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Calculation of end-winding forces of inverter fed drives

***Abstract**. End-winding forces can be approximated applying the Biot-Savart law. Usually constant speed and ideal sinusoidal currents are assumed. Applying these constrains a simulation of the forces for one electrical period is sufficient. However, for inverter fed drives, the harmonic content of the current is dependent on the operational load and speed. In this case the forces for each time step of the drive cycle of traction drive have to be calculated individually. This paper proposes a superposition of so called unit-current forces for each current harmonic. Still, the forces are calculated for one electrical period with unit-current amplitude based on Biot-Savart´s law. The calculation of the resulting forces is performed by means of a current superposition. This allows for a computational cost effective simulation of end-winding forces taking current harmonics due to inverter supply, speed and load dynamic operations into account.*

***Streszczenie.** Siły w połączeniach czołowych mogą byż wyznaczane przy użyciu prawa Biota-Savarta. Zwykle przyjmuje się stałą prędkość obwodową i sinusoidalny przebieg prądu. Te założenia są wystarczające dla symulacji sił w jednym okresie elektrycznym. Jednakże w sytuacji gdy napęd zasilany jest przekształtnika i występują zależne od obciążenia i prędkości harmoniczne siły muszą być obliczane indywidualnie dla każdego kroku czasowego w cyklu trakcyjnym. W artykule proponowana jest metoda superpozycji sił od jednostkowych prądów dla każdej harmonicznej prądu. Wówczas siły mogą być, podobnie jak dla przebiegów sinusoidalnych, obliczane w jednym okresie elektrycznym z wykorzystaniem prawa Biota-Savarta. Obliczanie siły wypadkowej wykonywane jest poprzez superpozycje prądów. Procedura taka pozwala na uwzględnienia prędkości i obciążenia w obliczeniach sił w połączeniach czołowych. (**Obliczenia sił w połączeniach czołowych napędów zasilanych z przekształtnika**).*

Keywords: Biot-Savart method, cage induction motors, electromagnetic forces, end windings
Słowa kluczowe: metoda Biota-Savarta, silnik indukcyjny klatkowy, siły elektromagnetyczne, połączenia czołowe.

Introduction

End-winding forces can mechanically damage the winding system. Classically, end-winding forces are investigated for large generators. On the one hand, these generators rotate at a constant speed and on the other hand they are designed to generate sinusoidal currents with a very low harmonic content. The start-up and breaking is of low importance in this application. Therefore, a simulation of the end-winding forces taking only one specific current shape into account is sufficient. This allows for the application of computational time cost expensive models, for example relying on three dimensional finite element method (3D-FEM). However in case of induction motors, where the start-up is of great importance, the computational effort prohibits to apply 3D-FEM. Further on, the development of high power converters yields to their application in large drives. The dynamic operation over a wide speed range increases the risk of exciting mechanical resonances with the end-winding forces. Additional significance of the end-winding forces is also given due to the higher harmonic content in the supply current. Not only current harmonics, which are generated by the machine itself, but also harmonics caused by the inverter switching can lead to additional forces. Machines, where end-winding forces are relevant, are usually driven with low switching frequencies in the range of several hundred hertz up to 1 kHz. These low switching frequencies lead to forces, which may be in the frequency range of mechanical resonances of the end-winding. Purpose of the presented research is a model for the simulation of end-winding forces in dynamic operation. 3D-FEM is known to deliver very accurate results, allowing for the consideration of the exact geometry and material parameters, but yields to unacceptable computational efforts for the investigation of the acceleration of an induction motor. The classical Biot-Savart approach assuming linear material properties allows for a good approximation of these forces [1]. In this paper a computational efficient procedure to calculate the end-winding forces is presented, relying on Biot-Savart´s law. The idea of the presented approach is to take the advantage of the linear modelling into account. A superposition of all current harmonics is proposed. The force calculation is then performed in frequency domain by means of convolutions. Thereby, the computational effort can be reduced, because the forces acting on the end-winding are only calculated for a unit current, i.e. a current with an amplitude of 1 A and a frequency of 1 Hz.

