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A PCG-PSO Image Reconstruction Algorithm For Electrical Capacitance Tomography System

Abstract:.*To solve the 'soft-field' nature and the ill-posed problem in electrical capacitance tomography (ECT) technology, this paper first presents a preconditioned conjugate gradient iterative algorithm for electrical capacitance tomography (PCG), Then, the results of the PCG algorithm using PSO method for imaging gray scale compensation.*

Streszczenie. *Artykuł przedstawia algorytm PCG (preconditioned conjugate gradient) do rekonstrukcji obrazu w elektrycznej pojemnościowe tomografii. Skala szarosci jest rekonstruowanoa przy wykorzystanoiu algorytmu PSO (particle swarm optimization) (**Rekonstrukcja obrazu w elektrycznej tomografii pojemnościuowej z wykorzystaniem algorytmów PCG i PSO**)*

Keywords: electrical capacitance tomography; image reconstruction; conjugate gradient; PSO.
Słowa kluczowe: tomografia pojemnościowa, rekonstrukcja obrazu

1. Introduction

Flow imaging is a new technique developed rapidly in recent years, which has great developmental potential and wide industrial application prospect [1-2]. Having many distinct advantages such as low cost, wide application field, simple structure, non-invade and better safety, Electrical Capacitance Tomography(ECT) has being the most popular research direction and the main development in flow imaging technique [3]. Due to ECT's inherent nonlinear characteristic, and the quantity of available independent measured capacitance values(projection data) is limited, much smaller than the pixel quantity needed for image reconstructing, no analytical solution exist for inverse problem. Meanwhile, because of the nonlinearity and "soft-field" effect, and because of ECT system's poor stability of solution and severe ill-condition caused by the error of measurement, it brings great difficulty to image reconstruction [4-5]. Image reconstruction algorithm has always been the main difficulty for practicing and further developing ECT technique[6], exploring good image reconstruction algorithm is important. At present, common methods used in ECT image reconstruction include: linear back projection algorithm (LBP), regularization, Landweber iterative method, projected-Landweber method and conjugate gradient (CG) [7] , etc.

2. ECT system

The imaging system of ECT is made up of the capacitance sensor, the data acquisition and signal processing, image reconstruction, as shown in Fig. 1. When the distribution pipes of medium change, the capacitance between the plates of the capacitor to change, this can be based on actual measured inversion capacitance plates inside the tubes of media distribution[8].

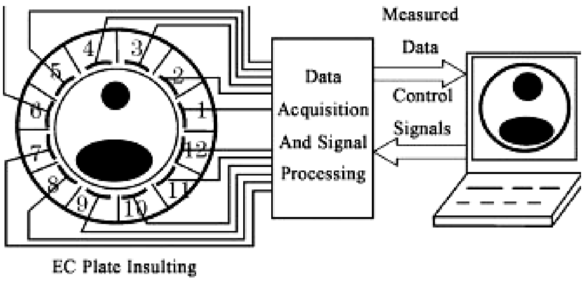


Fig.1 Structure of the ECT system

For an N-plate system, we can get the total number 'M' of independent electrode couples as follows:

$$M = C_N^2 = N(N-1)/2 \quad (1)$$

The subject investigate of this paper is major in a typical 12-electrode EC sensor, and we can acquire 66 independent measuring values C1, C2,… ,C66 .

At present, most of ECT imaging algorithm is the linear model which is based on the mapping from the dielectric constant to the capacitance, the model by discretization, linearization and normalization can be expressed as follows:

$$C = SG \quad (2)$$

where $C \in R^m$ is the normalization capacitance vector, $S \in R^{m\times n}$ is the coefficient matrix (sensitivity matrix), and $G \in R^n$ is the normalization medium distribution image vector. Where the task of ECT image reconstruction is as the given electrical capacitance value C for dielectric constant distribution G .

