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Study of Magnetic Levitation Spherical Joint with Decoupling Control

Abstract. *This paper presents a new magnetic levitation spherical reluctance driving joint. After the analysis of air-gap magnetic theory of the driving joint and the working mechanism of the magnetic levitation force and electromagnetic torque, its mathematical model and the inverse system decoupling model have been established. The decoupling linearization and the state feedback closed-loop control system have been achieved also. The result of system simulation provides us the decoupling characteristic and the dynamic characteristic of the driving joint system.*

Streszczenie. *W artykule zaprezentowano reluktancyjne połaczenie kuliste z lewitacja magnetyczną. Zaprezentowano model matematyczny i odwrotny model odsprzężenia. Przeanalizowano układ ze statycznym sprzężeniem zwrotnym. (Analiza kulistego połączenia z magnetyczną lewitacją i sterowaniem odsprzężonym)*

Key words: Magnetic Levitation Spherical Driving Joint, Mathematical Model, Inverse System, Decoupling Control
Słowa kluczowe: lewitacja magnetyczna, połączenie kuliste.

1. Introduction

Spherical joint, also called multi-freedom joint, by two or three rotating freedom, can rotate bypassing fixed axis space. It has high integration, and can instead of two or more single-degree-freedom motor drive mechanism in multiple degrees of freedom. So multi-freedom spherical joints have wide application prospects in robot, multi-axis machine center, space craft, medical equipment, etc [1].

The existing multi-freedom joint is made up of two or more motor drive mechanism of the device; therefore it is big, structure complex, and restricted [2]. In high-speed and ultra high-speed multi-freedom joint device, it also exists bearing supporting wear problem. And friction will also cause components fever. Therefore, based on the maglev technology and electromagnetic technology, the paper puts forward the two degrees or multi-degree- freedom magnetic levitation spherical reluctance driving joint and studies the working mechanism and the inverse system decoupling control of it. It has simple structure, high integration, wear friction-free, high accuracy and good dynamic performance, and can make the spherical rotor pitch, swing and deflect angles maximum [3, 4].

2. The basic structure and working principle of magnetic levitation spherical reluctance driving joint

Magnetic levitation spherical joints are made up of multiple joints with motor stator and joint of spherical rotor with turning arm etc, the three-dimensional structure signal is shown in figure 1. Joint rotor is surrounded by the joints of the motor stator; four stators are designed to be symmetric distribution in the equator line position of joint of spherical rotor, among the stators, the stator 1 and the stator 3 are symmetrical about spherical rotor and keep coaxial along the X axis, its function is driven rotor spinning around the X axis and make the rotor in the X axis stability of maglev force; the stator 2 and the stator 4 are symmetrical about spherical rotor and keep coaxial along the Y axis, its function is driven rotor spinning around the Y axis and make the rotor in the Y axis stability of maglev force; The stator 5 arranges the top of spherical rotor and keeps coaxial with the Z axis, its function is driven rotor spinning around the Z axis and make the rotor in the Z axis stability of maglev force.

Magnetic spherical driving joint rotor is the external envelope saliency sphere of spherical rotor, each joint stator is made up of several saliency columns, the tops of column are to constitute the internal envelope sphere and keep parallel with the external envelope sphere which keeps balance position. According to the theory of reluctance motor rotor, joint stator saliency is around polyphase winding which made rotor whirling. Each phase windings after electrify produces electromagnetic torque to drive spherical rotor and also provides radial magnetic levitation force for the spherical rotor.