Superposition of Unit Forces

The first step is the setup of a line conductor model for the estimation of the end-winding forces. The electromechanical forces are calculated analytically based on this model of the end-winding geometry using Biot-Savart´s law and Lorentz forces [1-3]. To obtain a unit force distribution, the forces are calculated once with this approach for one electrical rotation of the machine with a sinusoidal current of given amplitude $\hat{I}$ and frequency f_I. A Fourier transformation yields the amplitudes $\hat{a}$ of the harmonic force waves with fundamental frequency f_0 [4]

$$a_P(t) = \sum_{k=0}^{K-1} \hat{a}_{P,k} \cdot \cos\left(k \cdot 2\pi f_0 t - \varphi_{a,k}\right) \tag{1}$$

The appearing frequencies of the force waves arise from the multiplication of the current signal in the time domain which corresponds to a convolution in the frequency domain [5]

$$\mathcal{L}\{i[n] \cdot i[n]\} = I[r] * I[r] = \frac{1}{N} \sum_{s=0}^{N-1} I[s] \cdot I[r-s]. \tag{2}$$

Since a linear time invariant model is used, the resulting forces can be normalized to the amplitude of the given current. With

$$F \sim I^2 \tag{3}$$

the unit force a_e,k,P_q in point P_q is consequently determined by

$$a_{e,k,P_q} = \frac{\hat{a}_{P,k}}{\left(\hat{I} / \sqrt{2}\right)^2}. \tag{4}$$

The electromagnetic forces for various current patterns can be determined using weighted superposition of unit forces, taking into account the phasing of the current's frequency components. As in (1) the Fourier transformation of the squared current signal yields

$$\imath^2(t) = \sum_{m=0}^{M-1} \hat{\imath}_m \cdot \cos(m \cdot 2\pi f_s t - \varphi_{\imath,m}), \tag{5}$$

with fundamental frequency f_s and phase angle $\varphi_{t,m}$,m. The electromagnetic force $F_{Pq}(t)$ in one specific point P_q caused by a current $i(t)$ is then given by the convolution

$$F_{P_q}(t)=\sum_{k=0}^{K-1}\sum_{m=0}^{M-1}a_{e,k,P_q}\cdot\hat{i}_m\cdot\cos\left(m\cdot 2\pi f_s t-\left(\varphi_{\alpha,k}+\varphi_{t,m}\right)\right) \quad (6)$$

Eq. (5) and (6) reveal, that the frequencies of the forces can be calculated by the summation and derivation of the occurring current frequencies. The described procedure allows for the determination of forces generated by arbitrary current patterns. It solely relies on the calculation of the force responds obtained based on a unit current.

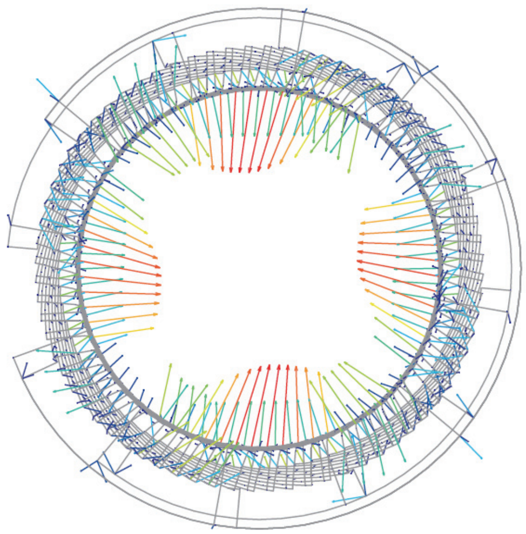

Fig.1. Forces calculated in 3D-modell, direction and amplitude indicated by arrows.

Further on, the forces can be mapped to specific current harmonics.

In order to obtain the spacial distribution, which is presented for one time step in Fig. 1, of the forces the described approach has to be conducted for all significant points of the end-winding. Doing so, it allows defining a ring on the tip of the end-windings. By the application of a second Fourier transformation the forces can be represented as one dimensional waves. These waves can be represented by

$$F(x,t)=\sum_{i=0}^{I-1}\sum_{j=0}^{J-1}\hat{F}_{i,j}\cos\left(i\cdot 2\pi f_s t-jx-\varphi\right) \quad (7)$$

whereby i represents the temporal frequency order and j the spacial one. The spacial order $2p$ (i.e. the number of poles is dominant).

