Force and torque study of magnetic levitation spherical driving joint with magnetic field segmentation method

Abstract. This paper presents a novel multi-degree-of-freedom spherical reluctance driving joint with magnetic levitation. The analysis of operation mechanism and the joint’s magnetic conduction of the gap are deduced by means of the way to magnetic segmentation. The relationship between electrical and mechanical energy conversion is analysed and then the radial suspension force and the tangential suspension torque are calculated. The simulation experiments show that the system of driving joint has well static and dynamic characteristics.

Keywords: magnetic levitation spherical driving joint, suspending force, electromagnetic torque, comprehensive control

1. Introduction

The movement of multi-freedom joints is traditionally composed of several single-degree-freedom joints. Thus, a multi-freedom joint system has complex structure, huge size, difficult manufacturing and assembling, slow response and bad dynamic performance [1, 2]. The driving joint system which based on the spherical driving joint with direct bearing driven by spherical motor is the multi-degree-freedom spherical driving joint. A more freedom spherical joints can simplify the structure of the mechanical system and reduce the volume and weight, etc, so as to improve system efficiency, accuracy, dynamic performance and high cost performance, get advantage in the trajectory planning and in control. But in order to realize the spherical motor rotating at different degrees, it generally uses the frame structure, each layer frame is supporting by mechanical bearing to spin around a coordinate, so it exist complex structure, large size and the restricted rotation angle, etc. Based on motor technology, magnetic levitation and robotics, the paper presents a novel multi-degree-of-freedom magnetic levitation spherical reluctance driving joints which domestic haven’t report, it uses magnetic levitation force to make spherical rotor suspend, makes it no friction with the stator and no abrasion, has high accuracy and good dynamic performance [3, 4]; and make mechanism study for it to produce the magnetic levitation force and the magnetic torque.

2. The basic components of magnetic levitation spherical driving joint

Under the active control of the controller, the magnetic levitation spherical driving joint can produce the electromagnetic levitation force supporting the joint rotor and the electromagnetic torque rotating the joint rotor, it owns two or three rotating freedom and takes rotating work with non-contact and abrasion free bypass the fixed point of the space axes. It is mainly made up of several motor stator and the spherical driving joint rotor with the tumbler, etc. Its three-dimensional structure is shown in figure 1. The joint rotor is surrounded by the joints of the motor stator, the four stators are symmetrical distribution at the equatorial line position, the stator 1 and 3 are symmetry for sphercial rotor and keep coaxial along the X-axis, its effect is to drive the rotor rotating around the X-axis and make rotor produce the stability magnetic levitation force in the Y-axis; the stator 5 arranges the top of spherical rotor and keeps coaxial with the Z-axis, its function is driven rotor spinning around the Z-axis and make the rotor produce the stability magnetic levitation force in the Z-axis.

Some shapes of saliency column distributed in the outside surface of magnetic levitation spherical joint (cylinder or Prism Object), the capital envelope spherical surface is rotor spherical surface; The stator joint contains a number of similar saliency shape salient pole, and according to principle of reluctance motor, multi-phase (m) winding winds around the salient pole which drives the rotor rotating, each phase windings have a pole. Each phase windings after electrify produce electromagnetic torque to drive the spherical rotor and also provides the radial levitation force for the spherical rotor.

3. The air-gap magnetic conductance and energy of spherical driving joint

Based on the driving of spherical reluctance motor and supporting of the spherical rotor with the tumbler, the joint realizes multi-freedom joints. Therefore, the study is the same of other analysis method of the reluctance, from the air-gap of the motor stator and rotor, the mechanical energy transformational relation is established, and then it get the relation between the electromagnetic levitation force suspending rotor and electromagnetic torque rotating rotor.

If the centre of sphere of the magnetic spherical reluctance driving joint doesn’t happen excursion, the radius of the stator salient pole envelope spherical surface is \( R \), the radius of the rotor salient pole envelope spherical surface is \( R_c \), the clearance between the stator salient pole...
envelope spherical surface and the rotor salient pole envelope spherical surface is $g_0 = R - R_r$. Make the symmetry axis of the stator to coincide with Z axis and build frame of axes $f(x,y,z) = F(R, r, \theta)$, as shown in figure 2. When the rotor joint is in the process of rotating, its salient pole surface is the overlap alignment with stator salient pole surface and it produces magnetic-pull (thrust) force to form electromagnetic torque, magnetic-pull force is the magnetic levitation force which drives the rotor suspend. Magnetic-pull (thrust) force is directly related to the size and distribution of the air-gap magnetic conductance.

