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Game-based Decentralized Charging Control for Large Populations of Electric Vehicles

***Abstract**. This paper proposes a game-based decentralized charging control strategy for large populations of electric vehicles (EVs). Assuming all EV owners make their own charging strategy according to the electricity price and the total electricity demand of the day before, the owners can be guided to actively participate in the game by a set of electricity pricing mechanism. The existence of Nash equilibrium and the global optimum (or 'Valley-filling') of the charging strategy are verified. Simulation results demonstrate the convergence to the Nash equilibrium within a few iterations.*

***Streszczenie.** W artykule zaproponowano strategię ładowania dla dużej populacji pojazdów elektrycznych bazująca na teorii gier. Strategia wykorzystuje informacje o cenie energii i prognozowanym zapotrzebowaniu. Zweryfikowano metody optymalizacji. (**Sterowanie ładowaniem baterii dużej populacji pojazdów elektrycznych bazujące na teorii gier**)*

Keywords: Decentralized Charging Control, Electric Vehicles (EVs), Nash Equilibrium, Valley-Filling.
Słowa kluczowe: pojazdy elektryczne, ładowanie baterii, optymalizacja.

Introduction

Electric vehicles (EVs) play a more and more important role in energy conservation, reducing greenhouse gas emissions, and travel convenience [1]. These vehicles will occupy the most part of the market in the next several years with the electrification of transportation [2, 3]. A series of challenges and problems arise. Some studies have been done to explore the potential influence of the EVs' increasing to the power grid [4-6]. In general, EV charging increases the randomness and uncertainty of the power demand side by increasing the electric loads. Therefore, it influences the stability of power grid.

Studies on EV's charging problems are split into two aspects: regular vehicle and irregular vehicle. Regular vehicle represents a vehicle whose trip mode is regular such as bus, sanitation car, postal vehicle etc.. L. Cheng [7] presented an intelligent control strategy of battery-electric bus based on the fuzzy comprehensive evaluation method. However, the irregular vehicles are major parts of the future EV market. It means that the charging time and the charging rate are both uncertain. Using electrovalence as the key basis to formulate control strategy is widely accepted. S. Shao [8] analyzed the impact of time-of-use (TOU) electricity rates on customer behaviors in a residential community, but it does not afford how to set the price of electricity. The coordinated charging strategies with the purpose of minimizing power losses and maximizing the main grid load factor have been studied [9, 10]. These centralized control strategies require a system structure to collect all EVs' information to get the charging rate of the EVs. Hence they all need a centralized controller with great communications and computational capability. Caramanis [11] introduced a decentralized control strategy by developing a decision support algorithm for optimal bidding to the existing wholesale as well as to the prospective retail/distribution market, but it dose not give the analysis of the optimality. Z. Ma [12-14] provided another decentralized strategy through a simulation-based study. This strategy receives good effects, as each EV owner has more autonomous right in deciding his own charging strategy.

In this paper, a charging method of the traditional charging station with parking space is proposed. Assuming there is a device which can show the electricity price and average charging rate curve of the day before beside every parking space so that each EV owner can decide the charging strategy according to these information. As the EV population grows, the influence of each EV on the average charging strategy is negligible. Under these conditions, the whole control strategy is a Nash equilibrium [15], if the following conditions are satisfied as

i. Each owner's control strategy is optimal for each owner with respect to the average charging curve.

ii. The average curve is reproduced, i.e. for each EV owner reluctant to change his own charging strategy.

By using a set of electricity pricing mechanism, the users are guided to actively participate in the game. Under suitable conditions, the Nash equilibrium will be the global optimum (or 'Valley-filling'). As shown in Fig.1, the EVs' demand fills the overnight valley of the power consumption curve.

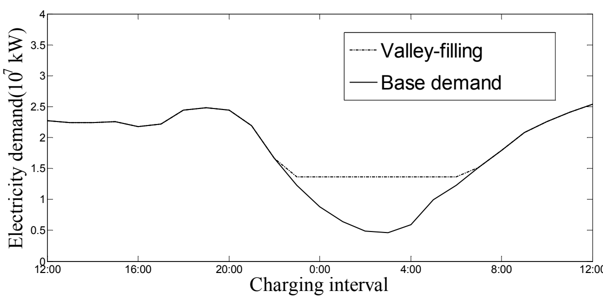

Fig.1. Global optimal charging

This paper investigates the decentralized control strategy receives good result in personal and global optimum under condition that the number of EV ownership is great. Section 2 formulates the decentralized charging control problem and provides the electricity pricing mechanism. Section 3 provides the certification procedure of the existence of the Nash equilibrium and demonstrate the Nash equilibrium condition is also global optimal. Numerical simulations are provided to illustrate these results in section 4, and conclusions and future works are presented in section 5.

