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Parametric curves in the specification of windings of applicators used in magnetotherapy

***Abstract**. Parametric description is a method of specification of geometrical figures, and is based on presenting the function that assigns points of the domain (a set of parameters) to points of space which the figure belongs to. The paper presents windings of frequently used applicators (magnetic field generators which can affect a part of the human body) by means of parametric curves. The presented method facilitates the approximation of path of the curve integral in the numerical calculation of magnetic field induction. Another benefit that cannot be overestimated, is the possibility to make any transformation of the shape, rotation and position, so as to adjust the position of the applicator to prescribed requirements.*

Streszczenie.** Opis parametryczny jest jedną z metod określania figur geometrycznych – polega na wprowadzeniu odwzorowania, które punktom dziedziny (zbioru parametrów) przyporządkowuje punkty przestrzeni, w której leży rozpatrywana figura. W artykule przedstawiono opis uzwojeń najczęściej stosowanych aplikatorów (generatorów pola magnetycznego, które może oddziaływać na fragment ciała ludzkiego) za pomocą krzywych parametrycznych. Prezentowany sposób ułatwia aproksymację drogi całkowania podczas numerycznego obliczania indukcji pola magnetycznego. Kolejną, nie do przecenienia korzyścią, jest możliwość dokonywania dowolnej transformacji kształtu, obrotów i pozycjonowania, tak aby dostosowywać położenie aplikatora do narzuconych wymagań. **(Krzywe parametryczne w opisie uzwojeń aplikatorów stosowanych w magnetoterapii).

Keywords: magnetic field, magnetotherapy, curve modelling, parametric curves, applicators.
Słowa kluczowe: pole magnetyczne, magnetoterapia, modelowanie krzywych, krzywe parametryczne, aplikatory.

Introduction

The exact calculation of the intensity of the magnetic field induced by applicators (magnetic field generators) used in magnetotherapy is necessary for several reasons. The first is the calculation of the distribution close to the windings (in some models it is equivalent to its distribution inside of the applicator). This type of distribution is associated with the therapeutic process. Precisely calculated magnetic field distribution is also required in solving problems related to the induction of eddy currents in magnetic field therapy.

We should also mention field distribution in the surroundings of an applicator – within the distance ranging from several centimeters to several meters – when determining the safety zones in the context of legal standards and labour regulations [1].

With the geometry of winding in space at one's disposal, any shape transformation (scaling) and rotation so as to adjust the position of the applicator to the adopted requirements can be performed. This saves the necessity of modeling the winding course in 3D space, in programs for calculation of field issues. Application of parametric curves is not limited to medical equipment and can be used in any problems related to the magnetic field and eddy currents, e.g. the impact of a field near power lines. The authors focused on the magnetotheraphy, because of the growing interest in this field of science both in the literature, as well as in industry, which can be seen in the growing market for physiotherapy services.

Applicators used in magnetotherapy

Kits used in magnetotherapy consist of magnetotherapy applicators and drivers. Applicators, devices used ine the treatment with the magnetic field – magnetic therapy – differ both in dimensions, maximal values of the magnetic field induced by a particular device (from a few μT up to 3 T in the case of Transcranial Magnetic Stimulation in treating depression [2]), and in the internal structure – the arrangement of windings conducting the current. External dimensions of applicators vary from a dozen to several hundred millimeters.

In addition, applicators have to be divided (which is most important in the description of the windings) in terms of possibility of shaping their geometry. The first group are applicators whose structure is fixed: so called solenoid applicators (fig. 1a), and e.g. applicators used in the treatment of depression – elliptical applicators (fig. 1b) and so-called eight-coils (fig. 1c) [3,4]. The casing of such applicators (usually plastic) does not allow any changes in their shape.

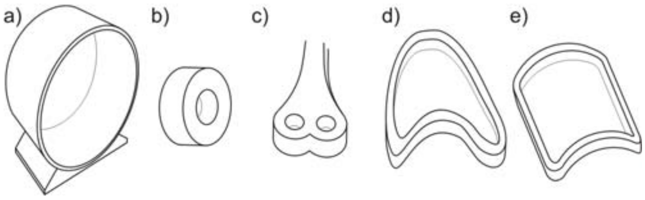

Fig.1. Applicators – (a) solenoid, (b) elliptical, (c) eight-coil, (d) portable, specified with the ellipse, (e) portable, with the rounded corners

The second group consists of the portable applicators, with the possibility of adjusting their shape to the size of limbs or trunk. They may be built built in the form of flexible hoops (fig.1d, 1e) [5,6] – and their geometrical representation is an ellipse wound around a side of cylinder.

