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Modeling and Parameter Identification for Rubber Bearings under Random Excitation

Abstract. *Based on the dynamic property test of rubber bearings a mathematical model of which is established under the condition of random excitation. The mathematical model shows that under random excitation the hysteretic restoring force is not only relevant to the displacement and the speed but also relevant to the mean value and the standard deviation of the random displacement. With the model the "force – displacement "and "force –time" relationship under different sampling segments and working conditions is reconstructed and matches well with test results.*

Streszczenie. *Przeprowadzono testy właściwości dynamicznych łożyskowania gumowego przy przypadkowym charakterze pobudzeń. Opracowano model matematyczny pokazujący że nie tylko właściwości histerezowe wpływają na przesunięcia ale ważne są też wartość średnia i rozkład przesunięć. (Modelowanie i identyfikacja parametryczna łożyskowania gumowego przy przypadkowych pobudzeniach)*

Keywords: Rubber Bearings, Random Excitation，Mathematical Model，Parameter Identification
Słowa kluczowe: łożyskowanie gumowe, model matematyczny

Introduction

For good damping and seal performance rubber bearings are applied widely in engineering fields. Generally speaking, when there are elastoplastic components or dry friction in a system its force - displacement or stress - strain curves behaviors hysteretic loops. And the main features of hysteretic non-linear rubber bearings are multi-valued and non-smooth. Because the forming mechanism of hysteretic non-linear force are different there are accordingly great differences within the utilized mathematical models. At present the widely utilized hysteretic non-linear models are as follows: Bilinear Model, Davidenkov Model, Bouc - Wen Model, Polynomial Model, Bingham Model, Hybrid Damping Model, Multiple Broken Line Model**,** Ramberg - Osgood Model, Meneyotto - Pinto Model, Trace Method Model and the improved models based on above models [1,2,3,4].

The parameter identification of the hysteretic non-linear system is also a problem paid long - term attention by many scholars and difficult to be completely resolved. At present methods of parameter identification in the hysteretic non-linear model include parameter separation identification method [5], optimal parameter identification method for hysteretic non-linear system in time domain with decimal coding genetic algorithm [6], undetermined coefficient method for identifying restoring force of shock absorber with the least square algorithm which considering the hysteretic nonlinearly restoring force only as the cubical function of displacement and speed [7], and the recently developing Neural network identification method, *etc* [8].

Because under different working conditions the stiffness and damping expressions of the hysteretic non-linear system have different forms only under particular conditions the above described methods for system modeling and parameter identification are tenable. Even though a lot of work has been carried out in this aspect the model still needs to be perfected. Besides, a great deal of research has been done under the condition of deterministic excitation. It was pointed out in reference [14] that there is very big difference of hysteretic non-linear modeling between deterministic and random excitations. In this paper the automobile metal rubber bearings with typical hysteretic non-linear property was taken as an example and the research on its dynamic property under random excitation was made. The force - displacement relationship can be obtained by the dynamic property test under simulated normal work condition. Then the work of modeling and parameter identification was carried out according to actual working conditions. Research in this paper has typicality and universality and the results can provide theoretical basis for dynamics study of vehicles and other large complex systems with hysteretic nonlinearity under random excitation.

1 Vibration Test of Rubber Bearings

In order to obtain the dynamic property of rubber bearings under random excitation the dynamic property test platform was built. The test platform shown in Fig.1 is composed of hydraulic vibrator table offered by Schenk Company in Germany, position transducers, force transducers and rubber bearings from Shanghai tire and rubber Co., LTD.

Fig.1. Layout of test

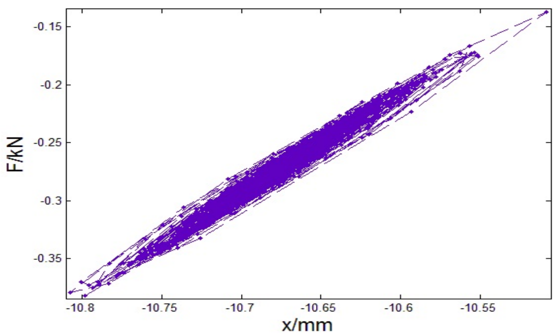

Fig.2. Force - displacement relation

The specimens for test are metal rubber bearing components assembled between vehicle body and sub frame. In order to simulate operating condition of rubber bearings as far as actually the random displacement excitation is taken as exciting signal. The specific test scheme is described as follows: when preloaded by force within 0-1kN the rubber bearings are subjected with the random displacement excitation with the amplitude from 0.1mm to 0.3mm. After sensor calibration, connection, installation and validation of test system the test data under different working conditions is obtained. Then force - time and displacement - time curves under different conditions can also be obtained by dealing with the test data in MATLAB and through further data processing the force - displacement relation graphs can be obtained. Fig.2 shows the force - displacement relation under random displacement excitation with 0.25kN preloading force and amplitude within 0.2mm. From Fig.2 we can see that the outline shape of the graph is nearly like an oval. The force - displacement relation graphs under other working conditions are similar to this figure except that the major and minor axes of the ellipse are different.

To contrast with the dynamic characteristics under determinate excitation tests under sine displacement excitation are also made. The test frequency scope is from 0Hz to 105Hz and the amplitude is from 0.1mm to 0.45mm. Fig.3 shows the force - displacement relationship curves under different working conditions when the rubber bearings are subjected to 50Hz exciting frequency with different amplitude. And Fig.4 shows the relationship when 0.3mm displacement exciting amplitude with different frequency.

