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Data processing algorithm for the Rayleigh fading channels

***Abstract**. The paper considers development of an adaptive algorithm for a measurement system in conditions of Rayleigh fading of signals in radio channels. The probability density functions (pdf) of time measurement variances are calculated. For these pdf an approximation of measurement noise distributions is proposed and an adaptive Kalman filter for processing transmitted data has been designed. The results of simulations of the proposed adaptive algorithm are presented.*

***Streszczenie.** W artykule zaproponowano adaptacyjny algorytm przetwarzania danych w systemach przesyłu informacji w warunkach fluktuacji amplitudy o charakterze Rayleigha. Zaproponowano wykorzystanie aproksymacji funkcji gęstości prawdopodobieństwa szumu pomiarowego oraz adaptacyjny filtr Kalmana. Przedstawiono wyniki badań symulacyjnych. (**Algorytm przetwarzania danych w systemach transmisji w kanale z fluktuacjami Rayleigha**)*

Keywords: Rayleigh fading channels, adaptive Kalman filtering.
Słowa kluczowe: adaptacyjny filtr Kalmana, fluktuacje o charakterze Rayleigha

The problem formulation

In measurement systems with using radio channels a pulse position modulation (PPM) is often used because of its simplicity and satisfactory performance characteristics [1]. In a process of propagation a signal amplitude can fluctuate so measurements of pulse time position (delay) have different signal-to-noise ratios (SNR). This can result in appearance of outliers and makes it difficult to design an optimal data processing algorithm. Usually as such an algorithm the Kalman filter is used.

For optimality of the Kalman filter it is necessary to know a current covariance matrix of a measurement noise $R(k)$ which is unknown due to amplitude fluctuations in radio channel. This uncertainty can be modelled using the following approach.

Let us consider a discrete-time stochastic equation which models an information process in the following form:

(1) $$x(k+1) = \Phi(k+1,k)x(k) + G_w(k)w(k)$$

where $x(k)$ is n dimensional state vector (information model), $\Phi(k+1,k)$ is the transition matrix, $w(k)$ is white Gaussian sequence with zero mean and covariance matrix $Q(k)$.

The measurement equation can be written as follows:

(2) $$y(k) = H(k)x(k) + v(k)$$

where $y(k)$ is s dimensional observation vector, $H(k)$ is the observation matrix, $v(k)$ is a zero mean noise with a covariance matrix $R(k)$ depending on the current value of SNR.

Under relatively high signal to noise ratio (SNR) the Cramer-Rao lower bound (CRLB) is widely used [2] because the CRLB can be obtained directly from the Fisher information matrix [3]. The time delay error variance for the single scalar measurement in this case can be written as follows:

(3) $$R(k) = \sigma_\tau^2 = \frac{1}{qF_e^2}$$

where q is the signal to noise ratio

(4) $$q = \frac{A^2}{2\sigma_n^2}$$

σ_n^2 is the noise variance at the output of a receiver, A is the signal amplitude and F_e is the signal effective bandwidth [4].

If the signal amplitude A has the Rayleigh distribution:

(5) $$f(A) = \frac{A}{\sigma_A^2} \cdot e^{-A^2/2\sigma_A^2}$$

then it can be shown that pdf of SNR q is of the following form:

(6) $$f(q) = \frac{\sigma_n^2}{\sigma_A^2} \cdot e^{-q\frac{\sigma_n^2}{\sigma_A^2}}$$

Using equation (3) and standard procedures of non-linear transformations for probability density functions [5] one can obtain a pdf for the variance of time delay measurement errors in the form:

(7) $$f(R) = \frac{1}{R^2 F_e^2 \sigma_A^2/\sigma_n^2} \cdot e^{-\frac{1}{RF_e^2\sigma_A^2/\sigma_n^2}}$$

The probability density functions of the variance $R(k)$ for different values $a = F_e^2\sigma_A^2/\sigma_n^2$ are presented in Fig. 1.