Problem formulation

Considering a certain place with the number of EVs is N. For each individual EV n, the charging state is defined as follows

(1) $$SOC_{n,t+1} = SOC_{n,t} + \frac{c_{n,t}}{b_n}, \quad t = T_0, ..., T-1,$$

where: $SOC_{n,t}$ – the state-of-charge of EV n at time t, $c_{n,t}$ – is the charging rate of EV n at time t, b_n – the battery capacity, T_0 and T – the initial charging time and the whole charging time.

The set of feasible charging control strategy for each individual EV is defined as

(2) $C_n \triangleq \left\{c_n \equiv (c_{n,T_0},...,c_{n,T-1}); \text{s.t.} c_{n,t} \ge 0, SOC_{n,T} = 1\right\}$,

where: $c_n \equiv \left\{c_{n,t}; t \in T\right\}$ – an admissible charging control strategy, which belongs to C_n.

The whole EVs' charging rates at time t is defined as $c_t \triangleq \left\{c_{n,t}; 1 \le n \le N\right\}$. The cost of the individual EV n is given by

(3) $$J_n(c_n;\overline{c}) \triangleq \sum_{t\in T}[p_t c_{n,t} + \varepsilon(c_{n,t} - \overline{c}_t)^2]$$

where:

$p_t \equiv p(B_t + \sum_{n\in N} c_{n,t}) = p(D_t) = [0.7 + (D_t - \overline{D}_{tb})\sqrt{|D_t - \overline{D}_{tb}|} \times 10^{-12}]$

– the electricity price at time t, $\overline{D}_{tb}$ – the average total electricity demand of the day before and 0.7 (Yuan RMB)is the benchmark electricity price. This definition implies that the electricity price is strictly increased depending on the base electricity demand and the total EV demand, $\overline{c}_t$ – the average charging rates of all EVs at time t, ε – the deviation factor of the individual charging rate and the average charging rates.

The base demand B_t can be predicted one day ahead and be approximately the same in short term. Then the algorithm of the electricity price guided game is shown as follows,

Step 1. Announce the intraday electricity price profile which is the function of the previous day's electricity demand at the Bulletin Board to all agents;

Step 2. Each agent calculates the cost function and makes the charging strategy according to the intraday electricity price profile and the electricity demand of the previous day;

Step 3. The control center adjusts the electricity price profile according to the intraday electricity demand;

Step 4. Repeat Step 1 – Step 3 every day, and it will reach to the Nash equilibrium few days later.

Because the action of individual EV on the whole system is negligible if the number of EVs is large enough, the decentralized control problem described above is a non-cooperative dynamic game15 the decentralized control problem described above is a non-cooperative dynamic game [15].

The optimal control strategy of agent n is given by

(4) $$c_n^*(c^{-n}) \triangleq \arg\min_{c_n \in C_n} J_n(c_n)$$

where: c^{-n} – the collection of charging strategies of the whole EVs without agent n.

DEFINITION 1. A collection of charging strategies $\left\{c_n^*; n \in N\right\}$ is a Nash equilibrium if each individual EV agent can benefit nothing by changing strategy when the average charging rates known to all agents, i.e.,

(5) $$J_n(c_n^*; c^{-n}) \le J_n(c_n; c^{-n})$$

for all $c_n \in C_n$, and all $n \in N$. This definition implies that the average charging strategy is reproducible, i.e.,

(6) $$\overline{c}^*(\overline{c}) = \overline{c}.$$

So far, the decentralized charging control problem is formulated as a dynamic game and the electricity pricing mechanism is provided. The tracking cost part $\varepsilon(c_{n,t} - \overline{c})^2$ in (3) is the key point to the decentralized control problem [12]. In reality, it can be the fee of the parking lot for the agent setting the charging strategy. The deviation progress of achieving individual and global optimum will be given in section 3. Section 4 will provide the simulation results.

