

# High Efficient Variable Phase PPK Modulation Scheme

**Abstract.** A kind of concise and higher bandwidth efficiency Variable Phase PPK (VP-PPK) modulation is proposed in this article. Base band pulse coding is analyzed, PSD expression for VP-PPK is deduced, and simulation is conducted. Coding realization of the system is given and performance simulation is given under the condition of AWGN channel. It is shown that bandwidth efficiency of VP-PPK can be  $\log_2 N$  times that of RPPK,  $N$  is the integer power of 2, and the 3-dB bandwidth efficiency of VP-PPK can reach 39.32bps/Hz calculated according to the definition.

**Streszczenie.** Zaproponowano nowy model modulacji szerokopasmowej nazwany Variable Phase PPK (pulse position keying). (Efektywny schemat modulacji Variable Phase PPK)

**Keywords:** UNB; Bandwidth efficiency; PSD; VP-PPK.

**Słowa kluczowe:** modulacja, kodowanie, PPK

## Introduction

Rapid increase in information and the degree of development of automation have led us to an era of interconnectivity, and all wired and wireless networks are developed rapidly, including sensors and sensor networks. Communication methods have become an important part of smart sensors and sensor network applications, especially under the conditions where battery power supply and spectrum resources are limited. Efficient communication methods can improve not only energy efficiency but also spectral efficiency. As a kind of exploration of high efficient modulation method, UNB was initiated and named by Dr.H.R.Walker [1]. As a kind of innovative modulation method, it has certain disadvantages. The key technology is the special filtering technology<sup>1</sup>, and there are a lot of difficulties to realizing and verifying it, while, as new modulation technology, it is still being explored continuously, such as (Very Minimum Chirp Keying) and so on [2]. A kind of random pulse position keying (RPPK) is proposed to try to reduce the difficulty of filtering existing UNB [3], but the study on PPK is not complete, the frequency spectrum is not fully utilized, and the complexity is high. In this article, a complete and concise Variable Phase-PPK (VP-PPK) modulation method is proposed, PSD analysis is presented, and both theoretical analysis and simulation results are shown to be consist. Based on the new modulation method, a system, which is much easier to realize digitally, is given; then demodulation performance is presented and bandwidth efficiency is analyzed. The article not only presents the -55dB bandwidth efficiency of 100bps/Hz [3], but also realizes the -60dB bandwidth efficiency of 100symbol/s/Hz. In addition, 3-dB bandwidth efficiency of VP-PPK can reach 39.32bps/Hz, which is much higher than that of the traditional methods.

The article is organized as follows: the principle of UNB is analyzed and the modulation function of VP-PPK is given in section 2. Then, in section 3, the PSD expression for VP-PPK is deduced. Next, the coding realization of the system is given. Performance simulation is also given based on the optimum receiver in the AWGN channel, and bandwidth occupation and the bandwidth efficiency of VP-PPK are analyzed. A comparison is done between VP-PPK and the existing method under the same conditions. Finally, the superiority of VP-PPK is discussed in the conclusion in section 5.

## VP-PPK Modulation

There are many modulation formats called UNB; the modulation process is similar. The base band coding method represented by Pulse Position Phase Reversal Keying (3PRK) is shown in the following[1][4],

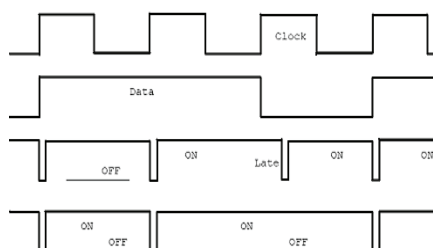


Fig. 1. Base band coding for UNB[4]

The modulation results are shown in Fig. 2. There are many line spectra, which block the bandwidth compression. The line spectra have nothing to do with the demodulation performance, so if these line spectra can be removed, the bandwidth occupation can be compressed greatly. Reference [5] presents the mathematic optimum methods; by using these, the line spectra are removed efficiently and the demodulation performance is not affected.