Application and results

In order to prove the proposed concept to calculate the end-winding forces, an induction motor with a rated power of 1 MW is investigated. The winding is a classic double layer winding with $q=2$ slots per pole and phase. The number of poles is $2p=4$. As an example the electromagnetic forces in radial direction acting on the end-winding are calculated based on the classical Biot-Savart and the proposed unit-current force superposition approach. A constant load and speed is assumed for this example. Special focus is on the consideration of current harmonics. Measured currents are used for the approximation of the occurring end-winding forces. The time signal of the investigated current and its Fourier transformation are shown in Fig. 2 and Fig. 3. In the investigated case, the fundamental current frequency is $f_0=106Hz$. The most significant current harmonic has the frequency $f_5=530Hz$. The forces are calculated based on the presented superposition approach. In order to evaluate the proposed approach, the obtained results are compared to the classical Biot-Sarvats' approach. The results of both approaches are presented in Fig. 4, which shows the obtained force distribution represented in frequency domain for one point at the tip of the end-winding next to a phase change. Both approaches show a good agreement in amplitude and phase of the estimated forces.

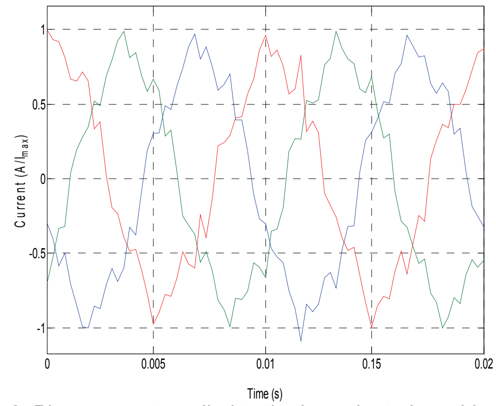

Fig.2. Phasecurrent applied to the investigated machine.

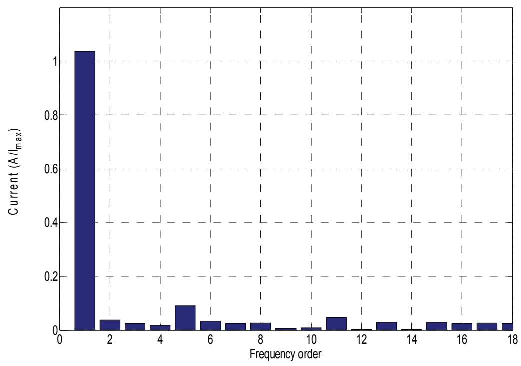

Fig.3. Fourier decompisition of applied phasecurrent.

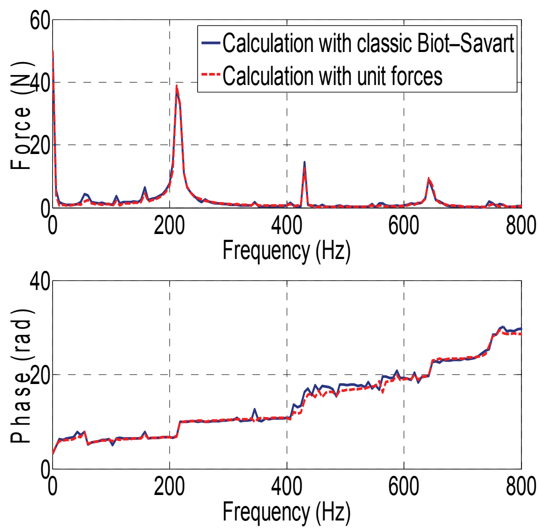

Fig.4. Comparison of the calculated forces. Presented is the force acting on the tip of the middle coil of one coil group.