Demonstrate of the optimum in decentralized control problem

A. Individual Optimal Charging Strategy [14]

The charging strategy minimizes the individual cost function (3) with respect to a fixed $\overline{c}$ by

(7) $$c_n^*(\overline{c}) \triangleq \arg\min_{c_n \in C_n} J_n(c_n;\overline{c}).$$

LEMMA 1. With a fixed average charging rate $\overline{c}_t$ the optimal individual charging strategy $c_{n,t}^*(\overline{c},\lambda) \in C_n$ can be denoted by

(8) $c_{n,t}^*(\overline{c},\lambda) = \dfrac{1}{2\varepsilon}\max\{0, \lambda + 2\varepsilon\overline{c}_t - p_t\}$, for all $t \in T$

where: λ is denoted $\lambda(\overline{c})$ and uniquely dependent on $\overline{c}$. With a particular value of $\lambda^*(\overline{c})$, the function (8) gives the optimal control trajectory minimizing the individual cost (3) of agent n.

B. Existence of the Nash Equilibrium

LEMMA 2 [14]. Assuming $p(B_t + \sum_{n\in N} c_{n,t})$ is continuous on $(B_t + \sum_{n\in N} c_{n,t})$, there is

(9)

$$\left\|c_n^*(\overline{c}) - c_n^*(\tilde{c})\right\|_1 \le 2\sum_{t\in T}\left|(\overline{c}_t - \tilde{c}_t) - \frac{1}{2\varepsilon}[p(B_t + \sum_{n\in N} c_{n,t}) - p(\tilde{B}_t + \sum_{n\in N}\tilde{c}_{n,t})]\right|$$

where: $\|X\|_1 \triangleq \sum|x|$ – the l_1 norm of X, $\tilde{c}$ – a different average charging control.

PROOF: Firstly, defining a charging control $\hat{c}_n(\tilde{c})$ satisfying that $\lambda(\hat{c}_t) = \lambda^*(\overline{c}_t)$ corresponding to $\tilde{c}$, so that,

(10)

$$\left\|c_n^*(\overline{c}) - \hat{c}_n(\tilde{c})\right\|_1 = \sum\left|(\overline{c}_t - \tilde{c}_t) - \frac{1}{2\varepsilon}[p(B_t + \sum_{n\in N} c_{n,t}) - p(\tilde{B}_t + \sum_{n\in N}\tilde{c}_{n,t})]\right|.$$

Then there are three cases to be considered:

i. $\sum c_n^*(\overline{c}) = \sum \overline{c}_n(\tilde{c})$. It implies that $c_n^*(\overline{c}) = \hat{c}_n(\tilde{c}) = c_n^*(\tilde{c})$. It follows immediately that $\left\|c_n^*(\overline{c}) - c_n^*(\tilde{c})\right\|_1 = 0$ satisfies (9).

ii. $\sum c_n^*(\bar{c}) < \sum \hat{c}_n(\tilde{c})$. Considering $\sum c_n^*(\bar{c}) = \sum c_n^*(\tilde{c})$, so that $\sum c_n^*(\tilde{c}) < \sum \hat{c}_n(\tilde{c})$. Hence
$$\begin{aligned}0 &\le \left\|\hat{c}_n(\tilde{c}) - c_n^*(\tilde{c})\right\|_1 = \sum \hat{c}_n(\tilde{c}) - \sum [c_n^*(\bar{c})] \\ &\le \left\|\hat{c}_n(\tilde{c}) - c_n^*(\bar{c})\right\|_1\end{aligned},$$
furthermore
$$\begin{aligned}\left\|c_n^*(\bar{c}) - c_n^*(\tilde{c})\right\|_1 &\le \left\|c_n^*(\bar{c}) - \hat{c}_n(\tilde{c})\right\|_1 + \left\|c_n^*(\tilde{c}) - \hat{c}_n(\tilde{c})\right\|_1 \\ &\le 2\left\|c_n^*(\bar{c}) - \hat{c}_n(\tilde{c})\right\|_1\end{aligned}.$$
So the equation (9) can be derived from (10).

iii. $\sum c_n^*(\bar{c}) > \sum \hat{c}_n(\tilde{c})$. A similar derivation process as (ii) can be used to obtain (9).

If $p(B_t + \sum_{n \in N} c_{n,t})$ is continuous on $(B_t + \sum_{n \in N} c_{n,t})$, a consequence that $c_n^*(\bar{c})$ is continuous on $\bar{c}$ can be derived from LEMMA 2, because the average of a continuous function is also continuous.