3. PCG algorithm for ECT

Conjugate direction method is a class method for or solving non-linear programming in unconstrained extremum problem. Conjugate gradient method allows the direction of steepest descent with the conjugate, thereby enhancing the effectiveness and reliability of the algorithm. the iterative conjugate gradient method formula can be expressed by:

$$x_{k+1} = x_k + \alpha_k d_k \quad (3)$$

The capacitance vector C, and image the distribution of vector non-linear relationship exists between G can be express as follow for ECT:

$$C = F(G) \quad (4)$$

To capacitance measurement values and calculated values of the squared error norm as the objective function with the following formula was established:

$$e(G) = \frac{1}{2}\|F(G) - C\|^2 = \frac{1}{2}(F(G) - C)^T(F(G) - C) \quad (5)$$

The gradient with h is expressed as:

$$h_k = (F'(G))^T(F(G) - C) \quad (6)$$

For Eq.6, replace $F'(G)$ with a sensitivity matrix S and replace $F(G)$ with SG_k , the equation can be express as follow:

(7) $$h_k = S^T(SG_k - C)$$

In ECT system, due to the sensitivity of non-positive definite matrix that is also asymmetric, in order to ensure the reconstruction algorithm has better convergence, the need for on Eq.2 regularization of follow.

Multiplied by a both ends of the type S^T, there are as follow:

(8) $$S^T C = S^T SG$$

To ensure the regularization properties of $S^T S$,on the right with a matrix λI , λ for the regularization parameter, multi-selection based on experience.

By Eq.3, the iterative formula of ECT system can be express:

(9) $$G_{k+1} = G_k + \alpha_k d_k$$

(10) $$d_0 = -g_0 = S^T(C - SG_0)$$

(11) $$g_k = S^T(SG_k - C)$$

(12) $$\alpha_k = \frac{-g_k{}^T d_k}{d_k{}^T(S^T S + \lambda I)d_k}$$

Further consideration of transformation:

(13) $$G = W^{\frac{1}{2}} z$$

Where W is the symmetric positive definite matrix, so $SG = C$ and Eq.14 are equivalent equations, Eq.14 can be express as follow:

(14) $$W^{-\frac{1}{2}}(S^T S + \lambda I)W^{-\frac{1}{2}} Z = W^{-\frac{1}{2}} C$$

Here the appropriate choice for W , So the condition number of Eq.15 as small as possible. Eq.15 can be express as follow:

(15) $$\tilde{w} = W^{-\frac{1}{2}}(S^T S + \lambda I)W^{-\frac{1}{2}}$$

Then the convergence rate will be significantly improved.Since $\tilde{w}$ and w^* is similar, w^* can be express as follow:

(16) $$w^* = W^{-1}(S^T S + \lambda I)$$

So here is equivalent to choose W so that Eq.16 of the condition number as small as possible.Preconditioned conjugate gradient algorithm can be express as follow some formula:

(17) $$\gamma_0 = SG_0 - C, \beta_{-1} = 0, d_{-1} = 0$$

(18) $$d_k = -W^{-1}\gamma_k + \beta_{k-1} d_{k-1}$$

(19) $$\alpha_k = \frac{\gamma_k^T W^{-1} \gamma_k}{d_k^T(S^T S + \lambda I)d_k}$$

(20) $$\gamma_{k+1} = \gamma_k + \alpha_k (S^T S + \lambda I)d_k$$

(21) $$\beta_k = \frac{\gamma_{k+1}^T W^{-1} \gamma_{k+1}}{\gamma_k^T W^{-1} \gamma_k}$$

The pre-conditions matrix w can be generated by Quasi-Newton correction, for the r vectors of the $\{s_j, y_j\}$, M satisfy the following equation:

(22) $$s_j = M_{j+1} y_j, \qquad j = 1, \cdots, r$$

Where s_j and y_j in Eq.22 can be expressed as follow:

(23) $$s_j = G_{j+1} - G_j$$

(24) $$y_j = h_{j+1} - h_j$$

As $(S^T S + \lambda I)s_j = y_j$ so there is the following formula:

(25) $$s_j = M_{j+1}(S^T S + \lambda I)s_j$$

Therefore, $M_{j+1}(S^T S + \lambda I)$ are r eigenvalue and its corresponding eigenvector $\{s_j\}$, So, M can be used in place of W^{-1} .

