functions are initialized to one and computed iteratively to have the same value for the outputs for the coarse and fine models.

At each iteration, a two-objective optimization is performed using ε-constraint method and the coarse corrected model in order to obtain a new Pareto front in a short time. $2^{(j+1)}$ solutions x^* belonging to the Pareto optimal set are chosen and computed by the fine model $f(x_j^*) \in \mathbf{R}^m$ and $g_f(x_j^*)$. Then the corrective functions are updated using the results of the fine model.

The algorithm stops when the following condition is checked:

$$\left\| \begin{bmatrix} c(x_j, \theta_{j+1}(\varepsilon)) \\ g_c(x_j, \theta_{j+1}(\varepsilon)) \end{bmatrix} - \begin{bmatrix} c(x_j, \theta_j(\varepsilon)) \\ g_c(x_j, \theta(\varepsilon)) \end{bmatrix} \right\| \leq \xi \qquad (5)$$

where ξ - the required accuracy. The algorithm does not stop until the corrected coarse model responses are close enough to the fine model ones.

In this study the conventional OSM technique is named 2L adapted OSM because only two models are used.

3-level OSM for bi-objective

The purpose of 3L-OSM algorithm [11] is to reduce even more the computation time compared to the conventional 2L-OSM. In order to do so, a model with a medium accuracy is added between the coarse and fine models and used within the OSM algorithm. The medium model has intermediate accuracy and computing time compared to the fine and coarse models.

A. 3-level output space-mapping

The medium-accuracy model is denoted by $m(x) \in \mathbf{R}^m$. In this case, the inputs of the three models are the same. The nonlinear constraints of the medium model are $g_m(x)$. The strategy of the OSM consists of aligning the coarse model and the medium model by the corrective coefficients $\theta \in \Theta \subset \mathbf{R}^m$. These coefficients are initialized with one and computed iteratively in order to have the same values for the outputs of the coarse and medium models.

$$\theta_{j+1} = \begin{bmatrix} m(x_j, \beta_j)/c(x_j) \\ g_m(x_j, \beta_j)/g_c(x_j) \end{bmatrix} \qquad (6)$$

$$\left\| \begin{bmatrix} c(x_j, \theta_j) \\ g_c(x_j, \theta_j) \end{bmatrix} - \begin{bmatrix} m(x_j, \beta_j) \\ g_m(x_j, \beta_j) \end{bmatrix} \right\| \leq \xi \qquad (7)$$

where β is used to correct the medium model as explained later. The space-mapping between the coarse and medium models stops when (8) is checked,

$$\left\| \begin{bmatrix} c(x_j, \theta_j) \\ g_c(x_j, \theta_j) \end{bmatrix} - \begin{bmatrix} m(x_j, \beta_j) \\ g_m(x_j, \beta_j) \end{bmatrix} \right\| \leq \xi \qquad (8)$$

where ξ - the required accuracy and β_j - constant during this part of the algorithm. The second step of the algorithm consists of calculating the outputs of the fine model with x_j and aligning the medium model with the fine one using the p scalar coefficients β as follows:

$$\beta_{j+1} = \begin{bmatrix} f(x_j)/m(x_j) \\ g_f(x_j)/g_m(x_j) \end{bmatrix} \qquad (9)$$

In the case of two-objective problems, as in the 2L adapted OSM the corrective coefficients β_{j+1} can also replaced by the corrective functions expressed by (10).

$$\beta = \begin{bmatrix} h_j \\ \vdots \end{bmatrix} \qquad (10)$$

where h_j - spline cubic interpolation functions which is updated using the $2^{(j+1)}$ solutions x^* belonging to the Pareto optimal set Pareto optimal.

The 3L-OSM algorithm stops when the discrepancy between the corrected medium model and the fine one is small enough:

$$\left\| \begin{bmatrix} m(x_j,\beta_j) \\ g_m(x_j,\beta_j) \end{bmatrix} - \begin{bmatrix} f(x_j) \\ g_f(x_j) \end{bmatrix} \right\| \le \xi \quad (11)$$

B. Proposed algorithm for bi-objective problem

To summarize, the algorithm carries out the following main steps:

0. Initialize $j=0$, β_0, $\theta_0=\mathbf{I}$
1. Build a Pareto front using the ε-constraint algorithm and the corrected coarse model $c(x_j, \theta_j(\varepsilon))$ to solve (2)
2. Choose (2^{j+1}) points on the Pareto front
3. Evaluate the medium model with chosen points to compute $m(x_j)$ and $g_m(x_j)$
4. Update the p corrective functions $\theta_{j+1}(\varepsilon)$ and $\beta_{j+1} = \beta_j$
5. If (5) go to the next step, else $j=j+1$ go to step 2
6. Evaluate the chosen points x_j with the fine model, i.e. $f(x_j)$ and $g_f(x_j)$
7. Update the p corrective functions β_{j+1} with (10)
8. If (11) stop the algorithm, else $j=j+1$ go to step 2

