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Hybrid Modeling and PID-PSO Control of Buck-Boost Chopper

***Abstract**. Due to its simplicity, low voltage stress, high reliability, low switch and inductor losses, and small inductor size, the Buck-Boost chopper has attracted a lot of attention in applications where it is necessary to step-up or step-down the DC voltage. In this paper a hybrid model of the Buck-Boost DC-DC converter using the PWA (piecewise affine) modeling framework is proposed, and then a PID controller is designed based on the PWA model. Finally, the particle swarm optimization (PSO) method is used to determine near optimum PID controller parameters. Designing the controller and analyzing the performance of the system based on the non-linear model are very difficult, so we used the PWA as an alternative solution. The proposed piecewise affine hybrid model lets decide about the control strategy and analyze the stability and performance of the closed loop control system using the classical control theory. Extensive simulations show the superiority of the PWA over the small signal linear model in prediction of the system behavior.*

***Streszczenie**. W artykule przedstawiono hybrydowy przekształtnik DC/DC typu Buck-Boost wykorzystujący modelowanie PWA) piecewise affine). Wykorzystano algorytm mrówkowy PSO do optymalizacji parametrów. (**Hybrydowe modelowanie i sterowanie PID-PSO przekształtnikiem Buck-Boost**)*

Keywords: Hybrid system, Buck-Boost chopper, piecewise affine (PWA) approximation, PID-PSO controller.
Słowa kluczowe: przekształtnik Buck-Boost, sterownik PID-PSO

Introduction

The theory of control includes controlling continuous systems. This theory is now combined with the digital computing science that comprises discrete models and the results of this combination are hybrid systems. As you can see in Fig. 1, hybrid systems are some kind of dynamic systems, in which the behavior of system is a combination of continuous dynamics, which is modeled with differential equations, and discrete dynamics, which is usually modeled with automatic machines. As a result, in recent years, the hybrid systems have attracted a lot of attentions [1].

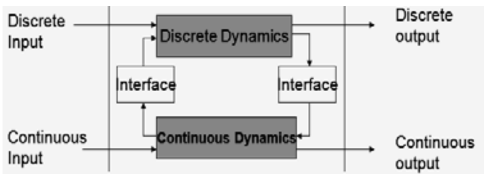

Fig.1. The structure of hybrid systems

Power electronic converters are the best choice for hybrid modeling and control; because the circuit parameters are continuous which lead to continuous dynamics and on the other hand, the switching behavior imposes discrete dynamics. Choppers are some kind of power electronic converters that are utilized in various industrial applications.

Choppers can be classified according to their control variable into two classes [2]. In the first class, the switching frequency is fixed and the system is controlled by changing the switches duty cycles through adjusting the active duration of switches in one switching period. In the second class, which includes the variable frequency converters such as resonant converters, the system is controlled by changing the switching frequency [3].

In this paper a new member of fixed switching frequency choppers, named Buck-Boost chopper is considered. This converter has found a lot of attentions in applications where it is necessary to step-up or step-down the DC voltage, because of its simplicity, low voltage stress, high reliability, low switch and inductor losses, and small inductor size. Equations that characterize these converters are non-linear.

Designing and analyzing controllers with non-linear equations are very complex. It is a common practice to use different linear approximation methods, because we want to use linear tools for the design and analysis.

PWA method is recently proposed for overcoming to such a problem [4, 5]. In this paper, PWA hybrid technique is employed to model the converter and decide about the control strategy and analyze the stability and performance of the closed loop control system using the classical control theory. In the following sections, at first, the Buck-Boost chopper is introduced. Then the PWA approximation method is defined and successfully applied to this converter. Next, a voltage loop PID controller is designed based on the PWA model. Finally, the particle swarm optimization (PSO) method is used to determine near optimum PID controller parameters. The effectiveness of this modeling and controller design method for the Buck-Boost chopper is verified by simulations.

Buck-Boost Chopper

This converter is a new member of choppers also known as DC to DC converters which can simultaneously increase or decrease the input DC voltage. The duty cycle is modified according to the difference between the measured and reference values of the output voltage [6].

Fig.2 shows the Buck-Boost chopper together with the control system.