BFGS formula used for correction of the M , so by the following formula:

(26) $$M_{k+1} = M_k + (1 + \frac{y_k^T M_k y_k}{s_k^T y_k})\frac{s_k s_k^T}{s_k^T y_k} - \frac{s_k y_k^T M_k + M_k y_k s_k^T}{s_k^T y_k}$$

4. Progressive Optimal-order of PCG algorithm

For formula $C = SG$,first order gives the best definition of progressive can be expression as follow[9]:

(27) $$M_{\nu,\Delta} = \{G = (S)^\nu z : \|z\| \le \Delta$$

Where $\nu > 0, \Delta \ge 0$, $|S|$ is Square root of S^*S , The following definitions:

Definition 1 Order $S: X \to Y$ is a bounded linear operator , $R(S)$ is Non-closure , $R_\delta : Y \to X$ is operator concerned to $\delta > 0$. If there is a constant $c_\nu > 0$, so for any of the $\Delta > 0, \delta > 0, G^* \in M_{\nu,\Delta}$ and $C_\delta \in Y$ satisfy the conditions $\|SG^* - C_\delta\| \le \delta$, has established the following formula:

(28) $$\|G^* - R_\delta C_\delta\| \le c_\nu (\Delta \delta^\nu)^{\frac{1}{\nu+1}}$$

Say $(R_\delta)_{\delta>0}$ is progressive optimal for $\nu > 0$ and S.The following main issues to consider gradient-based algorithm for ECT progressive anti-optimal order problem. In the ECT inverse problem solution, the set formula as follow:

(29) $$\|C - C_\delta\| \le \delta, \delta > 0, \nu_0 > 0$$

Criteria for the definition of a shutdown as described, $\nu > 0$ is a progressive on the best when $(R_\delta)_{\delta>0}$, so that $G^* \in \mathrm{N}(S)^\perp$ is the solution for the problem. Here a family of smooth construction sequence $\{G_\delta\}_{\delta>0}$, so

$G_\delta \to G^*$ when $\delta \to 0$ .Makes the structure of the sequence satisfy the following equation:

(30) $$G_\delta = |S|^{\nu_0} z_\delta, \|z_\delta\| \le \Delta_\delta$$

(31) $$\|S(G^* - G_\delta)\| \le (\tau - 1)\delta$$

(32) $$G_\delta \to G^*, \delta \to 0$$

(33) $$\Delta_\delta \delta^{\nu_0} \to 0, \delta \to 0$$

Other, if $G^* \in M_{\nu,\Delta}, 0 < \nu \le \nu_0, \Delta > 0$, then

(34) $$\left(\Delta_\delta \delta^{\nu_0}\right)^{\frac{1}{\nu_0+1}} \le c_{1,\nu}\left(\Delta \delta^{\nu}\right)^{\frac{1}{\nu+1}}$$

(35) $$\|G^* - G_\delta\| \le c_{2,\nu}\left(\Delta \delta^{\nu}\right)^{\frac{1}{\nu+1}}$$

Where $c_{1,\nu}$ and $c_{2,\nu}$ is two constants depends on ν , Supposed to have been well constructed such a sequence, set $C_\delta \in Y$ satisfied condition for $\|SG^* - C_\delta\| \le \delta$, by Eq.29 available:

(36) $$\|SG_\delta - C_\delta\| \le \tau\delta$$

By the Eq.30 available:

(37) $$\|G^* - R_{\tau\delta} C_\delta\| \le \|G^* - G_\delta\| + \tau^{\frac{\nu_0}{\nu_0+1}} c_{\nu_0}\left(\Delta_\delta \delta^{\nu_0}\right)^{\frac{1}{\nu_0+1}}$$

By the Eq.32 and Eq.33 available:

(38) $$\sup_{C_\delta \in Y: \|SG^* - C_\delta\| \le \delta} \|G^* - R_{\tau\delta} C_\delta\| \to 0, \delta \to 0$$

Futher, if $G^* \in M_{\nu,\Delta}$, then under Eq.33, Eq.34 and Eq.36 can be get follow expression:

(39) $$\sup_{C_\delta \in Y: \|SG^* - C_\delta\| \le \delta} \|G^* - R_{\tau\delta} C_\delta\| \le (c_{2,\nu} + c_{1,\nu}\tau^{\frac{\nu_0}{\nu_0+1}} c_{\nu_0})\left(\Delta \delta^{\nu}\right)^{\frac{1}{\nu+1}}$$

Conjugate gradient method of produced by each $p, 0 < p \le p_0$ for ECT inverse problem solution is the best gradual.