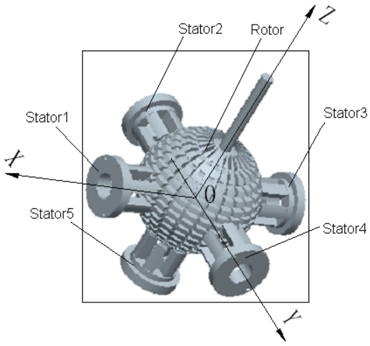


Fig.1. Structure drawing of spherical driving joint with magnetic levitation

3. The mathematical model of magnetic levitation spherical reluctance driving joint

Taking three-phase winding for example, if the magnetic levitation spherical reluctance driving joint in a coordinate direction (such as the X axis), the two stators which is symmetrical in rotor drive the stator suspend and rotate along the coordinates. Supposing the stability of the suspension rotor under the action of interference offset for x in the coordinate, In order to make the rotor restored to its original balance position, in the controller, the wind current i1 of the reduced stator which is near the clearance will decrease, the wind current i2 of the added stator which is near the clearance will increase. Magnetic levitation spherical reluctance driving joints in the X axis produce the electromagnetic levitation force is:

$$(1)\qquad F_x = \left[\frac{i_1^2}{2(g_0 - x\cos\varphi_0)^2} - \frac{i_2^2}{2(g_0 + x\cos\varphi_0)^2} \right] \cdot K_f(\psi)$$

Where g_0 is the radius clearance when the stator is concentric with the rotor, ψ is the angular displacement when the rotor rotates along the axis;

$$(2)\qquad K_f(\psi) = \frac{1}{2} N^2 R^2 \pi \cos\varphi_0 \sum_{k=0}^{2} K(\psi - \frac{2k\pi}{3})$$

Where N is the number the one phase windings; R is the radius of internal envelope sphere of the stator;

$$K(\psi)=\int_{\varphi_1}^{\varphi_2}\left[1+K\,cos\,2(\varphi+\psi)\right]sin\,\varphi d\varphi$$

Due to the excursion of the rotor, the total electromagnetic torque of the magnetic levitation spherical reluctance driving joint along the X-axis is:

$$(3)\qquad M_x=\frac{i_1^2K_m(\psi)}{2(g_0-x\,cos\,\varphi_0)}+\frac{i_2^2K_m(\psi)}{2(g_0+x\,cos\,\varphi_0)}$$

where, $K_m(\psi)=-N^2R^2K\pi\int_{\varphi_1}^{\varphi_2}sin\,\varphi\,sin\,2(\varphi+\psi--\frac{2k\pi}{3})d\varphi$

According to Newton's second law, the motion equation of the spherical rotor along the X-axis movement and rotating around the X-axis is:

$$(4)\qquad \begin{cases} m\cdot\dfrac{d^2x}{dt^2}=\dfrac{i_1^2K_f(\psi)}{2(g_0-x\,cos\,\varphi_0)^2}-\dfrac{i_2^2K_f(\psi)}{2(g_0+x\,cos\,\varphi_0)^2}+F_d \\ J\cdot\dfrac{d^2\psi}{dt^2}=\dfrac{i_1^2K_m(\psi)}{2(g_0-x\,cos\,\varphi_0)}+\dfrac{i_2^2K_m(\psi)}{2(g_0+x\,cos\,\varphi_0)}+M_d \end{cases}$$

Where m is the quality of the spherical rotor, J is the rotational inertia of the spherical rotor; F_d is disturbing force and M_d is the moment which the spherical rotor is to bear expect electromagnetic levitation force (moment).

So the magnetic levitation force of the magnetic spherical driving joints on the X-axis and the electromagnetic torque of the magnetic spherical driving joints around the X-axis are related to the clearance $g=g_0\pm xcos\varphi$ between the rotor convex poles and the stator convex poles and the resulting current winding i, therefore through changing winding currents it can control the stability of the suspension of the spherical rotor along the X axis and the rotation movement around the X axis.

The type 4 can describe the dynamic mathematical model of the magnetic levitation spherical reluctance driving joint without regard to the factors of the winding; thus, the system is the nonlinear strong coupling system.