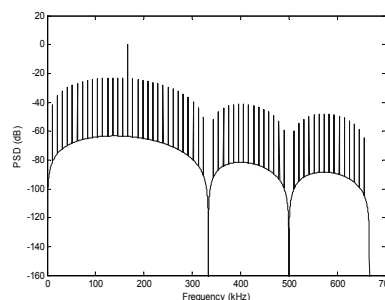


Fig. 2. PSD for 3PRK.

## Modulation Function For VP-PPK

Here, a kind of more concise and efficient modulation method realization is proposed and given in the following expression, defined as VP-PPK:

$$(1) \quad g_K(t) = \begin{cases} \sin 2\pi f_c t & 0 \leq t < \tau_1 \\ \sin(2\pi f_c t + \theta) & \tau_1 \leq t < \tau_2, 0 \leq \theta < \pi \\ \sin 2\pi f_c t & \tau_2 \leq t < T_s \end{cases}$$

$$K = m_1 + 2m_2 + \dots + 2^{n-1}m_n \quad K \in (0,1,2, \dots, 2^{n-1})$$

$$M = (m_1, m_2, \dots, m_n)$$

where  $M$  is the  $n$ -bit binary number, and  $K$  is the position number of the modulation pulse, which satisfies the above

mapping function. It can be seen that  $\tau_1$  controls the position of the modulation pulse.  $\tau_1 = KT_c$ ,  $\tau_2 = (K+1)T_c$ . The symbol is mapped into radio waves, and digital VP-PPK modulation is realized.  $N$  can also any number of integers to the power of 2.  $\theta$  is the variable phase[7].

### Discrete State-based PSD Analysis VP-PPK Signal

For the VP-PPK modulation shown in (1), under the condition of  $T_s(N)$  as determined, the number of states is also determined; for  $M$  symbols, the number of states is  $M$ . Based on this, VP-PPK modulation can be seen as the transfer of  $M$  states, as shown in Fig. 3.  $P_{ij}$  is the transfer probability among those states.

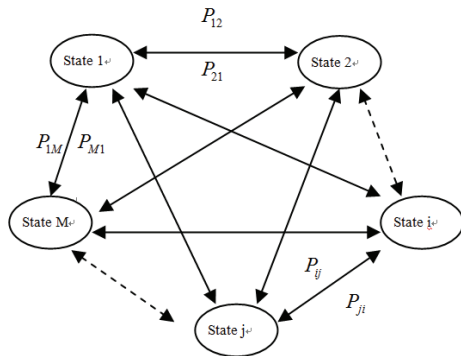


Fig. 3: State transfer of modulation proces

Since  $P_{ij}$  is the transfer probability among the symbols, define:

$$(2) \quad s_n(t) = g_i(t - nT_s), \quad i = 1 \sim M$$

Then, the modulated signal of VP-PPK can be represented as[6]:

$$(3) \quad s(t) = \sum_{n=-\infty}^{\infty} s_n(t)$$

The transmitted sequence is a Markov sequence, and the autocorrelation function of VP-PPK modulated waves is [7]:

$$(4) \quad R_s(\tau) = \frac{1}{T_s} \int_0^{T_s} R_s(t, t + \tau) dt = \frac{1}{T_s} \sum_{i=1}^M \sum_{j=1}^M \int_{-\infty}^{\infty} P_i s_i^*(u) [\sum_m a_{ij}^{(k)} s_j(u + \tau - kT_s)] du$$