Therefore, for ECT image reconstruction algorithms are gradient-based solution to a large extent the problem is not the real solution, only an approximate real solution, so after applying these algorithms to solve the problem even further approximation of the true anti-ECT solution.

5. Gray Compensation algorithm

PSO is a swarm intelligence-based heuristic global optimization algorithm[13], The specific mathematical description is as follows:

In the D-dimensional search space have m-particle, position of particle $i(i = 1,2,\ldots,m)$ is $X_i = (x_{i1}, x_{i2}, \ldots, x_{iD})$, it experienced the optimal location is recorded $P_i = (p_{i1}, p_{i2}, \ldots, p_{iD})$, also called individual extremum P_{best} ; Groups experienced the best of all particles for the $P_g = (p_{g1}, p_{g2}, \ldots, p_{gD})$, also called global extremum; the speed of particle i with $V_i = (v_{i1}, v_{i2}, \ldots, v_{iD})$ to represent. Then for every generation, by tracking two particles are extreme to update their own, that is, particles evolve according to the following formula:

(40) $$\begin{cases} v_{id}^{t+1} = \omega \cdot v_{id}^{t} + c_1 \cdot r_1 \cdot (P_{id}^{t} - x_{id}^{t}) + c_2 \cdot r_2 \cdot (P_{gd}^{t} - x_{id}^{t}) \\ x_{id}^{t+1} = x_{id}^{t} + v_{id}^{t+1} \qquad i = 1,2,\ldots,m \qquad d = 1,2,\ldots D \end{cases}$$

Where t is the number of iterations, ω is Inertia weight, c_1 and c_2 is acceleration constant, set $c_1 = c_2 = 2$, r_1 and r_2 is random function within the change [0,1].

In order to change the shortcomings of PSO algorithm to balance the global search ability of PSO algorithm and local capacity to improve the introduction of second-order particle swarm algorithm, denoted by SPSO, the particle velocity update formula was revised as follow:

(41) $$\begin{aligned} v_{i,d}(t+1) &= v_{i,d}(t) + c_1 r_1 (p_{i,d} - 2x_{i,d}(t) + x_{i,d}(t-1)) \\ &+ c_2 r_2 (p_{g,d} - 2x_{i,d}(t) + x_{i,d}(t-1)) \end{aligned}$$

Solutions to consider before using ECT imaging method for imaging, and then determine the boundaries of two media area, the border region selection method is to select grayscale image similar to some units. First post here on the image gray value matrix setting filtering threshold as t , select the regional parameters θ . the first the imaging unit of gray value is greater than $t + \theta$ demand convex geometry, then every point within the convex hull marking, denoted by S . Then on the gray value of the imaging unit demand greater than $t - \theta$ hull geometry, and then on each point within the convex hull marking, denoted by S' , into two convex part is surrounded by the border region (Fig. 2).

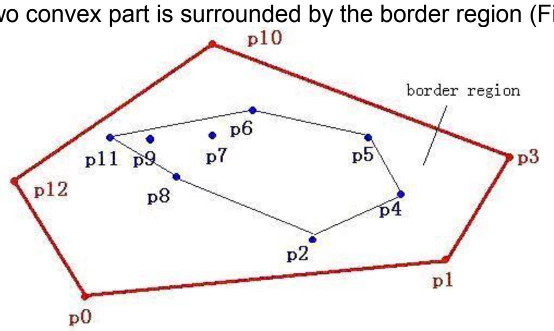


Fig.2 convex hull to determine the boundary region

In the algorithm process, the set G_k for the ECT imaging algorithm executed by the pixel area, set G_k' for our calculated boundary pixels area, there are $G_k' \subseteq G_k$. So each update particle velocity and displacement, update the gray in G_k its counterpart G_k' , according to Eq.(29) incremental norm to determine the minimum position of particle it is optimal.