The representation of the geometry of the applicator as an ellipse makes it possible to regulate the size of two semi-axes (of the ellipse), while maintaining a constant length of the coil.

Calculation of the magnetic field distribution

With the assumption, that in the analyzed area $\mu_r = \mu_0$, the external field can be calculated in any place outside the source area, by numerical integration of the following formula [7]:

$$\overline{B} = \frac{\mu_0}{4\pi} \int_V \frac{\overline{j} \times \overline{r}}{r^3} dV \tag{1}$$

The use of parametric curves is helpful in calculating the line integral above. The article emphasizes the approximation of course of the applicator's winding and therefore the line integral.

Parametric curves

Parametric description is a method of specification of geometrical figures, and is based on introduction of the function that assigns points of domain (set of parameters) to points of space, which this figure (in the paper – curve) belongs to [8]. The description of a curve in 3D space, urges to specify three functions, that defines coordinates of this curve:

(2) $$\bar{r}(t)=[x(t) \quad y(t) \quad z(t)]^T$$

In the case of applicators with adjustible shape, the formula (2) is obtained in two stages. The first stage requires the presentation of the parametric description of the windings on the plane (2D patterns in space, such as dependencies: 5 and 7). The second step is to present a parametric curve in 3D space, assuming that the applicator is applied to the side surface of a cylinder with a specific radius of the base. The cylinder is an equivalent of a limb or trunk of a specific size.

Parametric curve may be divided into sections of suitable length so that the numerical integration (necessary when using the Biot-Savart law) could be performed with accuracy required in a particular issue, minding the efficiency of the hardware on which the calculations are run. The Biot-Savart law is used to determine the magnetic field distribution of the applicator.

Solenoid and elliptical applicators

The solenoid applicators are the most common type, which is used in a wide range of diseases, particularly in facilitating and accelerating the treatment of orthopedic injuries. The applicator is depicted as a wire wound on a side of a cylinder.

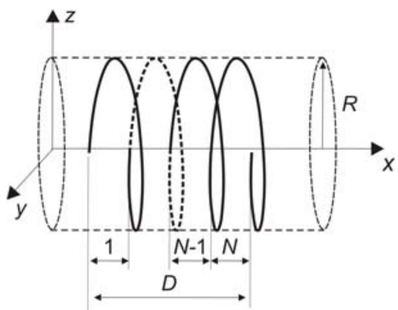

Fig.2. Solenoid applicators and characteristic parameters for description using parametric curves

The description needs a few parameters presented in figure 2, included in formula 3.

3D parametric equation is given in following formula:

(3) $$\bar{r}(t)=\begin{bmatrix} x(t) \\ y(t) \\ z(t) \end{bmatrix}=\begin{bmatrix} t \\ R\cos\frac{2\pi Nt}{D} \\ R\sin\frac{2\pi Nt}{D} \end{bmatrix}$$

where: $t \in \langle 0;D \rangle$ – set of domain points – parameter of value within a specific range, R – radius of the base of the cylinder around which the coil is wound, N – number of turns round the cylinder, D – height of the cylinder coated with windings (range of the windings).

An extended description of geometry is required in the case of the so-called elliptical coil [9], where the D-parameter (winding range) reaches approximately few centimeters, but there are many layers of windings used. Description is given by the equation with two parameters:

(4) $$\bar{r}(R,t)=\sum_{i=1}^{K}\begin{bmatrix} t \\ R_i\cos\frac{2\pi Nt}{D} \\ R_i\sin\frac{2\pi Nt}{D} \end{bmatrix}$$

where: $R=\{R_1,R_2,...,R_K\}$ – set of radii of consecutive cylinders around which are wound individual layers of the coil.

The eight coil (fig.1c) used in TMS (Transcranial Magnetic Stimulation) may be represented as two above-mentioned elliptical coils.

Flexible applicators

Within this group of applicators there is a possibility of description of two types of windings: ellipse imposed on the cylinder (fig. 3) [10] and the so-called rounded rectangle (rys.3b). In both cases, a planar description of the geometry of the applicators is required.

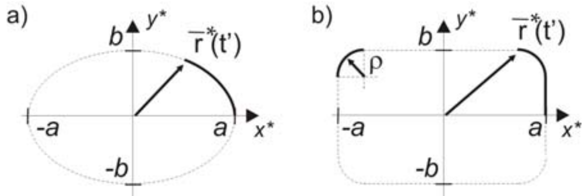

Fig.3. Planar model of two typed of flexible applicators (a) ellipse, (b) rounded rectangle

In case of the ellipse, 2D description is succinct:

(5) $$\bar{r}^*(t)=\begin{bmatrix} x^*(t) \\ y^*(t) \end{bmatrix}=\begin{bmatrix} a\cdot\cos t \\ b\cdot\sin t \end{bmatrix}$$

where:

a, b – semiaxes of ellipse, $t \in \langle 0;2\pi \rangle$.