Because the departure of the rotor from the equilibrium position is very small, the surface between the stator and the magnetic the rotor salient pole is considered the parallel surface, the magnetic circuit of the overlap between the stator gear and the rotor gear can be instead of the linear magnetic circuit; the elliptic curve approximately replaces the edge magnetic flow path. After simplifying like this, the calculation of the total magnetic conduction of the interaction between the pair poles for air-gap is:

$$
P(z, \theta) = \mu_0 (L - r_0)(L - z \sin \theta) + \frac{g}{g_0} 
$$

(2)

where, $\mu_0$ is the space permeability; $r$ is the inside circular arc radius of the stator magnetic pole; $L$ is the side length of the stator tooth notching; $g$ indicates the angle between the position of the stator tooth shape at normal and Z-axis; $g_0 = g_0 \pm z \cos \phi$ is the radial air gap length between the crest top land of the rotor salient pole and the crest top land of the stator salient pole(magnetic pole). When the rotor near the stator, to take "," when the rotor depart from the stator, to take "+", $g_0$ is the air-gap radius between the magnetic pole and the stator magnetic pole when the rotor doesn’t have offset.

If the stator motor joint has m-phase windings, each one-phase winding has a magnetic pole to work with electricity, according to the theory of motor, the calculation of the total energy which is stored in the air-gap magnetic field of the joint stator motor is:

$$
W_g = \frac{1}{2} L m^2 \frac{1}{2} K m^2 N^2 z^2 P(z, \theta)
$$

where, $L = K N^2 P(z, \theta)$ is the inductance which one-phase winding generates; $N$ is turns per coil of one-phase winding.

When the spherical rotor makes the offset along X-axis or Y-axis, similarly it can deduce the magnetic conduction air gap and the total magnetic energy of the joint which is the same structure of the type 2 and the type 3. According to the magnetic energy $W$ of the joint air gap, the magnetic levitation force of joint can be taken by the derivation of the offset of the rotor along the coordinate, the electromagnetic torque of the driving rotor which the joint generates can be taken by making the derivation of the rotor angle $\theta$.

4. The magnetic levitation force and electromagnetic torque of magnetic levitation spherical driving joint

According to type 3, it can take the magnetic levitation force and electromagnetic torque of the magnetic levitation spherical reluctance driving joint along the Z-axis by making the partial derivative of $z$ and $\theta$:

$$
T_z = \frac{1}{2} K m^2 \frac{1}{2} 4 \mu_0 r_0 L \sin \theta (g + 2r \theta)^2 \frac{\mu_0 (L - z \sin \theta)}{g} - \frac{2 \mu_0}{\pi} \ln \frac{2 \sin \theta + 2g \sin \theta + g^2}{g^2} + \frac{2 \mu_0}{\pi} \ln \frac{r g \sin \theta + 2g \sin \theta + g^2}{g^2}
$$

(3)

$$
F_z = \frac{1}{2} K m^2 \frac{1}{2} 4 \mu_0 r_0 L \sin \theta (g + 2r \theta)^2 \frac{\mu_0 (L - z \sin \theta) \cos \theta - g \sin \theta}{g^2} + \frac{2 \mu_0}{\pi} \ln \frac{2 \sin \theta + 2g \sin \theta + g^2}{g^2} + \frac{2 \mu_0}{\pi} \ln \frac{\mu_0 (L - z \sin \theta) \cos \theta - g \sin \theta}{g^2}
$$

(4)
According to the type 4, through controlling the supply current of the joints, it can adjust the size of magnetic levitation force and the electromagnetic torque.

For the X-axis (Y-axis) direction, due to the two stator electromagnet driving joint rotor to move, so its magnetic levitation force and the electromagnetic torque.

\[ F_s = F_{11} - F_{22} \]

\[ = \frac{1}{2} K_m N_1^2 \left( \mu_0 (L-r) [(L-x) \sin \phi - g_{11} \sin \phi] + \frac{4 \mu_0 (L-r)(g_{11} + 2 \sin \phi) g_{11} \sin \phi}{\pi g_{11}^2} + \frac{2 \mu_0 \sin \phi}{\pi g_{11}^2} 2x^2 \theta^2 + 2 g_{11} \theta + g_{11}^2 \right) \]

\[ + \frac{4 \mu_0 (L-r)(g_{11} + 2 \sin \phi) g_{11} \sin \phi}{\pi g_{11}^2} + \frac{2 \mu_0 \sin \phi}{\pi g_{11}^2} 2x^2 \theta^2 + 2 g_{11} \theta + g_{11}^2 \]

\[ + \frac{4 \mu_0 (L-x) \sin \phi (g_{11} + 2 \theta) \cos \phi}{\pi g_{11}^2} + \frac{4 \mu_0 (L-x) \sin \phi (g_{11} + 2 \theta) \cos \phi}{\pi g_{11}^2} \]

\[ = \frac{1}{2} K_m N_1^2 \left( \mu_0 (L-r) [(L-x) \sin \phi - g_{11} \sin \phi] + \frac{4 \mu_0 (L-r)(g_{11} + 2 \sin \phi) g_{11} \sin \phi}{\pi g_{11}^2} + \frac{2 \mu_0 \sin \phi}{\pi g_{11}^2} 2x^2 \theta^2 + 2 g_{11} \theta + g_{11}^2 \right) \]

\[ + \frac{4 \mu_0 (L-x) \sin \phi (g_{11} + 2 \theta) \cos \phi}{\pi g_{11}^2} + \frac{4 \mu_0 (L-x) \sin \phi (g_{11} + 2 \theta) \cos \phi}{\pi g_{11}^2} \]