Note:

$$(5) \quad R_s^{per}(\tau) = \frac{1}{T_s} \int_0^{T_s} E(s^*(t)) E(s(t + \tau)) dt = \frac{1}{T_s} \sum_{i=1}^M \sum_{j=1}^M \int_{-nT_s}^{(1-n)T_s} P_i s_i^*(u) [P_j s_j(u + \tau - kT_s)] du$$

which are the periodic components, because  $E(s(t)) = \sum_{n=-\infty}^{\infty} P_i s(t - nT_s)$  is the periodic function with period  $T_s$ . Therefore,  $E(s(t)) = \sum_k C_k e^{j2\pi kt/T_s}$ , where,

$$(6) \quad C_k = \frac{1}{T_s} \int_0^{T_s} E(s(t)) e^{j2\pi kt/T_s} dt = \frac{1}{T_s} \sum_{i=1}^M P_i S_i(\frac{k}{T_s})$$

Therefore:

(7)

$$R_s^{per}(\tau) = \frac{1}{T_s} \int_0^{T_s} \sum_k C_k^* e^{-j2\pi k t/T_s} \sum_l C_l e^{j2\pi l t/T_s} dt = \sum_m |C_m|^2 e^{j2\pi m \tau/T_s}$$

The Fourier transform of  $R_s^{per}(\tau)$  is:

$$G_{s_d}(f) = \frac{1}{T_s^2} \sum_m \left| \sum_{i=1}^M P_i S_i(\frac{m}{T_s}) \right|^2 \delta(f - \frac{m}{T_s})$$

Note that  $G_{s_c}(f)$  is the Fourier transform of

$R_s(\tau) - R_s^{per}(\tau)$ ; therefore there is

$$(8) \quad G_s(f) = \int_{-\infty}^{\infty} \frac{1}{T_s} \sum_{i=1}^M \sum_{j=1}^M \int_{-\infty}^{\infty} P_i s_i^*(u) [\sum_k (a_{ij}^{(k)} - P_j) s_j(u + \tau - kT_s)] du e^{-j2\pi f \tau} d\tau = \frac{1}{T_s} \sum_{i=1}^M \sum_{j=1}^M P_i S_i^*(f) S_j(f) [\sum_k (a_{ij}^{(k)} - P_j) e^{-j2\pi k/T_s}]$$

In summary:

(9)

$$G_s(f) = G_{s_d}(f) + G_{s_c}(f)$$

$$= \frac{1}{T_s} \sum_{i=1}^M \sum_{j=1}^M P_i S_i^*(f) S_j(f) [\sum_k (a_{ij}^{(k)} - P_j) e^{-j2\pi k/T_s}] + \frac{1}{T_s^2} \sum_m \left| \sum_{i=1}^M P_i S_i(\frac{m}{T_s}) \right|^2 \delta(f - \frac{m}{T_s})$$

because the input sequence  $\{a_{ij}^{(k)}\}$  is independent, that is:

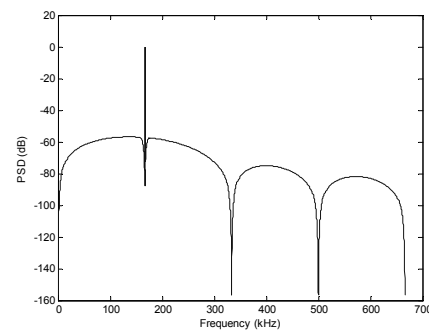
$$a_{ij}^{(k)} = \begin{cases} P_j, & k \neq 0 \\ \delta(i - j), & k = 0 \end{cases}$$

Then the PSD for the modulation signal is:

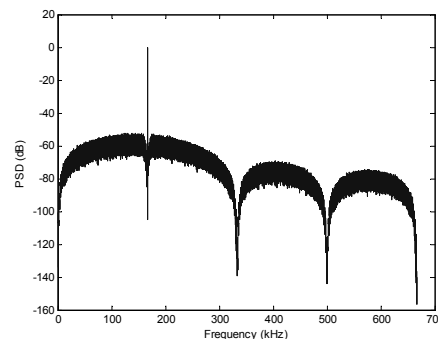
(10)

$$G_s(f) = \frac{1}{T_s} [\sum_{i=1}^M P_i |S_i(f)|^2 - |\sum_{i=1}^M P_i S_i(f)|^2] + \frac{1}{T_s^2} \sum_m \left| \sum_{i=1}^M P_i S_i(\frac{m}{T_s}) \right|^2 \delta(f - \frac{m}{T_s})$$

Take Fourier transforms of  $g_k(t)$  and substitute them into (11), PSD expression of VP-PPK can be achieved.