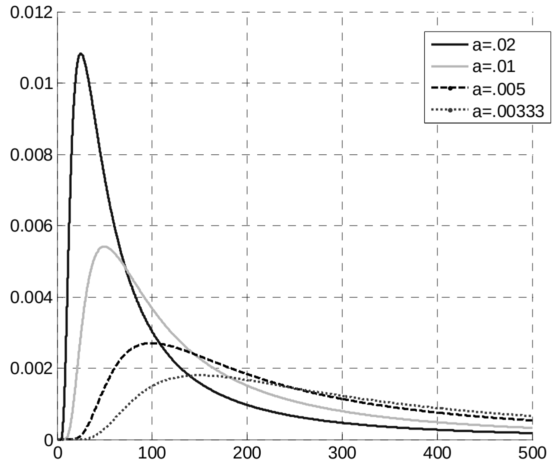

Fig. 1. Probability density functions $R(k)$

As it can be seen from the Fig. 1 the probability density functions of $R(k)$ have long “tails” which can result in appearance of anomalous errors (outliers).

A typical example of measurement noise realizations corresponding pdf presented in Figs. 1, 4 and 5 is given in Fig. 2.

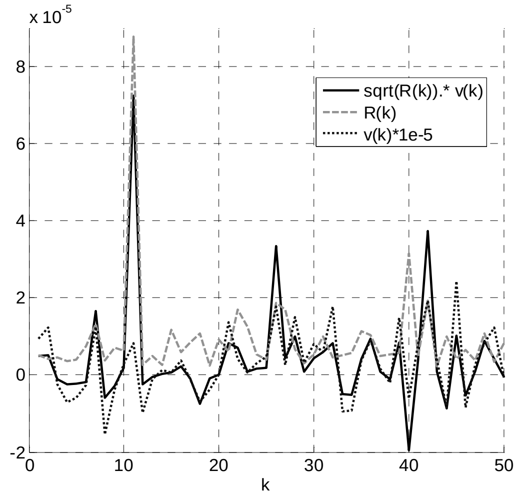

Fig. 2. Example of realization of $R(k)$, $v(k)$ and $sqrt(R(k)\bullet v(k))$

It can be seen the appearance of outliers at the time steps 11 and 40.

Probability that R/Rmean exceeds the given value is presented in Fig. 3.

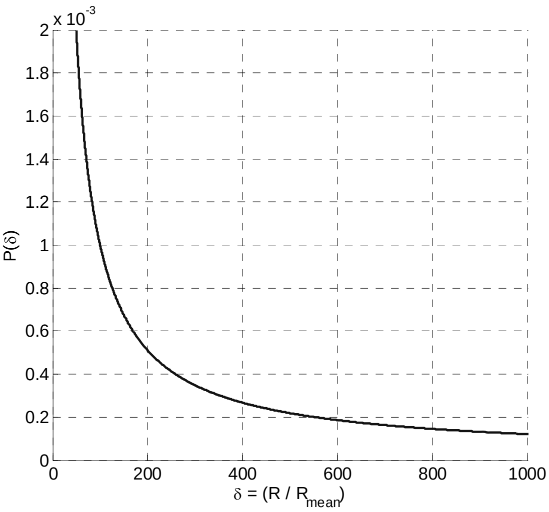

Fig. 3. Probability of exceeding by R/Rmean given value

The objective of the paper is designing a sub-optimal algorithm of data processing under conditions of fluctuating signals and presence of the outliers.

Main Results

The procedure of synthesis of the adaptive PPM processing algorithm in presence of fluctuating signals can be obtained using the nonlinear filtering approach [6]. The filter should take into consideration unknown changes of $R(k)$ sequence.

When the results of observations depend on noise with unknown $R(k)$ sequence it is necessary to use a general approach for calculating the system estimation. In this case the dynamic system state vector estimation can be found as a conditional mean of the following form [7]:

$$\hat{x}(k/k) = E[x(k)/Y_1^k] = \sum_{i \in 2^k} \hat{x}^i(k/k) P(\overline{R}_k^i / Y_1^k) \tag{8}$$

where: $Y_1^k = \{y(1), y(2), ... y(k)\}$ is the sequence of input data, $\overline{R}_k^i = \{R(1), R(2), ..., R(k)\}$ denotes the noise variances sequence and $\hat{x}^i(k/k) = E[x(k)/Y_1^k, \overline{R}_k^i]$ are the partial estimates of the state vector (1) that are calculated in correspondence with the equations of the Kalman filter.

In the measurement equation (2) a noise process can be considered as belonging to a set of Gaussian probability density functions of which variances have the pdf described by the equation (7). Because the pdf (7) is a continuous distribution, for practical purposes it is expedient to approximate it by a finite number of M Gaussian distributions.