As predicted from the theory, the forces occur with the sum and difference of the current harmonics. Here, the double fundamental frequency $f_{f0}=212Hz$ is dominant. The current harmonics generate additional force excitations with the frequency $f_{f3}=656Hz$. The coupled force at $f_{f5+0}=426Hz$ is also dominant. These additional force waves can be mapped to the third and fifth harmonic of the phase current.

In Fig. 5 the absolute deviation between the classical Biot-Savart's approach and the superposition approach is presented in amplitude and phase. A maximum deviation in the amplitude of *2.2* N is obtained, i.e. less than *5* %. The phase deviation is *3*° in maximum.

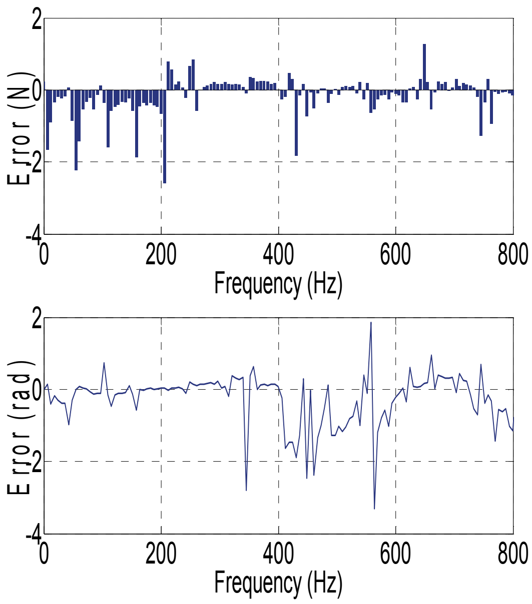

Fig.5. Deviation of the calculated forces, results of the unit-force model in relation to the classical Biot-Savart model. Presented are the force acting on the tip of the middle coil of one coil group.

Next steps and further prospects

In the presented work the proposed method is compared to classical approaches. The results are in good agreement. So far, only steady state operation is considered. However, the advantage of the presented approach is the consideration of speed and load dynamic operation. The proposed method is to be applied to defined test cycles in order to evaluate all important operation points. In combination with a modal analysis of the end-winding, electromagnetic excitations in the range of the mechanical resonances are to be indentified and thus to be reduced.

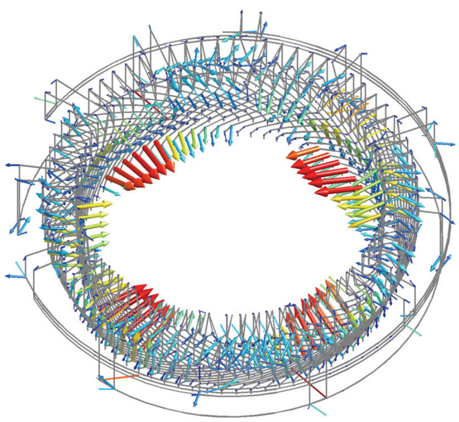

Fig.6. Forces calculated by the unit-force superposition model, direction and amplitude indicated by arrows.

In the presented work the end-winding forces are investigated at one specific coordinate. However the forces are calculated at all important points of the end-winding. The results of one time step are presented in Fig. 6. The results are stored in matrices. The deviation between the superposition model and the classical Biot-Savart's model for the non presented points are in the same range.

Summary

A unit-current force-superposition method is proposed as an easy to handle, time-efficient method to determine the electromagnetic forces on the end-winding of electrical drives. The proposed approach only requires the force calculation for one electric revolution. As excitation a current with a unit amplitude, for example *1A*, and a unit frequency, for example *1Hz*, is applied. In order to obtain a linear time invariant system, which is required for the unit force superposition, linear material properties are assumed. The conducted investigations have shown that this assumption is a valid simplification in the investigated case. The presented approach allows for the time efficient calculation of the end-winding forces for dynamic operation of the machine taking various current patterns into account. It is found that even small amplitudes of current harmonics can result in large force amplitudes. Since mechanical resonances are unavoidable for speed variable speed, a detailed knowledge of the excited forces is mandatory. An approach for mapping of the generated forces and the exciting current harmonics is presented. This mapping allows for the systematic reduction of specific force excitations.

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