(a) Theoretical plot



(b) Simulation plot

Fig. 4. PSD for VP-PPK when  $f_c = 166.67 \text{ kHz}$ ,  $N = 16$ ,  $\theta = \pi$

If  $f_c = 166.67 \text{ kHz}$ , then  $T_c = 1/f_c$ ;  $T_s = 16T_c$  ( $N = 16$ );  $\theta = \pi$ . With the above parameters, the theoretical PSD for VP-PPK is plotted in Fig. 4(a), and the corresponding

simulation result is shown in Fig. 4(b), and both are consistent. Because of the introduction of random pulse positions, the implicit periodic components don't exist in the sideband anymore; this compresses the bandwidth of the 3PRK modulation shown in Fig. 2 efficiently. If  $M$  is regarded as random hops, then PSD expression for RPPK can be achieved[3].

When different values of  $N$ ,  $\theta$  are chosen, the corresponding PSD for VP-PPK is shown in Fig. 5. For Fig. 5 (a),  $N=16$ , the symbol period is fixed, and  $\theta$  ranges  $0 \sim \pi$ . It can be seen that the sideband power level is lowered little by little with the reduction of  $\theta$ . When  $\theta > \pi/2$ , the reduction speed of the sideband power level is lower with the reduction of angle  $\theta$ , while when  $\theta < \pi/2$ , the reduction speed of the sideband power level is higher with the reduction of angle, and when  $\theta = \pi/2$ , the sideband attenuation can satisfy the rigorous -60dB bandwidth requirement of FCC. The -60dB bandwidth is only about 0.210 kHz in such a case. When  $\theta$  takes different values, the sideband power level is,  $\theta = \pi$ , -57.1 dB;  $\theta = 3\pi/4$ , -58.0 dB;  $\theta = \pi/2$ , -61.0 dB; and  $\theta = \pi/4$ , -67.1dB respectively. For Fig. 5 (b),  $\theta = \pi$  is fixed, and  $N$  changes; with the increasing of  $N$  value to be large, the sideband power level lowers gradually. When  $N=32$ , the sideband attenuation can satisfy the rigorous -60dB bandwidth requirement of the FCC, the -60dB bandwidth is only about 0.112 kHz in such a case. When  $N$  takes different values, the sideband power levels are  $N=8$ , -52.5 dB;  $N=16$ , -56.7 dB;  $N=32$ , -60.5 dB; and  $N=64$ , -63.8dB respectively.

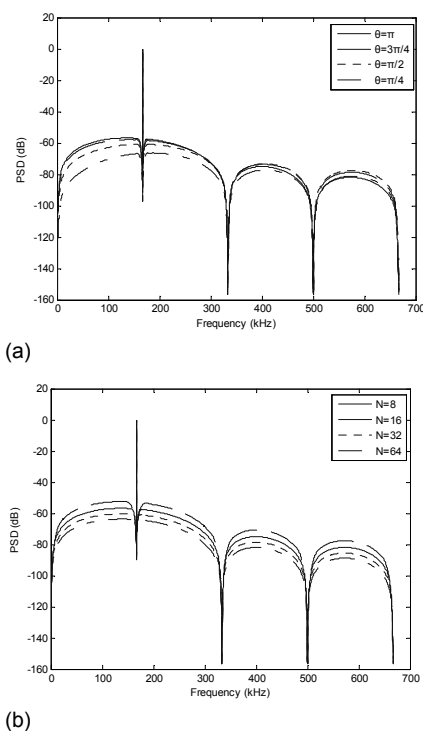


Fig. 5: (a) PSD characteristics with different  $\theta$  values ;(b) PSD characteristics with different  $N$  values