(42) $$\min \|C - SG_K\|$$

Further, We are going to put an amendment to image at each iteration process in accordance with the physical meaning, lead into a the estimated value x_k' within the scope of at 0 and 1 at each cycle. Especially when $x_k' > 1$, its value becomes 0, to ensure that particles continue to update. The Eq.40 has been modified into the following iterative projection:

$$v^{t+1}_{i,d}(t+1)=v^{t}_{i,d}(t)+c_1r_1(p^{t}_{i,d}-2x^{t}_{i,d}(t)+$$

(43) $$x^{t}_{i,d}(t-1))+c_2r_2(p^{t}_{g,d}-2x^{t}_{i,d}(t)+x^{t}_{i,d}(t-1))$$

$$x^{t+1}_{i,j}=P_{+}[x^{t}_{i,d}+v^{t+1}_{i,d}] \quad i=1,2,\cdots,m \quad d=1,2,\cdots D$$

where P_+ is non-negative convex projection, given by the following formula:

(44) $$P_{+}[x_k^{'}]=\begin{cases}0 & if \quad x_k^{'}\le 0\\ x_k & if \quad 0<x_k^{'}\le 1\\ 0 & \quad x_k^{'}>1\end{cases}$$

Projection operator to ensure that each iteration is converging to a convex set, so that each iteration of the solutions are non-negative and bounded constraints. Practice shows that the projection operator applied after the introduction of constraints on the physical sense, to a certain extent, can improve the reconstructed image quality and reconstruction speed.

6. Simulations and Analysis of Experimental Result

In order to verify the efficiency of algorithm, we perform an experiment on a 12-electrode system. The cross section of pipeline can be divided by a mesh of 32×32 when imaging, we can totally get 1024 pixel, which the effective area is 856 unit of imaging. To typical flow regime : stratified flow ,core flow and bubbly flow, which carried on to prepare a constitution experiment, when imaging we adopted statistical filter threshold. The PCG-PSO algorithm of this text elaborates by numerical simulation to reconstruct image, and carries on a comparison between the linear back-projection algorithm(LBP) and conjugate gradient algorithms, the simulation calculation is based on MATLAB which is on one computer that has PIV3.0G CPU and 2G memory.

Image reconstruction speed expressed by iteration times N, the N is bigger to then reconstruct time more long, which explain that the speed is slower. Because the LBP algorithm belongs to a single treatment, when N=0, the selection of iteration times N of iteration by numerical experiment.While experiment we usually do is iteration error satisfied

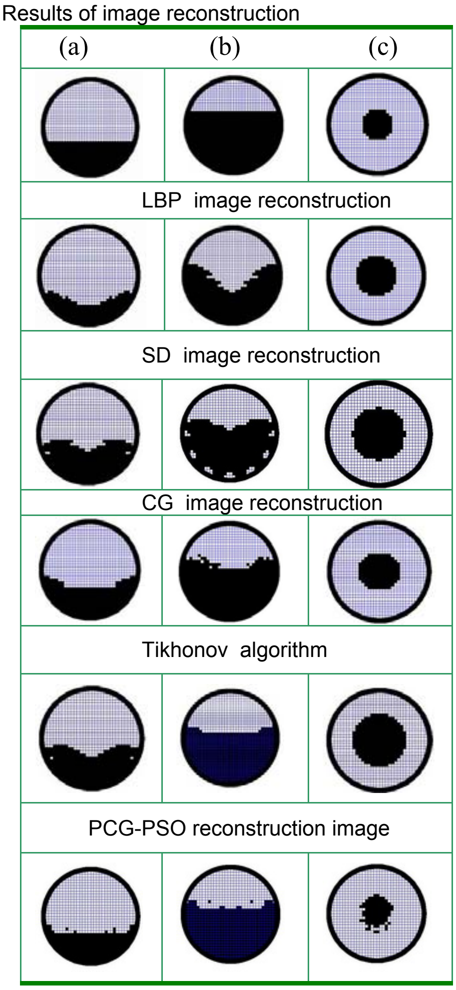
(45) $$\|SG_k-C\|<\xi$$

Stop iteration. we usually select iteration from more than ten steps (simple model) to several tens steps (complex model),so we can get better imaging quality, while analysis reconstructing image quality, selecting spatial image error as evaluation index of image quality, its definition as follows:

(46) $$\varepsilon=\frac{\sum_{i=1}^{n}\left|g_{i(img)}-g_{i(init)}\right|}{\sum_{i=1}^{n}g_{i(init)}}$$

where g_{img} is reconstructing image quality; g_{init} is the image vector of medium distribution prototype; i is imaging region fem element index; n is imaging region finite element numbers. The experiment results are shown as Table 1.