Safety isolating transformer

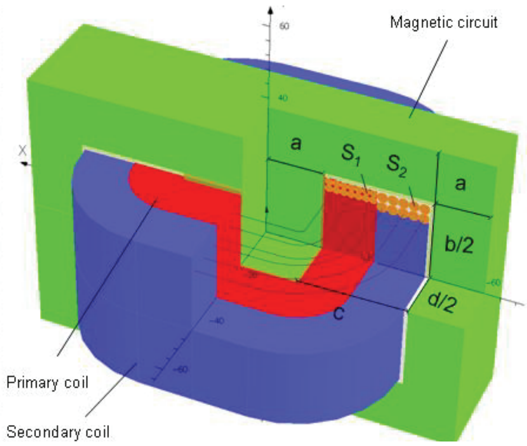

Fig.1 Transformer design variable

Fig. 1 presents the safety isolating transformer. It is a one-phase step-down transformer [10]. It has been selected as a simple practical example capable to provide a demonstration of the proposed algorithm procedure. Three models of different accuracy are presented and integrated within the 3L-OSM algorithm as follows.

A. Coarse model

An analytical model of the safety isolating transformer was previously developed in [5] and is used here as the coarse model. The physical phenomena inside the transformer are decomposed into thermal, electric and magnetic models. They are expressed in equations with consideration of three assumptions: uniform distribution of magnetic induction in the iron core, no voltage-drop due to the magnetizing current and vertical magnetic field in the coils. Due to the coupling between electrical and thermal phenomena, the model is an aggregation of a two set of equations.

B. Medium model

The medium-level model of the transformer is a 3D FEM with light mesh, i.e. 4,893 nodes and 27,024 elements. A static thermal analysis is weakly coupled with electromagnetic steady-state analysis at full-load and no-load. This model has a superior accuracy than the coarse model and it is faster than the 3D FEM with fine mesh: it has middle characteristics between those of the coarse model and the fine one. It gives acceptable results in a short simulation time.

C. Fine model

A high fidelity 3D FEM that includes 75,517 nodes and 442,118 elements was developed and used in order to simulate the behavior of the safety isolating transformer to be sized. This model is used for the evaluation of the efficiency, primary current, secondary voltage, copper and iron losses and temperature, magnetizing current and no-load voltage of the transformer.

D. Accuracy and computation time

An experimental transformer test bench is developed as shown in Fig. 2.

Fig.2 Transformer test bench

Table I presents the error of the three models compared to the measurement and computing time of each model for the existent device.

Table I. Model accuracy and computation time

type	Max error (%)	Time (s)
Coarse	9.04	0.76
Medium	6.96	65
Fine	2.77	850

The coarse model is 1118 faster than the fine one but it is the least accurate. The medium model is 13 times faster than the fine one but it is about 2.5 times less accurate. As expected, the fine model is the most accurate but it is the most time consuming. Based on these three models, a two-objective optimization problem is formulated in the following part and solved using the proposed 3L-OSM algorithm.

E. Optimization problem

The optimal sizing problem of the safety isolating transformer was proposed as an optimization benchmark and the detailed problem can be found in [10]. The optimization problem is expressed as:

$$\min\{mass, 1-\eta\}$$

$$\text{s.t.} \quad T_{copper} \le 120°C, \; T_{iron} \le 100°C, \; \frac{I_{10}}{I_1} \le 0.1, \quad \frac{\Delta V_2}{V_2} \le 0.1, \; f_2, f_1 \le 0.5 \quad (11)$$

$$\text{with} \quad a \in [3,30], \; b \in [14,95], \; c \in [6,40], \; d \in [10,80], \quad n_1 \in [200,1200], \; S_1 \in [0.15,19], \; S_2 \in [0.15,19]$$

The optimal sizing problem consists of 7 design variables: four variables defining the transformer iron core geometry (*a, b, c, d*) as shown in Fig. 1, two variables for the section of the enameled wires (S_1, S_2), and one variable for the number of primary turns n_1. The goal of the optimization problem is to minimize the mass of the iron and copper materials and to maximize the transformer's efficiency η with respect to six constraints: the copper and iron temperatures T_{copper} and T_{iron} should be less than 120°C and 100°C, respectively; The ratio between the magnetizing current I_{10} and the primary current I_1 and the ratio between the secondary voltage drop ΔV_2 and the secondary voltage V_2 should both be less than 0.1. The filling factors f_1 and f_2 of both coils should both be lower than 0.5.

F. Optimization results

The optimization is realized using both the conventional 2L-OSM and the proposed 3L-OSM. Fig. 3 shows the Pareto front obtained using these two algorithms.