Define a convex compact set $U \triangleq \left\{u_t; t \in T, 0 \le u_t \le \max_{n \in N}[b_n(1 - SOC_{n,0})]\right\}$, which implies that $C_n \subset U$ and $\bar{c}^*(\bar{c}) \in U$. As the analysis above, it can be deduced that $\bar{c}^*(\bar{c}) \in U$ holds for any $\bar{c} \in U$, in other words, $\bar{c}^*(\bar{c})$ maps a convex compact set to itself. There must be a fixed point $\bar{c} \in U$ satisfying $\bar{c}^*(\bar{c}) = \bar{c}$ by the Brouwer fixed point theorem16, by DEFINITION 1 this fixed point is a Nash equilibrium.

C. Method of Selecting the Deviation Coefficient ε

THEOREM 1. The decentralized charging control system converge to a unique Nash equilibrium if p_t is continuously and strictly increasing on $(B_t + \sum_{n \in N} c_{n,t})$, and

(11) $$\frac{1}{2}\max_{D_t \in (D_{\min}, D_{\max})} \frac{\mathrm{d}p_t}{\mathrm{d}D_t} \le \varepsilon \le k \min_{D_t \in (D_{\min}, D_{\max})} \frac{\mathrm{d}p_t}{\mathrm{d}D_t},$$

where: $D_t = (B_t + \sum_{n \in N} c_{n,t})$, $k \in (1/2, 1)$.

PROOF: According to (11),

(12) $$\begin{aligned}\left\|p(D_t) - p(\tilde{D}_t)\right\|_1 &\le \max_{D_t \in (D_{\min}, D_{\max})}\left\{\frac{\mathrm{d}p_t}{\mathrm{d}D_t}\right\}\left\|D_t - \tilde{D}_t\right\|_1 \\ &= \max_{D_t \in (D_{\min}, D_{\max})}\left\{\frac{\mathrm{d}p_t}{\mathrm{d}D_t}\right\}\left\|\bar{c} - \tilde{c}\right\|_1 \\ &\le 2\varepsilon\left\|\bar{c} - \tilde{c}\right\|_1\end{aligned},$$

and

(13) $$\begin{aligned}\left\|p(D_t) - p(\tilde{D}_t)\right\|_1 &\ge \min_{D_t \in (D_{\min}, D_{\max})}\left\{\frac{\mathrm{d}p_t}{\mathrm{d}D_t}\right\}\left\|D_t - \tilde{D}_t\right\|_1 \\ &= \min_{D_t \in (D_{\min}, D_{\max})}\left\{\frac{\mathrm{d}p_t}{\mathrm{d}D_t}\right\}\left\|\bar{c} - \tilde{c}\right\|_1 \\ &\ge \frac{\varepsilon}{k}\left\|\bar{c} - \tilde{c}\right\|_1\end{aligned},$$

Merging above two inequality (12) and (13) as

(14) $$\frac{1}{2k}\left\|\bar{c} - \tilde{c}\right\|_1 \le \frac{1}{2\varepsilon}\left|p(D_t) - p(\tilde{D}_t)\right| \le \left\|\bar{c} - \tilde{c}\right\|_1,$$

therefore

(15) $$(1 - \frac{1}{2k})\left\|\bar{c} - \tilde{c}\right\|_1 \ge \left\|\bar{c} - \tilde{c}\right\|_1 - \frac{1}{2\varepsilon}\left\|p(D_t) - p(\tilde{D}_t)\right\|_1.$$

Considering p_t is continuously and strictly increasing on $(B_t + \sum_{n \in N} c_{n,t})$, so that

(16) $$(1 - \frac{1}{2k})\left\|\bar{c} - \tilde{c}\right\|_1 \ge \left\|(\bar{c} - \tilde{c}) - \frac{1}{2\varepsilon}[p(D) - p(\tilde{D})]\right\|_1.$$

Combine (16) with (9), then it achieves that

(17) $$\left\|c_n^*(\bar{c}) - c_n^*(\tilde{c})\right\|_1 \le (2 - \frac{1}{k})\left\|\bar{c} - \tilde{c}\right\|_1.$$

Hence $c_n^*(\bar{c})$ is contraction mapping for $\bar{c}$ if $k \in (1/2, 1)$. Then the decentralized charging control system converge to a unique Nash equilibrium according to the contraction mapping theorem [17].