Because orthopedic injuries occupy an important place in the magnetotherapy [11], a description of the windings in the 3D space must take into account the need to position of the applicator around the limb whose the geometry approximates a cylinder.

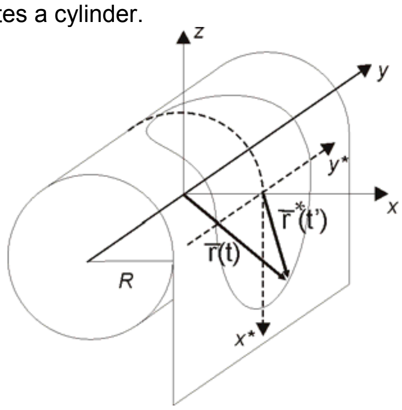

Fig.4. The surface containing an ellipse, wrapped around the side of a cylinder

After wrapping the parametric curve (5) around the cylinder (as shown in figure 4), the parametric curve is represented with the following relationship:

(6)
$$\bar{r}(t)=\begin{bmatrix}x(t)\\ y(t)\\ z(t)\end{bmatrix}=\begin{bmatrix}R\cdot\cos\left(\frac{\pi}{2}-\frac{a\cdot\cos t}{R}\right)\\ b\cdot\sin t\\ R\cdot\sin\left(\frac{\pi}{2}-\frac{a\cdot\cos t}{R}\right)\end{bmatrix}$$

where:

R – radius of the base of the cylinder, around which the winding is wrapped. Other parameters as in the planar model (relationship 5).

In Figure 5, the distribution of magnetic field induction on the surface of the bone and the geometry of the applicator (the curve from the formula 6) are shown.

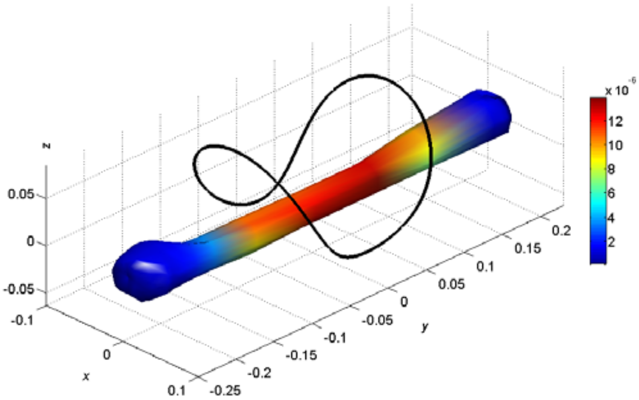

Fig.5. An example of the distribution of magnetic field induced by a flexible applicator. Applicator parameters: *R*=0,08 m, *a*=0,65π*R*, *b*=0,1 m.

The rounded rectangle in 2D space (fig.3b):

(7)
$$\bar{r}^*(t))=\begin{bmatrix}x^*(t)\\ y^*(t)\end{bmatrix}=\begin{bmatrix}\operatorname{sgn}(\cos t)\cdot a+\rho(\cos t-\operatorname{sgn}(\cos t))\\ \operatorname{sgn}(\sin t)\cdot b+\rho(\sin t-\operatorname{sgn}(\sin t))\end{bmatrix}$$

where:

$$t\in\left(0;\frac{\pi}{2}\right)\cup\left(\frac{\pi}{2};\pi\right)\cup\left(\pi;\frac{3\pi}{4}\right)\cup\left(\frac{3\pi}{4};2\pi\right),$$

2(*a*+ρ), 2(*b*+ρ) – rectangle extent, ρ – radius of the rounded corners, sgn – signum function

To obtain a description of the rounded corners (with a radius of ρ) after placing the applicator on a side of a cylinder (with the radius equal to *R*), the following parametric curve is to be used:

(8)
$$\bar{r}(t)=\begin{bmatrix}x(t)\\ y(t)\\ z(t)\end{bmatrix}=\begin{bmatrix}R\cos\left(\operatorname{sgn}\left(\sin\left(t-\frac{\pi}{2}\right)\right)\cdot\frac{(a-\rho)}{R}+\frac{\pi}{2}-\frac{\rho}{R}\cos t\right)\\ \rho\sin(t)+\operatorname{sgn}(\sin(t))\cdot(b-\rho)\\ R\sin\left(\operatorname{sgn}\left(\sin\left(t-\frac{\pi}{2}\right)\right)\cdot\frac{(a-\rho)}{R}+\frac{\pi}{2}-\frac{\rho}{R}\cos t\right)\end{bmatrix}$$

where:

$$t\in\left(0;\frac{\pi}{2}\right)\cup\left(\frac{\pi}{2};\pi\right)\cup\left(\pi;\frac{3\pi}{4}\right)\cup\left(\frac{3\pi}{4};2\pi\right)$$

The t-parameter takes values belonging to the four open intervals (each interval is used to describe one of the corners). Individual corners are connected by curves situated on the side of the cylinder (sample distribution and geometry of the applicator on figure 6).

