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The probability of collisions in Wireless Sensor Network with random sending

***Abstract**.. The goal of this work is to define and to test the assumptions for the accomplishment of a wireless sensor network (WSN) as a collective network (many-to-one) with random transmitting time of of sensor-senders to WSN (Wireless Sensor Network) receiving base. Innovativeness of this work lies in the concept of WSN network with random time of one-way transmission using one single radio frequency. Using PASTA system (Poisson Arrivals See Time Averages) for modeling WSN network, probability of transmission collision occurrence has been determined.*

***Streszczenie.** Celem pracy jest zdefiniowanie i zbadanie założeń realizacji bezprzewodowej sieci czujników w systemie simpleksowej transmisji na jednej częstotliwości radiowej, jako sieci zbiorczej z losowym czasem nadawania czujników-nadajników do stacji odbiorczej (Wireless Sensor Network - WSN). Wykorzystując strumień zgłoszeń Poissona do modelowania sieci zbiorczej WSN, wyznaczamy prawdopodobieństwo występowania kolizji w transmisji. **(Prawdopodobieństwo kolizji w bezprzewodowej sieci czujników z losowym nadawaniem**)*

Keywords: Wireless Sensor Network (WSN), Poisson Arrivals See Time Averages (PASTA system), probability of collision, random sending.
Słowa kluczowe: Bezprzewodowa sieć czujników, strumień zgłoszeń Poissona, prawdopodobieństwo kolizji, losowe nadawanie.

Introduction

The modern solutions of wireless communication networks are very attractive. They allow the communication to be accomplished under extreme conditions. However, the accomplishment of wireless networks requires a number of conditions to be met which are much more difficult to be made than in the case of a wire network. When considering a wireless network to be produced, it is necessary to solve some problems related to it [1], [2]. They cover following: the choice of network architecture (see [3], [4]) dedicated to the accomplishment of a determined task, working range of the network and autonomy of the network or its connecting to a larger communication system. The next assumption to be made is the choice of a stationary or mobile option of the Network [5]. If mobile, then its mobility must be determined somehow which is of influence on the solution of communication problems [6], i.e. for example, what is the migration of bases within the working range of the network. Principally, wireless networks are accomplished for two reasons, i.e. 1) network bases are assumed to be mobile, 2) we want to avoid cabling which is always a problem when building a network, with its components to be fixed. This fact is related with the problem of power supply for a base in the network. When giving up the traditional cabling there usually appears the problem of power supply for a base. Therefore, in such cases it is of great importance to solve the problem of saving the power supply source. Modern technologies of high-economy electronic systems are very helpful in this respect but the problem of power supply is also on the side of network architecture, communication procedure and spatial extensiveness [7], [8], [9]. Radio transmission based communication procedure constitutes the key problem of the discussed networks [10].

The aim of this work is to work out and to test the communication procedure for a certain class of wireless networks. We are considering the architecture of a wireless collective network (many-to one) for measuring purposes. We assume the quantity of information sent from particular sensors not to be high. However, the number of sensors with transmitting systems, which will be referred to as „sensor-senders", may be significant. Moreover, we have only one single radio frequency (carrier wave) for the whole network. Under such circumstances there appears the problem of undisturbed transmission when each of sensor-senders must send the information to the receiving base collecting the information from particular sensors.

There is a large demand for this class of networks in different fields of science and technology. The concept of the solution presented in the paper has been motivated by the need of monitoring some climatic and biophysical parameters of a certain piece of land which was out of any technological and communication infrastructure.

Network modeling PASTA system (Poisson Arrivals See Time Averages, see [11], [12]) has been used for the need of working out the concept of operation of Wireless Sensor Network (WSN) aimed at the above mentioned purpose. Independent operation of sensor-senders is one of significant assumptions. The operation of the whole network is not dependent on a single sensor. No procedures are required in order to connect an additional sensor-sender. No synchronization or return channel are required for supplying the data from particular sensors to the receiving base. Network mobility should not affect its operation.

In Section 2 we have presented the assumptions for the proposed network, description of the network stochastic model and the most important features of Poisson's process which have been used for the analysis of the proposed network. Conditional collision probability has been given in the paper [13]. In this paper the unconditional collision probability has been given (without any additional assumptions and conditions) in two cases, i.e. 1) for protocol transmission time t_p, 2) for any length of observation period. The obtained results have been plotted. Finally, we summarize results and draw conclusions.