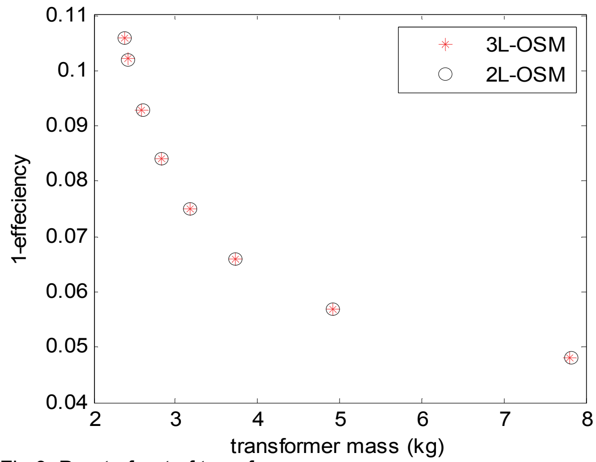

Fig.3. Pareto front of transformer

The 8 solutions for the both Pareto fronts are very close and well distributed.

The table II presents the comparison between the 2L-OSM and 3L-OSM in terms of the computing time.

Table II. Comparison between 3L-OSM and 2L-OSM with and without cubic interpolation of correctors

	Algorithm	Computing time (s)
With cubic interpolation	2L-OSM	19456
	3L-OSM	6686
Without cubic interpolation	2L-OSM	32134
	3L-OSM	11722

The 3L-OSM is 2.9 times faster than the 2L-OSM using the cubic interpolation function, and 2.7 times faster without the cubic interpolation function. Therefore, the 3L-OSM with the cubic interpolation of correctors is more efficient.

Conclusion

Space-mapping techniques are used in the optimization of the safety isolating transformer. Three models with different accuracy levels are combined to achieve satisfactory results in a short time. By using the ε-constraint method with a set of corrective functions, the bi-objective optimization problem of the transformer is solved by the OSM technique. Both conventional and three-level OSM algorithms converge to the same solutions. However, a decrease of 2.9 of times of the computation time is obtained using the cubic interpolation function, and 2.7 times without the cubic interpolation function. The proposed 3L-OSM technique can provide a more efficient way to build an accurate Pareto front.

AcknowledgeMENT

This project was supported by the Laboratory of Electrical Engineering and Power Electronics (L2EP) of Ecole Centrale de Lille, France. One of the authors, Jinlin Gong, wishes to gratefully thank the Chinese Scholarship Council (CSC) for the funding of this work.

REFERENCES

[1] Neittaamäki, P., Rudnicki, M., and Savini, A., "Inverse problems and optimal design in electricity and magnetism*", Oxford University Press Inc*., New York, pp. 325-348, 1996

[2] Bandler, J.W., Biernacki, R.M., Chen, S.H., Grobelny, P.A. and Hemmers, R.H., "Space Mapping Technique for Electromagnetic Optimization", *IEEE Trans. Microw.Theory Tech*, vol. 42, no. 12, pp. 2536-2544, 1994

[3] D. Echeverria, D. Lahaye, L. Encica and P.W. Hemker. "Optimization in electromagnetic with the space mapping technique". *Compel*, vol. 24, no. 3, pp.952-966, 2005.

[4]D. Echeverria, D. Lahaye, L. Encica, E.A. Lomonova, P.W. Hemker, and A.J.A. Vandenput." Manifold-mapping optimization applied to linear actuator design". *IEEE Tranactions on Magnetics*, vol. 42, no. 4, pp. 1183–1186, April 2006.

[5] T.V. Tran, F. Moussini, S. Brisset and P. Brochet, "Adapted Output Space-Mapping Technique for a Bi-Obejtive Optimization", *IEEE Transactions on Magnetics*, vol. 46, no. 8, pp. 2990-2993, March 2010.

[6] L. Encica, J.J.H. Paulides, E.A. Lomonova and A.J.A. Vandenput, "Electromagnetic and thermal design of a linear actuator using output polynomial space mapping", *IEEE Transactions on Industry Applications*, vol. 44, no. 2, pp. 534-542, 2008.

[7] P. D. Barba, Multiobjective Shape Design in Electricity and Magnetism, Springer Press, Dordrecht Heidelberg London New York, 2010

[8] A. Murano, A. Passaro, N. M. Abe, A. J. Preto, and S. Stephany, "Multi-objective optimization of electro-optic modulators by using the ε-constraint method", *presented at the CEFC*, Athens, Greece, May, 2008.

[9] K. Deb, A. Pratap, S. Agarwal and T. Meyarivan, A Fast and Elitist Multi-objective Genetic Algorithm: NSGA-II, *IEEE Transactions on Evolutionary Computation*, Vol. 6, No. 2, pp. 182-197, April 2002

[10] T.V. Tran, Safety Isolating Transformer benchmark. Available online at: http://l2ep.univ-lille1.fr/come/benchmark-transformer.htm

[11] R. Ben-ayed, J. Gong, S. Brisset, F. Gillon, P. Brochet, Proposal of a Three-Level Output Space Mapping Strategy, *IEEE Transaction on Magnetic*, Vol. 48 (2), pp. 671-67, Feb. 2012

Authors:

Dr. Jinlin Gong , L2EP, Ecole Centrale de Lille, 59650, Villeneuve d'Ascq, France, E-mail : jinlin.gong@ec-lille.fr