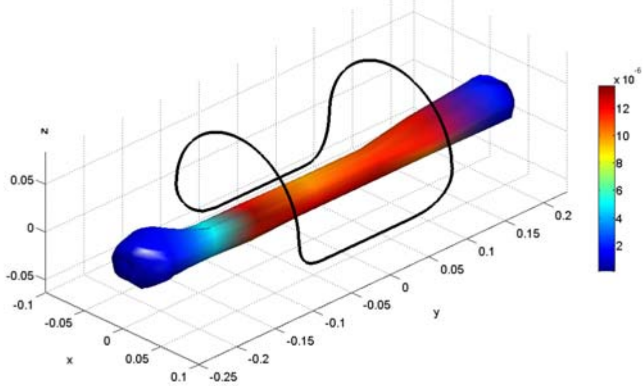

Fig.6. An example of the distribution of magnetic field induced by a flexible applicator. Applicator parameters: *R*=0,08 m, *a*=0,65π*R*, *b*=0,1 m, ρ=0.04 m

It is also possible to obtain a parametric description of windings wrapped around an elliptic cylinder. Then, the appropriate components (with currently adopted designations: *x* and *z*) have to be properly scaled.

Spiral planar coils

Spiral planar coils (in form of mats) belong to a group of applicators used for regeneration (especcially in case of dermal diseases), and the magnetic field reaches the values used in magnetic stimulation, up to tens of microteslas.

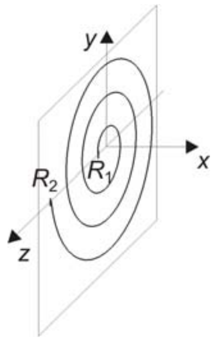

Fig.7. Model of spiral planar coil

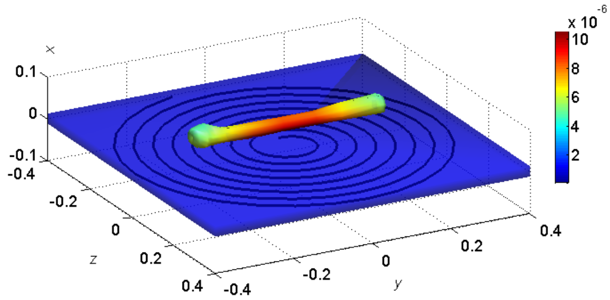

Fig.8. An example of magnetic field distribution around an applicator used for the regeneration. Spiral planar coil of parameters: R_1=0,05 m, R_2=0,05 m, *N*=8

Mats designed to stimulate the trearment with the magnetic field are built as spiral coils located planarly [12]. In this case, characteristic parameters, used to present curve, are listed sequentially:

R_1 – inner radius, R_2 – outer radius, N – number of winding (in presented case: round the x-axis).

Curve (fig.7) is given by following parametric equations:

$$\bar{r}(t)=\begin{bmatrix} x(t) \\ y(t) \\ z(t) \end{bmatrix}=\begin{bmatrix} 0 \\ t\cdot\sin\left(2\pi N\left(\frac{t-R_1}{R_2-R_1}\right)\right) \\ t\cdot\cos\left(2\pi N\left(\frac{t-R_1}{R_2-R_1}\right)\right) \end{bmatrix} \tag{9}$$

where:

$t\in\langle R_1;R_2\rangle$

Figure 8 shows the geometry of applicator and sample distribution of the magnetic field on the bone surface.

Conclusions

The article presents the advantages of using the parametric curves. This form of description can be particularly helpful, when the appropriate size of applicator providing the optimal field distribution from a medical point of view is to be selected.

Each type of winding, has only a few parameters, by which it can be unambiguously presented in 3D space - so these parameters can be used in the process of shape optimization, after the set of the criteria is established.

Parametric curves may be a good way of data compression, and since windings of applicators are described with just a few variables rather than a capacious set of points, which is important e.g. for communication between different applications used to solve field problems.

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