In Figs. 4 and 5 there are presented the approximations of the function (7) for M=3 and M=10 respectively. For another values of M the approximations are presented in Appendix.

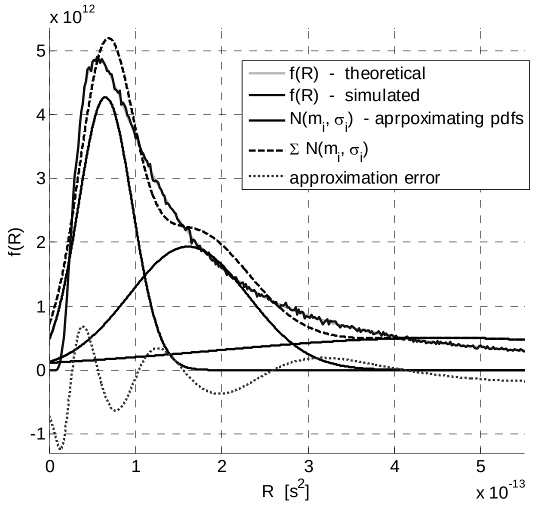

Fig. 4. Approximation of $f(R)$ for $M=3$

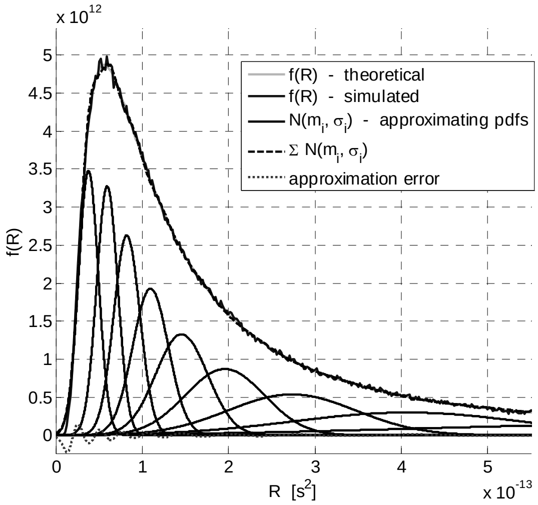

Fig. 5. Approximation of $f(R)$ for $M=10$

The probability density function of the estimates (8) can not be defined exactly because of infinitely growing memory for their calculations. That is why for calculating of the probability density function of $f(x(k)/Y_1^k)$ it is worthwhile to use the Gaussian approximation approach [7]. In such an approach the state vector estimates $\hat{x}(k/k)$ can be expressed as the weighted sum of N partial estimates

$\hat{x}^i(k/k)$ corresponding to the certain level of the SNR affecting noise variance in the current measurement. Each of these partial estimates can be calculated by prediction of the estimates obtained at the previous time steps.

The a posterior probability of the measurement channel state $p(j/k) = P[R(k) = R_j / Y_1^k]$ depends on the noise stochastic characteristics. In general case the Markov chain description should be used when there exists time correlation between consecutive SNR level. In uncorrelated case a posterior probabilities $p_{j/k}$ [5, 6] can be found with constant a priori probabilities $q_j(k)$ as following:

$$(9)\quad p_{j/k} = \frac{f(y(k)/R(k) = R_j,\ Y_1^{k-1})\ q_j(k)}{\sum_{i=1}^{N} f(y(k)/R(k) = R_i,\ Y_1^{k-1}) q_i(k)}\ ;\quad j = 1,\ldots,N$$

where

$$f(y(k)/R(k) = R_i, Y_1^{k-1}) =$$
$$N\{H(k)\hat{x}(k/k-1),\quad H(k)P(k/k-1)H^T(k) + R_i\},\quad i = 1,\ldots,N$$

denotes the Gaussian density function of the predicted estimates and $P(k/k\text{-}1)$ is the corresponding covariance matrix.

Finding a posteriori probabilities of measurement errors and incorporating that information in nonlinear filtering method should decrease estimation error.