The averaging state-space method is a generalized analysis tool which is readily applicable to either simple circuits or complex structures in order to achieve a continuous model. For the converter of Fig.2, there are two distinct switching states as follows [6]:

1. S conducts ($0 < t < dT$):

$$a:\begin{cases}\dfrac{dV_C}{dt}=\dfrac{1}{C}[0-\dfrac{V_C}{R}]\\[2mm]\dfrac{di_L}{dt}=\dfrac{1}{L}[V_{in}]\end{cases}\quad\Rightarrow \tag{1}$$

$$\begin{pmatrix}\dfrac{dV_C}{dt}\\[2mm]\dfrac{di_L}{dt}\end{pmatrix}=\begin{pmatrix}\dfrac{1}{RC} & 0\\ 0 & 0\end{pmatrix}\begin{pmatrix}V_C\\ i_L\end{pmatrix}+\begin{pmatrix}0\\ \dfrac{1}{L}\end{pmatrix}V_{in} \tag{2}$$

2. S is off state ($dT < t < T$):

(3) $$b: \begin{cases} \dfrac{dV_C}{dt} = \dfrac{1}{C}[i_L - \dfrac{V_C}{R}] \\ \dfrac{di_L}{dt} = \dfrac{1}{L}[-V_{in}] \end{cases} \Rightarrow$$

(4) $$\begin{pmatrix} \dfrac{dV_C}{dt} \\ \dfrac{di_L}{dt} \end{pmatrix} = \begin{pmatrix} \dfrac{-1}{RC} & \dfrac{1}{C} \\ \dfrac{-1}{L} & 0 \end{pmatrix} \begin{pmatrix} V_C \\ i_L \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \end{pmatrix} V_{in}$$

The state-space averaging technique is a generalized analysis tool which is readily applicable to either simple circuits or complex structures in order to obtain a continuous model. If the switching frequency is high enough, then these distinct models can be averaged over a switching period to compose an approximate but continuous model of the converter which represents the average behavior of it over a switching period:

(5) $$\begin{pmatrix} \dfrac{dV_C}{dt} \\ \dfrac{di_L}{dt} \end{pmatrix} = \begin{pmatrix} \dfrac{-1}{RC} & \dfrac{1}{C} \\ \dfrac{-1}{L} & 0 \end{pmatrix} \begin{pmatrix} V_C \\ i_L \end{pmatrix} + \begin{pmatrix} \dfrac{-i_L}{C} \\ \dfrac{V_C + V_{in}}{L} \end{pmatrix} d$$

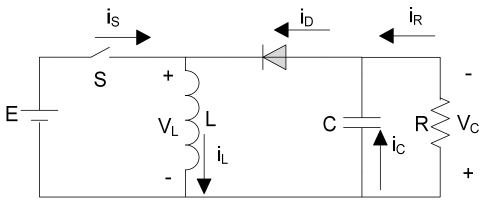

Fig.2. The Buck-Boost converter

Piecewise Affine Approximation

The class of nonlinear systems considered in this work is described by:

(6) $$\begin{bmatrix} \dot{x} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} A_{xx} & A_{xz} \\ A_{zx} & A_{zz} \end{bmatrix} \begin{bmatrix} x \\ z \end{bmatrix} + \begin{bmatrix} b_x \\ f(x,z) \end{bmatrix} + B(x,z)u$$
$$y = C \begin{bmatrix} x \\ z \end{bmatrix}$$

where, $b_x \in R^{n_x}$ is a constant vector, $B = \left[B_x^T B_z^T \right]^T$, $x \in R^{n_x}$ contains the state variables, $z \in R^{n_z}$ contains the state variables associated with the nonlinear dynamics and $u \in R^{n_u}$ is the control input. All the matrices are assumed to have the appropriate dimensions [7, 8]. In the case where there is no variable with affine dynamics, the description of the system is:

(7) $$\dot{z} = f(z) + B_z(z)u$$

The polytopic cells are denoted by $R_{i,} i \in I$, where I is a finite index set [7].

Each cell is constructed as the intersection of a finite number pi of half spaces given by the following inequalities [8]:

(8) $$R_i = \left\{ x \left| h_{ij}^T x < g_{ij}, j = 1,...,p_i \right. \right\}, H_i^T x - g_i < 0$$

Where $H_i = \left[h_{i1} h_{i2} ... h_{ip_i} \right]$ and $g_i = \left[g_{i1} g_{i2} ... g_{ip_i} \right]$.