### Performance Analysis of the System

Fig. 6 is the principle of realization of VP-PPK modulation. In the transmitter, for the 16-VP-PPK case, the binary data stream is converted to a 4-bit code group by a serial-parallel converter; it is  $\log_2 N$  bit for the  $N$ -VP-PPK case. Then the 4-bit code group is converted to a value corresponding to 0~15, the value of  $K$ . Then, the value of

$\tau_1, \tau_2$  are calculated according to  $K$ . Further more, the pulse modulation position is determined in order to produce the RF modulated waves. The part in the dashed line completes the base band coding of VP-PPK, as shown in Fig. 1. Then the base band signal is modulated as BPSK modulation progress by the 'Modulator' in Fig. 6 (a).

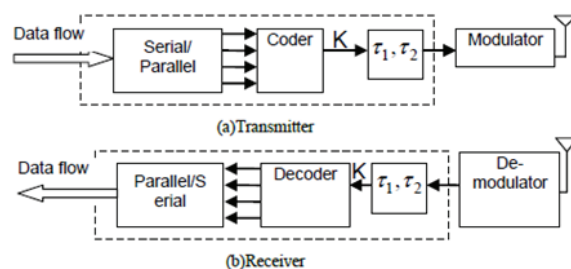


Fig. 6. System, principle

In the receiver, the received waves are first demodulated into a base band signal by the 'Demodulator' in Fig. 6 (b). After synchronization, the base band signal is then grouped according to the symbol period,  $\tau_1, \tau_2$  are determined, and  $K$  is computed. Then  $K$  is converted to 4-bit binary group. After passing parallel-serial converter, the sequential binary data stream is recovered. A relative simple method is a look-table method. The optimum receiver in the AWGN channel is used in the detection [8]. It is stated as the following:

Digital information is transmitted using  $M$  signal waveforms  $\{g_K(t), K=0,1,\dots,M-1\}$ ; during each symbol period  $0 \leq t \leq T_s$ , the received signal is represented as:

$$r(t) = g_K(t) + n(t), \quad 0 \leq t \leq T_s$$

$n(t)$  is the sample function of Gauss white noise with a PSD of  $N_0/2$ .  $\mathbf{r}$  represents the received signal vector, and  $\mathbf{g}_K$  ( $K=0,1,\dots,M-1$ ) represents the transmitted symbol vector. The optimum receiver decides which symbol is transmitted among  $\mathbf{g}_K$ , according to  $\mathbf{r}$ , to make the probability that the correct decision is the largest. That is:

$$P(\mathbf{g}_K | \mathbf{r}), \quad K = 0,1,\dots,M-1$$

is the largest, which is called the MAP principle. According to the Bayes principle,  $M$  symbols have equal probability, so the correlated measure is achieved:

$$C(\mathbf{r}, \mathbf{g}_K) = \int_0^T r(t) \mathbf{g}_K(t) dt - \frac{1}{2} E_K, \quad K = 0,1,\dots,M-1$$

$E_K$  is the symbol energy, and  $E_K$  s are equal for VP-PPK, so the optimum receiver for VP-PPK is:

$$C(\mathbf{r}, \mathbf{g}_K) = \int_0^T r(t) \mathbf{g}_K(t) dt, \quad K = 0,1,\dots,M-1$$

### Simulation

Parameters: the carrier frequency is  $f_c = 166.67 \text{ kHz}$ , the modulation is 16-VP-PPK, and the angle is  $\theta = \pi$ . Then the symbol rate is 10.42 kHz, and the binary data rate is 41.67 kHz. The filter given in [9] is used, whose 3-dB bandwidth is only 1.06 kHz (Fig. 7 (a)), the bandwidth efficiency of which can be 9.83sybol/s/Hz or 39.32bps/Hz. The corresponding Symbol Error Rate (SER) and Bit Error Rate (BER) are

shown in Fig. 7 (b). The principle of the filter is as that of Zero Group Delay (ZGD) filter presented in [1];