Network model

We analyze a network consisting of n sensors which are able to send information about the measured physical magnitude on one selected radio frequency to the receiving base, quite independently of each other. Duration of communication protocol is t_p, the sensors send the information to the receiving point in randomly selected moments, every T s. at an average. Beginning and cessation of transmission of a particular sensor takes place in random moments of time but these moments are relatively rare. It is a one-way transmission, i.e. from sensors to the receiving base. The sensors are completely independent of one another and their on or off state is of no influence on the operation of the network. All the sensor-senders or a part of them may be mobile provided that their senders have been left within the radio range of the receiving base. If one or more senders start sending while protocol transmission of t_p time is going on from another sensor, then such a situation is called collision. Collision excludes the possibility of the correct receiving of information by the receiving base. Such a disturbed signal is ignored. The receiving base rejects the erroneous message and waits for a re-transmission to be made after the average time T. We must accept a certain loss of information in exchange for simplicity in respect of both system and equipment.

As mentioned in the Introduction, we used to modeling our wireless network a Poisson process. Poisson process is the stochastic process in which events occur continuously and independently of the one another Mathematically the process N is described by so called counter process N_t or $N(t)$ (see [14], [13]) of rate $\lambda > 0$. The counter tells the number of events that have occurred in the interval $[0,t]$ $(t \geq 0)$. N has independent increments (the number of occurrences counted in disjoint intervals are independent from each other), such that $N(t)-N(s)$ has the Poisson ($\lambda(t-s)$) distribution (mean $\lambda(t-s)$), for $t \geq s \geq 0,\ j = 0,1,2,\dots,$

(1) $$P\{N(t)-N(s)=j\} = e^{-\lambda(t-s)}\frac{[\lambda(t-s)]^j}{j!}.$$

The rate parameter λ is the expected number of events that occur per unit time. A counting process has two corresponding random sequences, the sequence of count times (T_j) and the sequence of inter count times (U_j), such that $U_1 = T_1$ and $U_j = T_j - T_{j-1}$, for $j \geq 2$.

In the following two propositions we give the well-known characterization of a Poisson process ([14]).

Proposition 1. Let N be a counting process and let $\lambda > 0$. The following are equivalent:

a) N is a Poisson process with rate λ.

b) The inter count times $U_1, U_2, ...$ are mutually independent and each is exponentially distributed with parameter λ (mean $1/\lambda$).

Proposition 2. Let $\lambda > 0$, $t > 0$, $n = 1,2,\dots$. Then assuming $N_t = j$:

a) the times of n counts during $(0,t]$ are the same as n-independent uniformly distributed on the interval $[0,t]$ random variables, reordered to be non-decreasing. That is the random vector $(T_1,\dots,T_j)$ is uniformly distributed on the set $\Omega_t = \{(t_1,...,t_j) : t_1,...,t_j \geq 0,\ 0 \leq t_1 \leq t_2 \leq ... \leq t_j \leq t\}$ with the conditional density $f(t_1,\dots,t_j \mid N_t = j) = j!/t^j$, for $(t_1,\dots,t_j) \in \Omega_t$, and 0 else,

b) the random vector $(U_1,\dots,U_j)$ is uniformly distributed on the set $\Omega_t^* = \{(u_1,\dots,u_j) : u_1,\dots,u_j \geq 0, u_1 + \dots + u_j \leq t\}$, with the conditional density $f(u_1,\dots,u_j \mid N_t = j) = j!/t^j$, for $(u_1,\dots,u_j) \in \Omega_t^*$, and 0 else.