Each polytopic cell has a finite number of facets and vertices. Any two cells sharing a common facet will be called neighboring cells. A parametric description of the boundaries can then be obtained as:

(9) $$\overline{R_i} \cap \overline{R_j} \subseteq \left\{ l_{ij} + F_{ij} q \left| q \in IR^{n_x + n_z - 1} \right. \right\}, \ i = 1,...,G, \ j = 1,...,N_i$$

where, G is the cardinality of this set and N_i is neighboring cells of i and $F_{ij} \in R^{(n_x+n_z) \times (n_x+n_z-1)}$, $l_{ij} \in R^{(n_x+n_z)}$. The objective is to find the matrix A_i and the vector b_i. At each vertex of each simplex i, a linear equation of the form $f(\alpha)^T = [\alpha^T|1]\theta$ can be written, where θ is defined by

(10) $$\theta = \left[A_i^T \middle| b_i^T \right]$$

and α represents the n×1 vector with the coordinates of the vertex. If all the values $f(\alpha)$ are stacked in a matrix F and all the rows $[\alpha^T|1]$ are in a matrix X then the solution is given by:

(11) $$\theta = X^{-1} F$$

At this point, the matrix B_i should be found for each cell i. the point chosen for evaluating this function is Chebychev center $\omega^i_{(cheb)}$ for each cell i, that is the center of an Euclidean ball with the maximum radius that can be fit inside the polytopic cell, using the following approximation:

(12) $$B_i = B(\omega^i_{cheb})$$

Model Derivation

The proper shape and number of cells are determined according to [7, 8, 9]. Based on explanations in the previous section, equation (5) is rewritten as follows

(13) $$\begin{pmatrix} \dfrac{dV_C}{dt} \\ \dfrac{di_L}{dt} \end{pmatrix} = \begin{pmatrix} \dfrac{-1}{RC} & \dfrac{1}{C} \\ \dfrac{-1}{L} & 0 \end{pmatrix} \begin{pmatrix} V_C \\ i_L \end{pmatrix} + \begin{pmatrix} 0 & \dfrac{-i_{L_{cheb}}}{C} \\ \dfrac{V_{in_{cheb}}}{L} & \dfrac{V_{C_{cheb}}}{L} \end{pmatrix} \begin{pmatrix} d_1 \\ d_2 \end{pmatrix}$$

where, variables with a *"cheb"* subtitle are calculated from Chebychev center of each cell. The variations of the duty cycles d_1 and d_2 are not independent, so all the results are plotted versus d, which is defined in equation (14).

(14) $$d = d_1 = d_2 + \alpha = 0.51 + 0.25\sin(200\pi t)$$

Each cell is defined according to the following equations

(15) $$R_i = \left\{ z \left| H_i^T x - g_i < 0 \right. \right\}, \ i = 1,...,M$$

where, $H_i = \begin{bmatrix} -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix}$

and, $g_i = \left[-x_1^-(R_i) \ \ x_1^+(R_i) \ \ -x_2^-(R_i) \ \ x_2^+(R_i) \right]^T$

and $x_2^-(R_i)$ and $x_1^-(R_i)$ represent the lower bounds and $x_1^+(R_i)$ and $x_2^+(R_i)$ represent the upper bounds of x_1 and x_2 in the cell R_i respectively.

PID-PSO Controller Design

To arrive at a PWA approximation of the converter, the state space has been divided into 10 cells. The proposed method based on the procedure described in the previous sections is depicted in Fig. 3, where, k_M is the M^{th} linear controller that should be designed. The proper switching between different controllers is governed by the state *x(t)*. Existing control techniques for power electronic converters mainly rely on PID controllers.

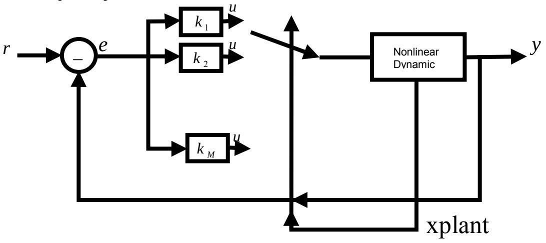

Fig. 3. The closed loop control procedure for nonlinear system with PID controller

Because of its simplicity and robustness, a proportional–integral–derivative controller (PID controller) is widely used for feedback controller systems. A block diagram of a PID controller is shown in Fig. 4. A PID controller calculates an "error" value as the difference between a measured process variable and a desired set point. The controller attempts to minimize the error by adjusting the process control inputs [11, 12].