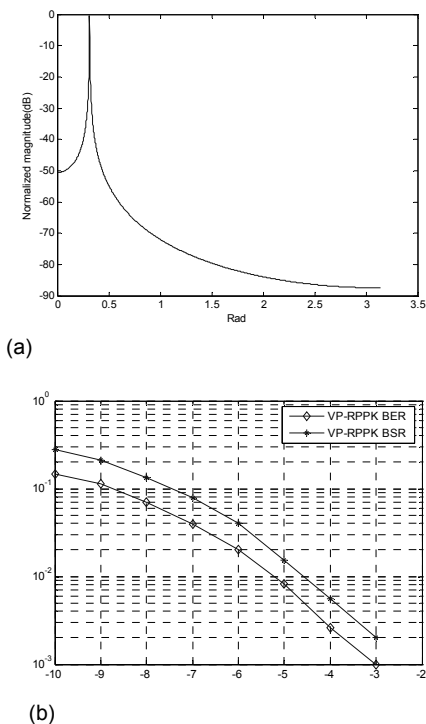


Fig. 7: (a) Filter magnitude characteristics[9]; (b) Bit error rate and symbol error rate

### Bandwidth Efficiency

There are several different definitions of bandwidth, such as the -55 dB bandwidth or the more rigorous -60 dB bandwidth required by FCC[10]. For VP-PPK modulation, the -60 dB bandwidth is used, which can satisfy more rigorous spectral mask requirement. At the same time, the definition of -55 dB is also used. When the modulation parameter  $N=16$ , and  $\theta$  takes different values, -55 dB and -60 dB bandwidth are shown in Table 1.

Table 1: Bandwidth when different  $\theta$  are taken

| $\theta$<br>Bandwidth(kHz)<br>Definition | $\pi$ | $3\pi/4$ | $\pi/2$ | $\pi/4$ |
|--|-------|----------|---------|---------|
| -55 dB                                   | 0.102 | 0.101    | 0.098   | 0.091   |
| -60 dB                                   | 155   | 135      | 0.106   | 0.099   |

As given in[3], when  $\theta = \pi$ , -55 dB bandwidth is about 0.10 kHz, because  $f_c = 166.67\text{kHz}$ , and the symbol rate is  $f_c / N = 166.67\text{kHz} / 16 = 10.4169\text{kHz}$ . Further, the bandwidth efficiency is 102.126 Symbol/s/Hz, and the bit bandwidth efficiency should be  $\log_2 N = \log_2 16 = 4$  times the symbol bandwidth efficiency. But the bandwidth efficiency is only about 100 bits/s/Hz using the RPPK method [3]. In order to

satisfy a more rigorous sideband attenuation requirement with the same modulation parameters, let  $\theta < \pi / 2$ ; the -60 dB sideband attenuation can be satisfied, as shown in Table 1, the bandwidth efficiency is still 98.272 Symbol/s/Hz and 105.221 Symbol/s/Hz, while the -60 dB bandwidth efficiency of RPPK in such a case is only about 0.0672 Symbol/s/Hz, that is, 0.2688 bit/s/Hz. Similarly, the case is that  $\theta = \pi$  is fixed and  $N$  changes.

### Conclusions

A more easily digitalized VP-PPK modulation format is proposed in this article. Following this, based on Markov discrete states, concise PSD expression for VP-PPK is achieved. In addition, PSD characteristics are given and analyzed at different parameters. In AWGN channels, VP-PPK shows good demodulation performance using the optimum receiver. Under the same conditions, the binary data rates of VP-PPK can be  $\log_2 N$  times that of RPPK, while the complexity is much less. 3-dB bandwidth efficiency of VP-PPK can reach 39.32bps/Hz, which is much higher than that of the traditional modulation methods. As a kind of new modulation method, it is hoped to be a potential candidate for communication in spectrum- and energy-limited case in addition to use in power line communication [1].

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