Let us state our main assumptions. There are n identical sensors observing a dynamical system and reporting to a central location over the wireless sensor network with one radio channel. For simplicity, we assume our sensor network to be a single hop network with star topology. We also assume every node (sender-sensor, shortly sensor) always has packet ready for transmission. We assume that sensors send probe packets at Poissonian times. The average time between sending (the wake-up- times) of a sensor is T (the epoch period), and the duration of the on-time t_p (the awake interval). Assume that the wake-up-times corresponding to sensors are independent each of other. Let N be the Poisson process representing the time counter of sending sensors. Let $T_1, T_2, \dots$ be the sending times (the wake-up- times) of sensors, $U_1, U_2, \dots$ the inter sending times. Then the average time between sending of sensors is T/n, the average number of sending sensors in the time interval of T length equals to n. We say that a collision occurs in the time interval of t_p length , if at least two sensors start sending within this interval. We say that a collision occurs in time interval s, if there exist at least two sensors which start sending within this interval with the difference between the beginning of their sending time not exceeding the value of t_p.

Then the Poisson process N has the rate $\lambda = n/T$. By (1)

(2) $$P(N_t = j) = e^{-\lambda t}\frac{[\lambda t]^j}{j!}\ (j = 0,1,\dots).$$

Let $p(j;t_p)$ $(j = 0,1,2,\dots)$ be the probability that the number of sensor transmissions that have occurred in the interval $[0,t_p]$ equals j. From (2), we obtain, for $j = 0,1,2,\dots$

(3) $$p(j;t_p) = e^{-\lambda t_p}\frac{[\lambda t_p]^j}{j!} = e^{-n\frac{t_p}{T}}[n\frac{t_p}{T}]^j / j!.$$

In particular cases, for $j = 2,3,\dots$, $p(j;t_p)$ can be regarded as the probability of exactly j collisions in the interval $[0,t_p]$, and consequently by the stationarity of Poisson process, on every interval $[t, t+t_p]$ $(t > 0)$.

Let A_s, $A_s^{'}$ be the events that denote the collisions occur and the lack of collisions, respectively, on the interval of s length $(s > 0)$.

Now we are going to calculate a probability of collisions on the t_p-long interval . Taking into account that a collision occurs in the time interval of t_p length, if at least two sensors start sending within this interval, by (3) we obtain:

$$P(A_{t_p}) = \sum_{j=2}^{\infty} p(j;t_p) = 1 - p(0;\ t_p) - p(1;t_p) =$$
$$= 1 - e^{-n\frac{t_p}{T}} - n\frac{t_p}{T}e^{-n\frac{t_p}{T}}.$$

Note, that the number of sensor transmissions that have occurred in the interval $[0,t_p]$, i.e. N_{t_p} has a Poisson distribution with the rate $\lambda t_p = n\frac{t_p}{T}$. Consequently

(4) $$EN_{t_p} = n\frac{t_p}{T} = \sum_{j=0}^{\infty} j\,p(j;t_p) = \sum_{j=1}^{\infty} j\,p(j;t_p).$$

Let Y_{t_p} be the number sensor transmissions in collision, in the interval $[0,t_p]$. Then we have

$$P(Y_{t_p}=0)=p(0;\,t_p)+p(1;t_p)=e^{-n\frac{t_p}{T}}+e^{-n\frac{t_p}{T}}[n\frac{t_p}{T}],$$

$$P(Y_{t_p}=j)=p(j;\,t_p)=e^{-n\frac{t_p}{T}}\frac{(n\frac{t_p}{T})^j}{j!},\qquad j=2,3,....$$

Consequently, by (4), we obtain:

$$E(Y_{t_p})=\sum_{k=0}^{\infty} jP(Y_{t_k}=j)=\sum_{k=2}^{\infty} j\,p(j;t_p)=$$

$$=\sum_{k=1}^{\infty} j\,p(j;t_p)-p(1;\,t_p)=n\frac{t_p}{T}-e^{-n\frac{t_p}{T}}n\frac{t_p}{T}=$$

$$=(1-e^{-n\frac{t_p}{T}})n\frac{t_p}{T}.$$

A summary of the above discussion gives the following theorem on a probability of collisions (see Fig. 1).

<u>Theorem 3.</u>

a) The probability of collisions in the interval of t_p length, is given by the formula

$$P(A_{t_p})=1-e^{n\frac{t_p}{T}}-n\frac{t_p}{T}e^{-n\frac{t_p}{T}}.$$

b) The average of the number of sensor transmissions in collision, in the interval of t_p length, is given by

$$E(Y_{t_p})=(1-e^{n\frac{t_p}{T}})\cdot n\frac{t_p}{T},$$

where n is the number of sensors, T is the average time between sending of a sensor and t_p is the duration time of a protocol.