D. Global Optimum (or Valley-filling)

DEFINITION 2. The whole charging strategy is Valley-filling means for any pair of charging instants t_1 and t_2 ($t_1, t_2 \in T$), then the following equation holds as

(18) $$B_{t_1} + c_{t_1} = B_{t_2} + c_{t_2}.$$

To prove the Nash equilibrium analyzed in part B of this section is Valley-filling, the contradiction method can be used. Suppose that there exist two time instants $t_1, t_2 \in T$ which holds $B_{t_1} + c_{t_1} \ne B_{t_2} + c_{t_2}$, i.e. $B_{t_1} + \bar{c}_{t_1} \ne B_{t_2} + \bar{c}_{t_2}$. It means that $B_{t_1} + \bar{c}_{t_1} = B_{t_2} + \bar{c}_{t_2} + \xi$, for some $\xi \ne 0$. Then there must exist a constant $\tilde{\xi}$ satisfying $\tilde{\xi} < \xi$ such that $B_{t_1} + c_{n,t_1}^* = B_{t_2} + c_{n,t_2}^* + \tilde{\xi}$ for agent n. Since $c_{n,t}^* \ge 0$ for all $t \in T$, assume a charging strategy c_n^δ for agent n with $c_{n,t_1}^\delta = c_{n,t_1}^* + \delta$, $c_{n,t_2}^\delta = c_{n,t_2}^* - \delta$ and $c_{n,t}^\delta = c_{n,t}^*$ for all $t \in T \setminus \{t_1, t_2\}$.

According to function (3),

(19) $$\begin{aligned}J_n(c_n^*; \bar{c}) - J_n(c_n^\delta; \bar{c}) = \delta(p_{t_2} - p_{t_1}) + 2\varepsilon\delta[(c_{n,t_2}^* - c_{n,t_1}^*) \\ -(\bar{c}_{t_2} - \bar{c}_{t_1})] - 2\varepsilon\delta^2\end{aligned}.$$

Noting that $B_{t_1} + \bar{c}_{t_1} < B_{t_2} + \bar{c}_{t_2} + \xi$ and p is strictly increasing on $(B_t + \sum_{n \in N} c_{n,t})$, so that $p_{t_2} - p_{t_1} > 0$, $(c_{n,t_2}^* - c_{n,t_1}^*) - (\bar{c}_{t_2} - \bar{c}_{t_1}) = \xi - \tilde{\xi} > 0$ and $2\varepsilon\delta^2$ is tiny. Therefore $J_n(c_n^*; \bar{c}) - J_n(c_n^\delta; \bar{c}) > 0$ i.e. $J_n(c_n^*; \bar{c}) > J_n(c_n^\delta; \bar{c})$ can be derived. However, $J_n(c_n^*; \bar{c})$ is a Nash equilibrium so that $J_n(c_n^*; \bar{c})$ should be the minimum. There is a contradiction so that $B_{t_1} + c_{t_1} = B_{t_2} + c_{t_2}$ is proved. The Nash equilibrium obtained above is Valley-filling, which is proved by DEFINITION 2.

Numerical simulations

The examples use a base demand profile refers to another literature [12] combined with the statistics of Beijing's electricity demand shows in Fig.2, which shows a normalized one-day electricity demand in a huge city such as Beijing China and the dashed line shows the demand of the EVs.

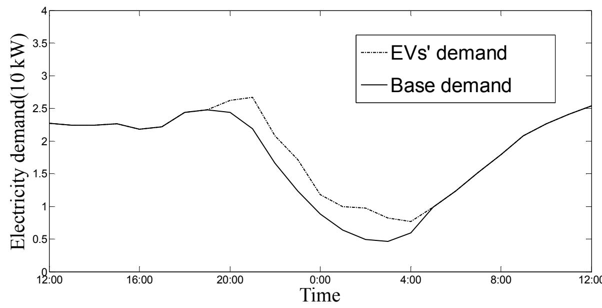

Fig.2. A normalized one-day electricity demand

Considering the identical battery size is 10kWh and the initial SOC is 15% for all EVs, the number of EVs is about $N=10^6$ and the charging time covers 12 hours from 20:00 to 8:00 on the next day, the continuously and strictly increasing price function is as follows,

(20) $$p(D_t)=[0.7+(D_t-\overline{D}_{tb})\sqrt{\left|D_t-\overline{D}_{tb}\right|}\times 10^{-12}].$$

From Fig.2 it can be verified that

(21) $$\begin{cases} D_{\min}=0.89\times 10^7\,\mathrm{kW} \\ D_{\max}=2.99\times 10^7\,\mathrm{kW} \end{cases}.$$

Hence

$$\frac{1}{2}\max_{D_t\in(D_{\min},D_{\max})}\frac{\mathrm{d}p_t}{\mathrm{d}D_t}=4.10\times 10^{-9}\le\varepsilon\le k\min_{D_t\in(D_{\min},D_{\max})}\frac{\mathrm{d}p_t}{\mathrm{d}D_t}=4.47k\times 10^{-9}$$

with some $k\in(1/2,1)$ so that the decentralized control problem converges to a unique Nash equilibrium on the basis of (13).