To using state space model system, magnetic levitation spherical reluctance joint is described:

$$(5)\qquad \begin{cases} \dot{X}=f(X)+g(X)U \\ Y=h(X) \end{cases}$$

where, $X=\begin{bmatrix}x & \dot{x} & \Psi & \dot{\Psi}\end{bmatrix}^T=\begin{bmatrix}x_1 & x_2 & x_3 & x_4\end{bmatrix}^T$

$$U=\begin{bmatrix}i_1^2 & i_2^2\end{bmatrix}^T=\begin{bmatrix}u_1 & u_2\end{bmatrix}^T$$

$$f(X)=\begin{bmatrix}x_2 & \frac{1}{m}\cdot F_d & x_4 & \frac{1}{J}\cdot M_d\end{bmatrix}^T$$

$$h(X)=\begin{bmatrix}x_1 & x_4\end{bmatrix}^T$$

$$g(X)=\begin{bmatrix} 0 & 0 \\ \dfrac{1}{m}\cdot\dfrac{K_f(x_3)}{2(g_0-x_1\,cos\,\varphi_0)^2} & -\dfrac{1}{m}\cdot\dfrac{K_f(x_3)}{2(g_0+x_1\,cos\,\varphi_0)^2} \\ 0 & 0 \\ \dfrac{1}{J}\cdot\dfrac{K_m(x_3)}{2(g_0-x_1\,cos\,\varphi_0)} & \dfrac{1}{J}\cdot\dfrac{K_m(x_3)}{2(g_0+x_1\,cos\,\varphi_0)} \end{bmatrix}$$

The type 5 describes the original system model of the magnetic levitation spherical reluctance driving joint system's nonlinear dynamics coupling system. According to the prototype model, we can get the inverse system model, so as to decoupling control for joint system.

4.The inverse system model of magnetic levitation spherical reluctance driving joint

Using literature [5], it makes inversion for:

$$(6)\qquad \begin{cases} \ddot{y}_1=\dfrac{1}{m}\cdot\dfrac{u_1K_f(x_3)}{2(g_0-x_1\,cos\varphi_0)^2}-\dfrac{1}{m}\cdot\dfrac{u_2K_f(x_3)}{2(g_0+x_1\,cos\varphi_0)^2}+\dfrac{1}{m}\cdot F_d \\ \dot{y}_2=\dfrac{1}{J}\cdot\dfrac{u_1K_m(x_3)}{2(g_0-x_1\,cos\varphi_0)}+\dfrac{1}{J}\cdot\dfrac{u_2K_m(x_3)}{2(g_0+x_1\,cos\varphi_0)}+\dfrac{1}{J}\cdot M_d \end{cases}$$

Take matrix $A(U)=\begin{bmatrix}\ddot{y}_1 \\ \dot{y}_2\end{bmatrix}$, to calculate:

$$\frac{\partial A(U)}{\partial U}=\begin{bmatrix} \dfrac{K_f(x_3)}{2m(g_0-x_1\,cos\varphi_0)^2} & \dfrac{-K_f(x_3)}{2m(g_0+x_1\,cos\varphi_0)^2} \\ \dfrac{K_m(x_3)}{2J(g_0-x_1\,cos\varphi_0)} & \dfrac{K_m(x_3)}{2J(g_0+x_1\,cos\varphi_0)} \end{bmatrix}$$

Because $rank[\partial A(U)\ /\ \partial U]=2$ is a nonsingular matrices, the relative degree of the magnetic levitation spherical reluctance driving joint is $\alpha=(2\ 1)$, and $\sum_{i=1}^{2}\alpha_i=3<n(=4)$, so the inverse system exists.

Order

$$(7)\qquad :\begin{bmatrix}\ddot{y}_1 \\ \dot{y}_2\end{bmatrix}=\varphi$$

where $\varphi=\begin{bmatrix}\varphi_1 \\ \varphi_2\end{bmatrix}$ indicates the new system input.

According to the type 6 and the type 7, it can get the control discipline of magnetic levitation spherical reluctance driving joint with the inverse system:

$$(8)\qquad \begin{cases} u_1=\dfrac{2(g_0-x_1\,cos\,\varphi_0)^2(mD\varphi_1+JB\varphi_2+BM_d+DF_d)}{K_f(x_3)K_m(x_3)g_0} \\ u_2=\dfrac{2(g_0^2-x_1^2\,cos\,\varphi_0{}^2)^2(-mC\varphi_1+JA\varphi_2+AM_d-CF_d)}{K_f(x_3)K_m(x_3)g_0} \end{cases}$$

where $\begin{cases} A=\dfrac{K_f(x_3)}{2(g_0-x_1\cos\varphi_0)^2}, & B=\dfrac{K_f(x_3)}{2(g_0+x_1\cos\varphi_0)^2} \\ C=\dfrac{K_m(x_3)}{2(g_0-x_1\cos\varphi_0)}, & D=\dfrac{K_m(x_3)}{2(g_0+x_1\cos\varphi_0)} \end{cases}$