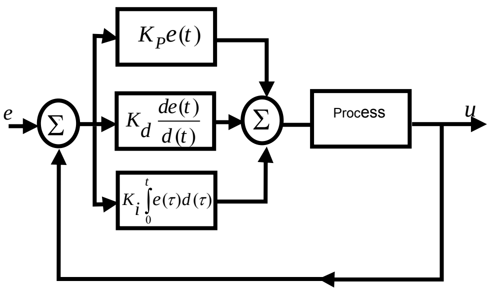

Fig 4. The block diagram of a control system with a PID controller

Several optimization-based methods to obtain the proper PID controller parameters are available [13]. The main drawbacks associated to these techniques include: 1) the performance is very dependent on the starting point, and 2) the non-ability in the case of complex systems. Particle swarm optimization (PSO), as an advanced and intelligent optimization algorithm, has been considered as a successful solution to determine the coefficients of PID controllers [14].This technique is easy to implement and can find the global optimums with a lower computational burden and more stable convergence characteristic than other methods such as genetic algorithms (GA).

In this paper, the PSO method is used to determine the optimum PID controller parameters in terms of the following time-domain cost function:

$$\begin{aligned} J = (C\,(&\text{steady-state error} - \text{steady-state error}_{optimum})\,^\wedge 2 + \\ &(\text{overshoot} - \text{overshoot}_{optimum})\,^\wedge 2 + \\ &(\text{rise time} - \text{rise time}_{optimum})\,^\wedge 2 + \\ &(\text{settling time} - \text{settling time}_{optimum})\,^\wedge 2) \end{aligned} \tag{16}$$

where $C = 10$. The above criterion tries to minimize the overshoot, rise time, settling time, and steady-state error, simultaneously. Due to the importance of the steady-state error compared to other constraints, a weighting coefficient (C) is used for it. Also through several simulations, it was concluded that three other terms create numbers close together, so they are weighted equally. PSO optimization problem is formulated for each cell, and consequently the PID controllers are designed for each cell separately.

Table 1. The parameters of the non-inverting Buck-Boost converter

Output Voltage	10V
Load	R=5Ω
Filter capacitance	C=47 µF
Inductor	L=0.1 mH
Input voltage	Vin=10V
Switching frequency	f_s=100kHz

Table 2. The partitions used in derivation of PWA

Parameter	Min	Max	Step
Inductor current(A)	0	40	20
Output voltage(V)	0	20	4

Table 3. Controllers designed for each cell

Cell	*Controller*
1	$\frac{7.28S^2+48.57S+1005.47}{S}$
2	$\frac{3.88S^2+91.27S+106.64}{S}$
3	$\frac{1.88S^2+54S+118}{S}$
4	$\frac{30.9S^2+27.05S+76.04}{S}$
5	$\frac{0.95S^2+12.33S+93.25}{S}$
6	$\frac{77.68S^2+64.99S+352.77}{S}$
7	$\frac{79.29S^2+74.9S+39.58}{S}$
8	$\frac{28.5S^2+58.62S+48.41}{S}$
9	$\frac{9.96S^2+114.9S+34.71}{S}$
10	$\frac{24.45S^2+37.47S+37.93}{S}$

Table 4. Performance characteristics of the PID-PSO controller

Rise Time[Sec]	0.002
Overshoot [%]	0.000
Steady-State Ripple %]	0.120
Settling Time [Sec]	0.6
Steady-State Error [%]	0.19

Proper tuning of the PID controller using the PSO brings the following results. The transient performance is improved due to the stable zero and increased phase margin of the closed-loop system. Also, the steady-state characteristic, regarding the type of the system is enhanced, causing the step signals can be tracked with zero steady state error. The optimum zero and pole placement through minimizing the cost function, equation (16), has an important role in improving the stability margins. The combination of the PID controller, as a robust technique, with an intelligent method results in a robust and optimal control solution with increased stability margins, which significantly improves the stability of the closed loop system.

Simulation results

The parameters used for simulations are summarized in Table 1. In such a two dimensional space, cell dimensions are chosen according to Table 2. The validity and accuracy of each approximation is examined through simulations. For the Buck-Boost chopper under study the optimum local controller for each cell is obtained as shown in Table 3. The performance of the nonlinear and the small signal linearized as well as the PWA approximation models are compared in Figs. 5 and 6. The simulation results show how the PWA model pricisely matches the nonlinear model. On the other hand, it is evident that the small-signal linear model is unable to predict the system performance under large variations. Simulated result of the PID-PSO controller is shown in Fig. 7and Table 4.Since the system is non-minimum phase the waveform has some undershoots as expected. It is evident that the PID-PSO provides satisfactory performance and possesses good robustness (no overshoot, minimum rise time, minimum settling time, and minimum steady state error).