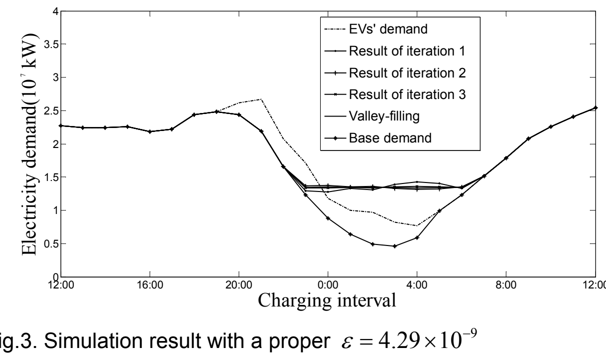

Fig.3. Simulation result with a proper $\varepsilon=4.29\times 10^{-9}$

Fig.3 shows the game process when choosing a proper ε. It converges to the Nash equilibrium after several rounds of game and gets the Valley-filling. In order to demonstrate the effect of the convergence, the relative difference of the average charging demand between two iterations is defined as $\mu=\left\|\overline{c}_{i,t}-\overline{c}_{i-1,t}\right\|_1/N$, where $\overline{c}_{i,t}$ is the average charging demand of all EVs at time instant t in the ith iteration. If the difference is less than 1‰, the Nash equilibrium is considered to be achieved. Fig.4 shows the difference μ with respect to iterations and illustrates that μ is less than 1‰ after the 3rd iteration as 0.557‰. However if the deviation factor is improper it can be seen in Fig.5 that it can not converge to a unique Nash equilibrium. Fig.3 and Fig.5 illustrate that the effect of convergence is sensitive to ε.

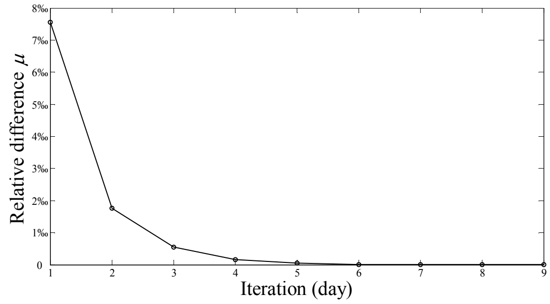

Fig.4. The relative difference μ of the average charging demand between two iterations

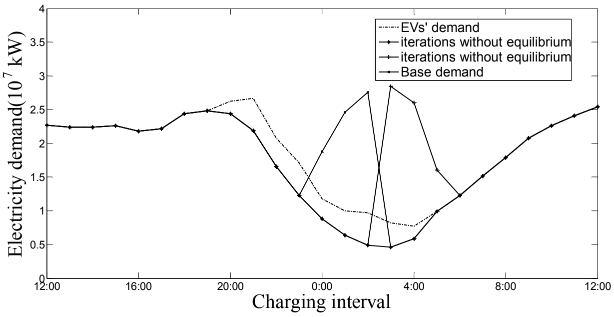

Fig.5. Simulation result with an improper $\varepsilon=6.20\times 10^{-9}$

Conclusions and future works

In this paper, a game-based decentralized charging control strategy to fill the overnight electricity valley has been studied. This control strategy does not need complex central computational controller and communication device. This problem is formulated as a large-population dynamic game on a finite charging interval. The existence of the Nash equilibrium and the global optimal valley-filling are analyzed. It can be converged to a unique Nash equilibrium through several rounds of game which is also verified. Simulation results demonstrate the fast convergence of the algorithm. In order to simplify the analysis, it is assumed that all EVs share the same charging interval with the same initial state-of-charge, the base demand B_t can be predicted and be approximately the same in short term. Future works will get rid of these assumptions and consider the V2G (vehicle-to-grid) capability developed recently. This control strategy will also take effect in the game-based grid-friendly appliance and smart micro-grid system research.

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