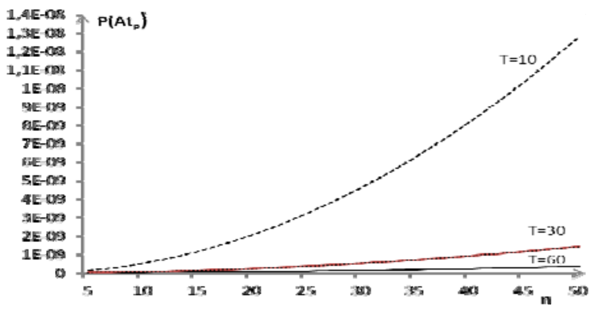

Fig. 1. Collision probability within t_p- long interval, depending on the number of sensors in the network, where $T=10$ s., 30 s., 60 s..

<u>Theorem 4.</u> The probability of collisions in the interval of s length $(s>0)$ is given by

$$(5)\qquad P(A_s)=\sum_{j=2}^{\infty} e^{-n\frac{s}{T}}\frac{(n\frac{s}{T})^j}{j!}[1-(1-j\frac{t_p}{s})^j],$$

where n is the number of sensors, T is the average time between sending of a sensor and t_p is the duration time of a protocol.

<u>Proof</u> Consider the interval $[0,s]$, where $s>t_p$. Assume $N(s)=j$, i.e. the number of sensor transmissions equals to j $(j\geq 1)$. By Proposition 2, the random vector $(U_1,\ldots,U_j)$ of inter transmission-times is uniformly distributed on the set $\Omega_t^*=\{(u_1,\ldots,u_j):u_1,\ldots,u_j\geq 0,u_1+\ldots+u_j\leq s\}$ with the conditional density $f(u_1,\ldots,u_j\,/\,N(s)=j)=j!/s^j$, for $(u_1,\ldots,u_j)\in\Omega_s^*$, and 0 else. Then the conditional probability of the lack of collisions in the interval $[0,s]$, assuming $N(s)=j$, is
By the formula of total probability we have

$$P(A'_s\,/\,N(s)=j)=P(U_1>t_p,\ldots U_j>t_p)=(1-j\frac{t_p}{s})^j.$$

Consequently,

$$(6)\qquad P(A_s\,/\,N(s)=j)=1-(1-j\frac{t_p}{s})^j$$

for $j=2,3,\ldots$, with the convention $P(A_s\,/\,N(s)=0)=P(A_s\,/\,N(s)=1)=0$. From (2), we have

$$(7)\qquad P(N(s)=j)=e^{-n\frac{s}{T}}\frac{(n\frac{s}{T})^j}{j!}(j=0,1,2,\ldots).$$

By the formula of total probability we have

$$P(A_s)=\sum_{j=0}^{\infty}P(N(s)=j)\cdot P(A_s\,/\,N(s)=j).$$

Then taking into account (6) and (7), we obtain (5). This completes the proof.

Applying the Bayes theorem, which gives a mathematical representation of how the conditional probability of event A given B is related to the converse conditional probability of B given A: $P(A/B)=P(B/A)\cdot P(A)/P(B)$, with $A:=\{N(s)=j\}$ and $B:=A_s$, we obtain the following theorem.

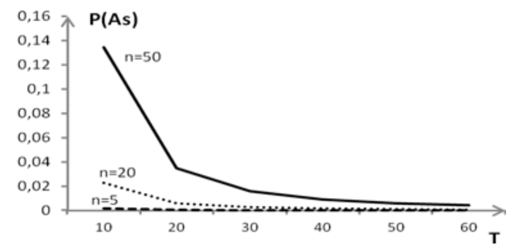

Fig.2. Collision probability depending on the average time interval between transmissions for the sensor, where $n=5, 20, 50$. Observation time $s=180$ s..

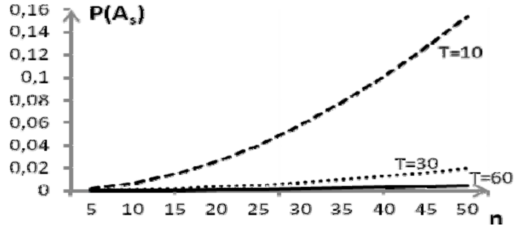

Fig.3. Collision probability depending on the number of sensors, where $T=10$ s., 30 s., 60 s.. Observation time $s=180$ s.