Finally the suboptimal estimate $\hat{x}(k/k)$ can be calculated as following:

$$(10)\quad \begin{aligned}\hat{x}(k/k) &= \hat{x}(k/k-1) + \\ &+ \sum_{j=1}^{N} p(j/k) K_j(k)[y(k) - H(k)\hat{x}(k/k-1)]\end{aligned}$$

where the filter matrix gain have to be calculated with taking into consideration a posteriori probabilities of the observation channel state and corresponding partial gains $K_j(k)$:

$$(11)\quad \begin{aligned}K_j(k) &= P(k/k-1)H^T(k) \times \\ &\times [H(k)P(k/k-1)H^T(k) + R_j]^{-1}\end{aligned}$$

Covariance matrices of errors can be recursively computed in a following way:

$$(12)\quad \begin{aligned}P(k/k) &= P(k/k-1) - K_\Sigma(k)H(k)P(k/k-1) + \\ &+ \sum_{j=1}^{N} p(j/k)[K_j(k) - K_\Sigma(k)] \cdot S(k) \cdot [K_j(k) - K_\Sigma(k)]^T\end{aligned}$$

where

$$S(k) = z(k/k-1) \cdot z^T(k/k-1)\,,$$

and

$$(13)\quad \begin{aligned}&P(k/k-1) = \\ &= \Phi(k,k-1)P(k-1/k-1)\Phi^T(k,k-1) + Q(k-1)\end{aligned}$$

The gain matrix $K_\Sigma(k)$ is a resultant gain matrix:

$$(14)\quad K_\Sigma(k) = \sum_{i=1}^{N} p(j/k) K_j(k)$$

Simulation results

The performance of the proposed method was investigated by using 500 Monte Carlo runs. The first-order system was simulated. The noises in the observation channel (3) was assumed to be described by $R(k)$ depending on SNR with the signal effective bandwidth $Fe=0.1MHz$. Signal amplitude fluctuations was modeled with Rayleigh distribution resulting with pdf of $R(k)$ as presented in Fig. 4 and 5 and with appearance of outliers. The objective of the simulation was to evaluate the proposed method algorithm in comparison with traditional Kalman filter with average signal power (KFASP) and with optimal filter (OF). Estimation in the latter filter is carried out with full knowledge of the $R(k)$ sequence values. That is of course possible in simulations only, but enables to obtain the lower bound of the estimation accuracy.

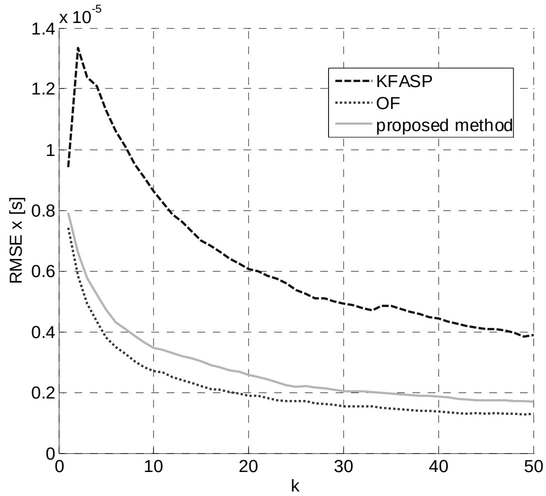

Fig. 6. The RMSE of data processing for the information process (SNR = 0 dB, M = 3)

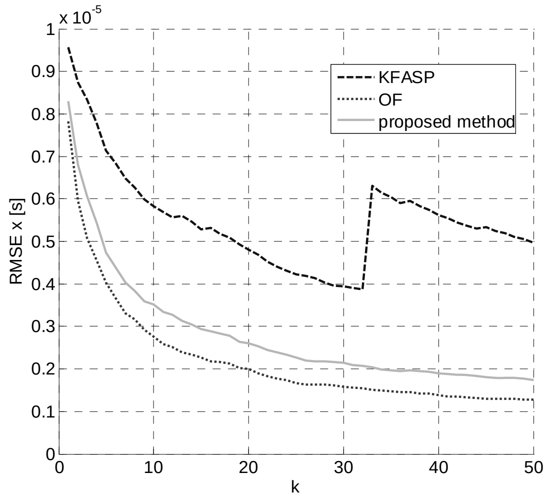

Fig. 7. The RMSE of data processing for the information process for the realization (SNR = 0 dB, M = 3)