The steady state output voltage for the PID-PSO controlled converter system at the nominal load is shown in Figs. 8 and 9. This simulation results show that output voltage with the controller has a low distortion even in present of disturbances.

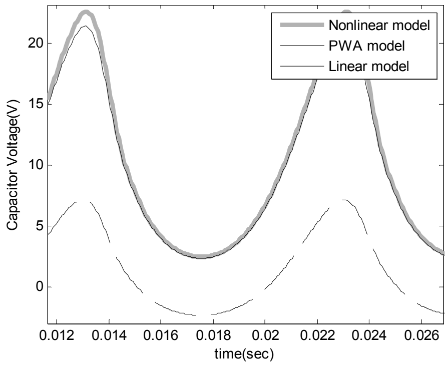

Fig. 5. Comparison of the output voltage waveform of nonlinear, PWA and linear models

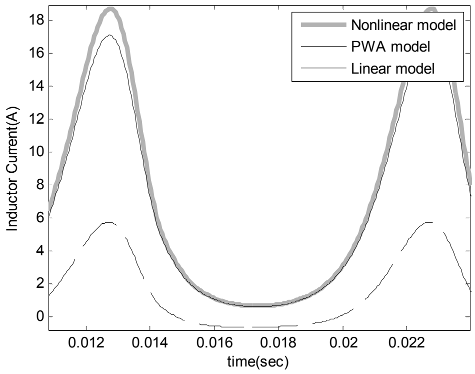

Fig.6. Comparison of the inductor current waveform of nonlinear, PWA and linear models

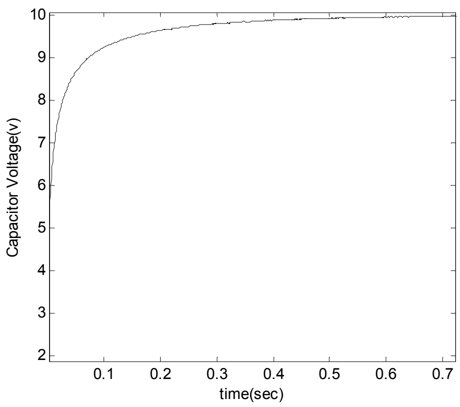

Fig.7. Output voltage at start-up

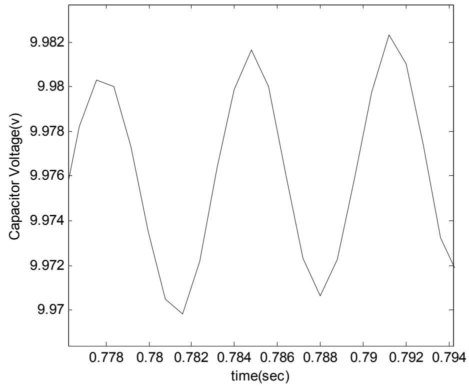

Fig.8. Steady state output voltage

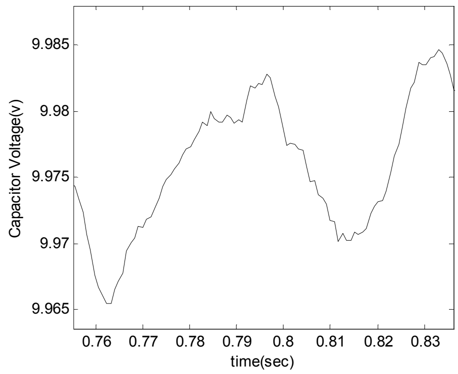

Fig.9. Steady state output voltage in response to highly disturbed input voltage (10% white noise and 10% sixth order harmonic).

Conclusion

Simulation results confirm the validity and precision of the PWA model to estimate the system's real behavior. On the other hand, it was shown that the small-signal linear model is unable to predict the system performance under large variations. The main application of the presented PWA approximation is in controller design procedure, which brings about the possibility of using linear control methods. This technique is successfully adapted to one of the new power converter topologies, namely Buck-Boost chopper. The PWA model of the converter was utilized to design the PID-PSO controller and simulations were carried out to confirm the validity of the proposed scheme. The results show that the designed control method ensures good performance and that it guarantees a stable operation under ill conditions. Although this control method was investigated on a Buck-Boost converter, it can also be applied to other converters, such as Buck, Boost, Buck-Boost, Cuk and SEPIC.

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