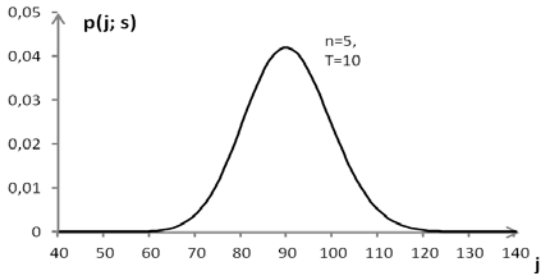

Fig. 4. Probability of j transmissions (for n = 5, T = 10s.). Observation time s = 180 s..

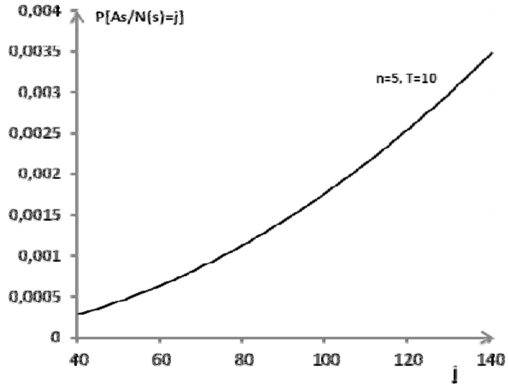

Fig. 5. Conditional probability of collision, where transmission number equals to j (for n = 5, T = 10s.). Observation time s = 180 s

Theorem 5. The conditional probability of j sensor transmissions ($j = 2,3,..$), assuming the collisions occur, is given by

$$P(N(s) = j / A_s) = e^{-n\frac{s}{T}} \frac{(n\frac{s}{T})^j}{j!}[1-(1-j\frac{t_p}{s})^j]/P(A_s).$$

Conclusions

Two dependencies for the probability of a collision have been presented in the paper. The first formula determines the probability of a collision in a short protocol sending duration t_p by defining the probability of undisturbed sending of the protocol (Fig. 2). The second formula has been derived on the basis of the features of Poissson's process in respect of collision probability in any length of transmission duration. Probability of a collision has been graphically presented in diagrams as depending on the number of sensor-senders with the defined average time between subsequent message transmissions (Fig. 4); moreover, dependence has been shown on the average protocol transmission time where the number of sensor-senders is defined (Fig.3). Using the diagrams optimum values of the parameters can be found which influence the transmission correctness (n,T, t_p). It should be determined to which extent the quality of transmission is maintained at the present level or for which values (n,T, t_p) collision probability is growing rapidly. Range of values of collision probability can be read for optionally selected parameters: e.g. for t_p = 3,2 x 10^{-5}, the number of sensor-senders equal to 10 and transmitting, whereby the average transmission time every $T = 60\ s$, the probability of a collision amounts to 1,65 x 10^{-4}. It is possible to obtain so surprisingly good conditions of operation of a network with quite a large number of sensor-senders, when protocol duration t_p is by several ranges shorter than the average time interval between transmissions. Due to that the network can be qualified for a certain class of applications where such transmission parameters are acceptable. On the other hand, it can be seen that the assumptions such as one-way transmission (Simplex), maximally short transmission protocol, which does not require any synchronization or additional extension of protocol framework by additional bytes related to access procedures, control of flow, etc., enable to reach such an advantageous situation where the transmission protocol may be short. It is also extraordinarily advantageous from the point of view of battery saving which give power supply to sensor-senders (maximum saving of power supply batteries). Certainly, it is possible only where the quantity of information sent from particular sensors is small. In the analysis carried out within this paper protocol duration time has been assumed to be t_p = 3,2 x 10^{-5}.

On the basis of the formulas derived in the paper conditions of network correct operation can be defined for all requirements regarding the configuration of such a type of network (n,T, t_p) (collision probability). Such a surprisingly good result of transmission quality in the proposed network solution is possible only for random transmission times. There is a certain similarity to the random access control by means of CSMA method (Carrier Sense Multiple Access) in Ethernet computer network.

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