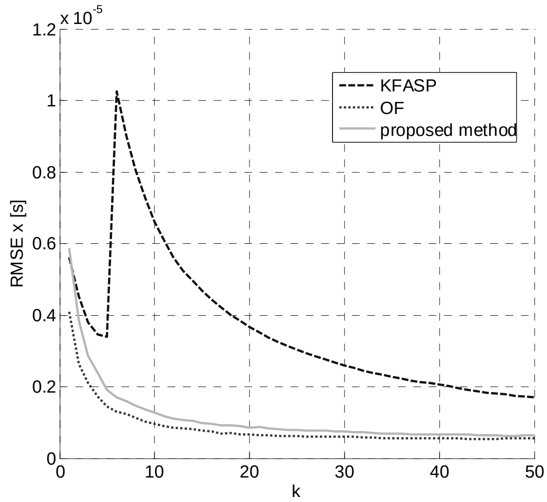

Fig. 8. The RMSE of data processing for the information process (SNR = 10 dB, M = 5)

As it can be seen from the figures 6-8 the proposed algorithm reveals a considerable performance improvement with respect to the usual Kalman filter which is designed for the mean value of the signal amplitude and practically is as optimal as in a case of exact knowledge of current amplitude. In a case of appearance of outliers the developed algorithm considerably improves performance characteristics of the data processing unit.

Conclusions

The paper considers development of an adaptive algorithm for telemetric measurement systems in conditions of Rayleigh fading in transmitting channel. The probability density functions of time-delay variances are calculated and approximation is made which make it possible to apply nonlinear filtering approach to the synthesis of the adaptive processing filter. This filter calculates current a posteriori probability of the measurement variance and uses it for estimation of the information process. The simulation results have revealed a high efficiency of the proposed algorithm and a high immunity with respect to the outliers.

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Appendix: Approximations of the probability density function of R(k) for various M

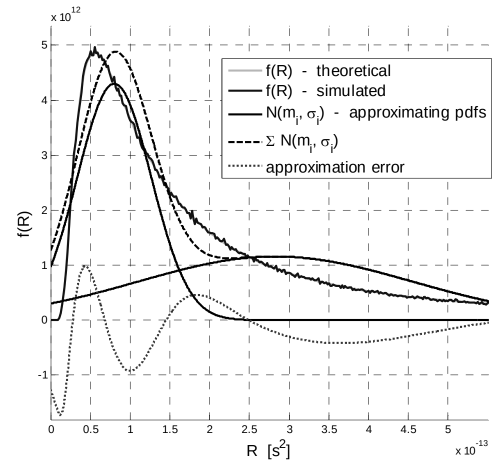

Fig. A1. Approximation of *f(R)* for *M=2*

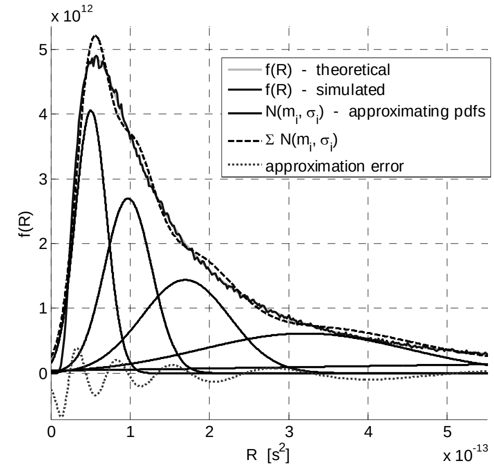

Fig. A2. Approximation of *f(R)* for *M=5*

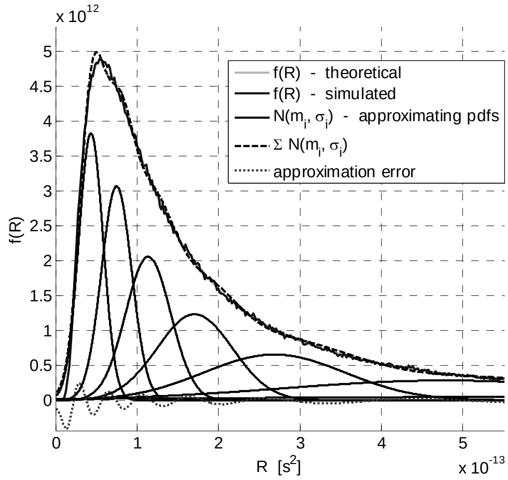

Fig. A3. Approximation of *f(R)* for *M=7*

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