Table 1 Results of image reconstruction



From Table 1 and Table 2 can be seen that PCG-PSO algorithm is very close to the original flow pattern for the core flow and laminar flow. Relative to the LBP,SD and CG algorithm, iterative steps of PCG-PSO algorithm is large, and iterative steps of Tikhonov algorithm is largest number. From the above analysis we can see that the use of the PCG-PSO image reconstruction algorithms, for simple flow patterns and complex flow patterns of its imaging accuracy and quality than LBP, Steepest Descent algorithm (SD), CG and Tikhonov algorithm is better, but the iteration step number is large.

Table 2 Image error(%)

prototype	(a)	(b)	(c)
LBP	40.32	49.68	86.38
CG	26.61	38.66	63.63
SD	34.67	40.19	172.73
Tikhonov	32.25	39.72	190.91
PCG-PSO	21.37	18.43	35.22

Table 3 Number of iteration(time)

prototype	(a)	(b)	(c)
LBP	0	0	0
CG	7	9	6
SD	26	7	24
Tikhonov	310	300	280
PCG-PSO	100	100	200

7. Conclusions

This paper first presents a preconditioned conjugate gradient iterative algorithm for electrical capacitance tomography (PCG), Then, the results of the PCG algorithm using PSO method for imaging gray scale compensation. Based on the analysis of the basic principles of electrical capacitance tomography, deduced PCG-PSO method of iteration formulas and calculation steps and discussed the application of the algorithm is the feasibility of ECT. This algorithm programs simply, need low memory capacitance,which has the advantage of image high precision and easy satisfied with convergence condition.The numerical experiment shown that PCG-PSO algorithm image reconstruction quality is better than LBP algorithm, Numerical experiments show that the algorithm of image reconstruction quality is far better than the LBP algorithm, better than the conjugate gradient algorithm (CG), Steepest Descent algorithm (SD) and Tikhonov algorithm, which reconstructing image more close to raw flow regime,thus shown a new effective method of ECT image reconstruction.

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REFERENCES

[1] Shi L., Jing L., Mathematical Imaging and Vision. 39,269(2011)
[2] Smolik Waldemar T., Accelerated levenberg-marquardt method with an optimal step length in electrical capacitance tomography.2010 IEEE International Conference on Imaging Systems and Techniques,(2010)July 1-2;Thessaloniki, Greece
[3] Decai Lu., Fuqun Shao., Zhiheng Guo.,Yan Wang., The research of electrical capacitance tomography system for large industrial equipments.2009 IEEE Circuits and Systems International Conference on Testing and Diagnosis,(2009) April 28-29;Chengdu,China
[4] Lei, J., Liu, S., An image reconstruction algorithm based on the semiparametric model for electrical capacitance tomography. Computers and Mathematics with Applications.61(2011),n,9, 2843-2853
[5] Deyun C., Yu C., Lili.W., Xiaoyang Y., A novel Gauss-Newton image reconstruction algorithm for Electrical Capacitance Tomography System.4,37(2009)
[6] Yao C, xiaowei W., Study on image reconstruction algorithm for electrical capacitance tomography system.2009 9th International Conference on Hybrid Intelligent Systems,(2009)
[7]Huaxiang W.,Xueming Z.,Lifeng Z., Conjugate gradient algorithm for electrical capacitance tomography. Journal of Tianjin University,2005,38(1):1-4
[8]Hua Y.,Jun W.,Yinggang Z.,An electrical capacitance tomography system with double measurement channels.2011 International Conference on Industry, Information System and Material Engineering,(2011) April 16-17;Guangzhou, China
[9] Plato R.,Optimal Algorithms for linear Ill-Posed problems yield regularizations methods, Numerical Functional Analysis and Optimization,11(1990), 111-118
[10] Peng N.,Ji G.,Zhanghuan Q., Self-adaptive inertia weight PSO test case generation algorithm considering prematurity restraining, International Journal of Digital Content Technology and its Applications.9(2011), n 9, 125-133

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