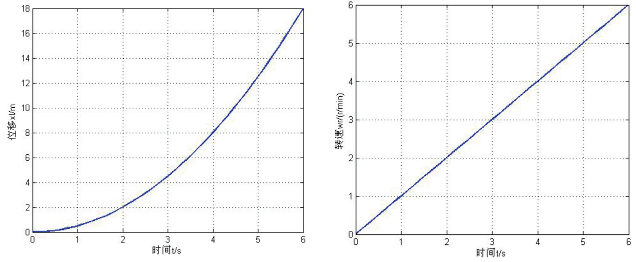
The type 8 constitutes the magnetic levitation spherical reluctance driving joint system which makes φ_1, φ_2 for input and u_1, u_2 (or $i_1=\sqrt{u_1}$, $i_2=\sqrt{u_2}$) for output with the state feedback linearization control algorithm of 4 order integral inverse system decoupling

5. The linear decoupling control and simulation performance analysis of the system

According to the type 5 and 8, simulation model is established, the structure parameters which results into the system on the simulation system can be used to make simulated solving of the system and get the simulation curve of inverse system decoupling. As the shown in the figure 2(a) and the figure 2(b), they are the radial displacement of spherical rotor along the X-axis and the curves of angular velocity changing with time around the X-axis when the system input is $\varphi_1=\varphi_2=1$. As the shown in the figure 4(c) and the figure (d), they are the radial displacement of spherical rotor along the X-axis and the curves of angular velocity changing with time around the X-axis when the system input is $\varphi_1=2$, $\varphi_2=1$.

(a)when, $\varphi_1=1$, the radial displacement of spherical rotor along the X-axis

(b) when, $\varphi_2=1$, the curves of angular velocitychanging with time around the X-axis



(c)when, $\varphi_1=2$, the radial displacement of spherical rotor along the X-axis

(d) when, $\varphi_2=1$, the curves of angular velocity changing with time around the X-axis

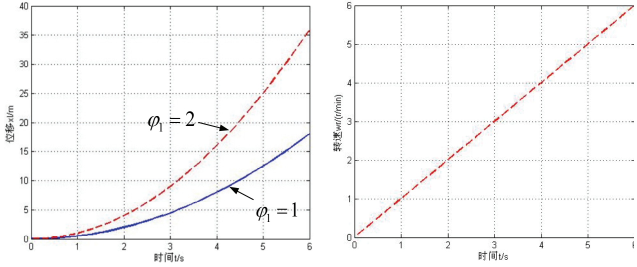


Fig.2. the simulation curve of spherical driving joint with inverse system decoupling

To compare simulation results, it only influences the input y_1 (the radial displacement of spherical rotor along the X-axis) and don't influence the angular velocity y_2 of the rotor around the X-axis when it only change the control input φ1 which is the radial displacement along the X-axis; otherwise, it only influences the input y_2 (the angular velocity of the spherical rotor around the X-axis) and don't influence the radial displacement y_1. So magnetic levitation spherical reluctance driving joint system changes from nonlinear system to linear system. From the figure 2, we can find that two independent systems are all divergent unstable system after decoupling, we should regulate it by the controller.

According to the type 5 and 8, the simulation model of magnetic levitation spherical reluctance driving joint with inverse system decoupling and comprehensive correction is established, to substitute the specific parameters for system simulation, it can get the simulation curve which likes figure 3 to 5.