The air gap between the two stator and rotor magnetic poles is: \( g_{11} = g_{21} = x \cos \phi \), \( g_{21} = g_{11} + x \cos \phi \).

Similarly the total electromagnetic torque of the two joint stators which drive the spherical rotor rotating is:

\[ T_s = T_{11} + T_{21} \]

\[ = \frac{1}{2} K_m N_1^2 \left( \frac{4 \mu_0 (L-x) \sin \phi (g_{11} + 2 \theta) \cos \phi}{\pi g_{11}^2} + \frac{4 \mu_0 (L-x) \sin \phi (g_{11} + 2 \theta) \cos \phi}{\pi g_{11}^2} \right) \]

\[ = \frac{1}{2} K_m N_1^2 \left( \frac{4 \mu_0 (L-x) \sin \phi (g_{11} + 2 \theta) \cos \phi}{\pi g_{11}^2} + \frac{4 \mu_0 (L-x) \sin \phi (g_{11} + 2 \theta) \cos \phi}{\pi g_{11}^2} \right) \]

According to the type 5 and 6, the magnetic levitation force and electromagnetic torque mutually interfere and influence which the joint generates, the whole system is the nonlinear strong coupled system and needs nonlinear decoupling control.

5. The control system and the simulation analysis of the magnetic levitation spherical reluctance driving joint

In the same coordinate direction, two reluctance motor stator windings of magnetic levitation spherical reluctance driving joints can be alternative controlled according to the control strategy of the reluctance motor, the subsystem which constitutes two reluctance motor can synchronous drive sphere rotor rotating. Take two of the three-phase (\( m=3 \)) motor driving in the X-axis for example, it detects the offset signal of the rotor by displacement sensor, to convert it into current (or voltage) by after filtering and amplification and suspension controller, etc, after making feedback to the input of the windings control and comparing it with the displacement of the input signals \( x^* \), then to go through decoupling controller decoupling with the rotate speed signals of the rotor and transform the decoupling signal into the control signals of the three-phase circuit by \( 2 \dot{\theta}/3 \dot{\theta} \), so as to change the current size of the stator winding and achieve suspension control and rotating control, the principle of control system is shown in figure 4.

According to the voltage equation of the joint stator winding and the kinematics equations of the spherical rotor, the nonlinear coupling dynamic mathematical model of magnetic levitation spherical reluctance driving joint, as shown in the figure 4, the control system begins to emulate magnetic levitation spherical reluctance driving joint and then can analyze the performance of the system.

![Fig.4 Control principle of spherical driving joint with magnetic levitation](image)

1. The simulation results of the joint rotor which is disturbed

After the driving joints achieved stabilize suspension, the simulation result of impulse interference in the positive and negative direction is shown in figure 5 which the joint rotor input when it is at 0.5s and 0.8s. According to the figure, if the magnetic levitation spherical reluctance driving joint received the radial disturbance, it can restore to the equilibrium position in a short period of time, the rising time is all under 0.2s and the overshoot is small, it doesn’t have the shocking phenomenon.

2. The rotating speed response of the spherical rotor

After the inverse system decoupling and comprehensive correction, figure 6 shows the rotating speed step response of the spherical rotor spinning around the X-axis, the expected rotating speed is \( 1.2 \times 10^2 \text{ rpm} \). According to the results of the simulation and the rotating has less overshoot, the settling time is 0.5s, the subsystem of the rotating speed has good performance index.

![Fig.5 Radial offset with interference of Laplace domain signals](image)

![Fig.6 The dynamic response of the rotor speed](image)
6. Conclusions
This paper presents a multi-degree-of-freedom magnetic levitation spherical reluctance driving joint which has simple structure, high precision and good dynamic performance. According to the air-gap magnetic energy, it established the transformational relation of the joint mechanical energy and studies the mechanism of the joint to produce electric magnetic levitation force and electromagnet-tic torque; according to this, the comprehensive control system of the joint is established and it made the system simulation analysis of it, the simulation results show that the driving joint system has good dynamic and static performance and the anti-interference ability.

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REFERENCES

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