(1) The float process of spherical rotor

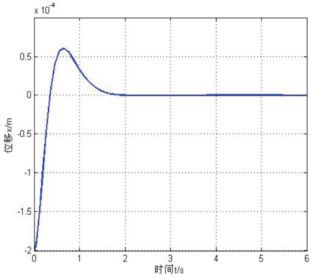


Fig.3: The dynamic response of the rotor position

Figure 3 is the spherical rotor float from natural place after inverse system decoupling and comprehensive correction, when to be unloaded, the initial value of the dynamic response of the direction of a degree of freedom is $x_0=2\times10^{-4}$m along the X axis. According to the result of the simulation, we can find the overshoot of the system is 22% and the accommodation time is 1.7s. The radial position subsystem of the spherical rotor of the magnetic levitation spherical reluctance driving joint decoupling control system has good dynamic performance and static performance.

(2) The rotate speed response of spherical rotor

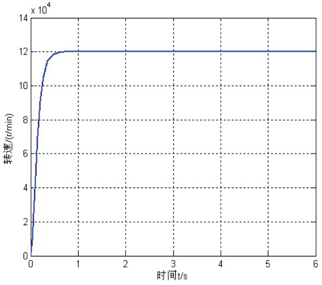


Fig.4: The dynamic response of the rotor speed

Figure 4 is the step response of the spherical rotor rotating along the X-axis after inverse system decoupling and comprehensive correction, the expect speed is 1.2×10^5 r/min. According to the result of the simulation, the rotating speed doesn't have overshoot and the accommodation time is 0.5s, the speed subsystem have good performance index.

(3) Dynamic decoupling performance

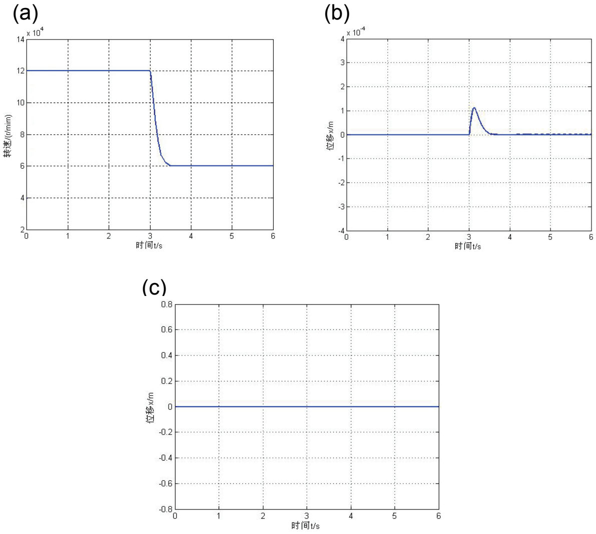


Figure 5: the radial displacement by the influence of the rotating speed

Inverse system decoupling control made full use of all the system state variables; because considering the coupling between subsystems, the dynamic performance is better. The rotating speed influences the radial position which likes figure 5. Figure 5(a) is the rotor speed state chart, after the rotating speed get the stable value (1.2×105 r/min), it cuts down to 6×10^4 r/min when $t = 3$s; when in the station of the puzzled decoupling, after using PID control strategy the dynamic process of the radial rotor position changing with the rotating speed likes figure 5(b); after inverse system decoupling ,the dynamic process of the spherical rotor position changing with the rotating speed by using the state feedback control strategy likes figure 5(c).

According to the result of the simulation, if using PID control strategy, the radial rotor position will undulate when the rotating speed are changing. When using the state feedback control strategy, the radial position is unacted on the rotating speed and always in balance.

6. Results

The paper presents a new magnetic levitation spherical reluctance driving joint, based on the principle of magnetic reluctance motor, it Analyses the air-gap magnetic energy of the driving joints and the mechanism of generating electromagnetic levitation force energy and electromagnetic torque, the mathematical model of driving joint system and inverse system decoupling model are established, it makes decoupling linearization of driving joint system and closed-loop control of station feedback and study the decoupling characteristics and dynamic response characteristics of the magnetic levitation spherical reluctance driving joint system.

Acknowledgements

This work was supported in part by the China Natural Science Foundation under Grant Nos. 50975249, and the Fundamental Research of Natural Science Foundation for Universities of Jiangsu Province under Grant Nos. 08KJB460008 and 09KJD460006.

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