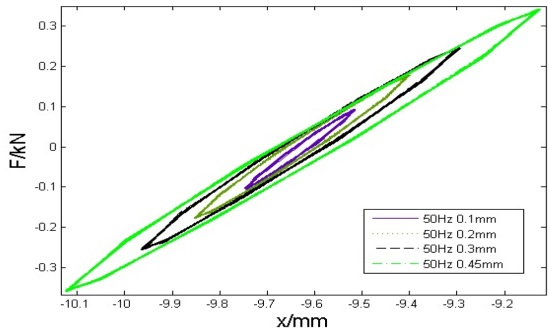

Fig.3. Force - displacement relationship under sine excitation

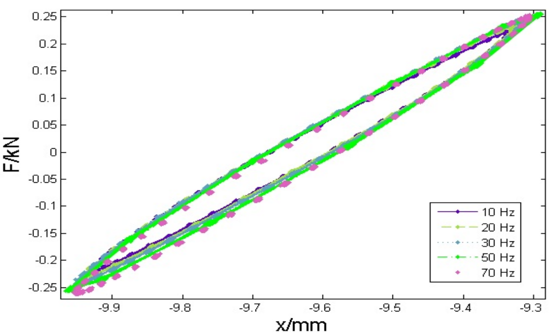

Fig.4. Force - displacement relationship with different frequency

2 Modeling and parameter identification of the metal rubber bearings

2.1 Modeling under Sine Excitation

To study the mathematical model of metal rubber bearings under random excitation the dynamic property of model under sine excitation was studied at first. From Fig. 3 we can see that the loop curves can be divided into the upper curve and the lower curve and they correspond to two states of speed greater than zero and less than zero respectively. In the case of same nature of rubber bearings and installation geometrical symmetry the two parts can be considered as displacement anti-symmetry. Therefore the two restoring force curves can be expressed by power polynomials. The polynomial for the upper curve is as follows:

(1) $$F_U(x) = \sum_{i=0}^{n} a_i x^i \qquad \dot{x} > 0$$

And according to the principle of displacement anti - symmetry, $F_U(x)=-F_L(x)$, the polynomial for the lower curve can be described as follows:

(2) $$F_L(x) = \sum_{i=0}^{n} (-1)^{i+1} a_i x^i \qquad \dot{x} < 0$$

where: $F_U(x)$ – restoring force corresponding to the upper curve, $F_L(x)$ – lower curve separately, x – displacement, $\dot{x}$ – speed, a_i – the coefficient of polynomial, and n – is the number of polynomial terms.

If *e.q.*(1) and *e.q.*(2) are combined after separating even item and odd item of the polynomial the total restoring force can be expressed as follows:

(3) $$F(x,\dot{x}) = \sum_{i=1}^{(n+1)/2} a_{2i-1} x^{2i-1} + \sum_{i=0}^{(n-1)/2} a_{2i} x^{2i} \operatorname{sgn}(\dot{x}) = F_K(x) + F_C(x,\dot{x})$$

where: n – an odd number, and $sgn(\dot{x})$ is expressed as:

(4) $$\operatorname{sgn}(\dot{x}) = \begin{cases} 1 & (\dot{x} > 0) \\ 0 & (\dot{x} = 0) \\ -1 & (\dot{x} < 0) \end{cases}$$

After mathematical treatment as above the hysteretic restoring force of rubber bearings can be decomposed into two parts: $F_K(x)$ and $F_C(x, \dot{x})$. In geometric respect $F_K(x)$ is a non-linear function curve with single value, and $F_C(x, \dot{x})$ is a non-linear closed curve with double value. And in physical respect $F_K(x)$ stands for the anhysteretic non-linear elastic part of the hysteretic restoring force and $F_C(x, \dot{x})$ stands for the pure hysteretic non-linear damping part. Fig.5 shows the $F_K(x)$ and $F_C(x, \dot{x})$ decomposed from the hysteretic restoring force when frequency is 55 Hz and amplitude is 0.3mm under sine excitation.

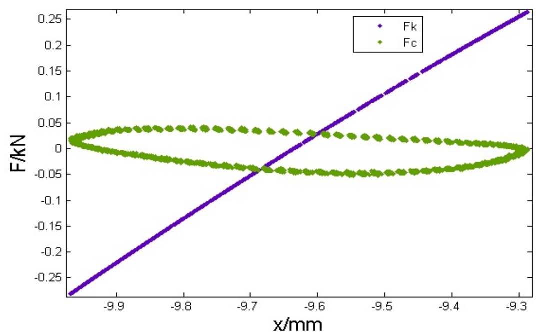

Fig.5. Hysteretic restoring force decomposed into $F_K(x)$ and $F_C(x, \dot{x})$

After decomposition as above models of the elastic force and the damping force are established respectively according to the dynamic property of the metal rubber bearings. The model in this paper comes from a kind of trace method model which is a hysteretic non-linear model related to displacement, speed and cube of them. And the expression is as follows:

(5) $$F = K_1 x + K_3 x^3 + C_1 \dot{x} + C_3 \dot{x}^3$$

Because the technique of modeling and parameters identification for non-linear system of rubber bearings under sine excitation is relatively mature there is no more discussion in this paper.

2.2 Modeling under Random Excitation

In Fig.2 the force - displacement relationship shows a shape of ellipse-like, which shows hysteretic non-linear characteristic with uncertain random characteristics. Besides, the outline of this graph is very similar to graphs under sine excitation. The specimen should have same dynamic properties no matter under sine excitation or under random excitation. So the modeling under sine excitation can be used for reference here and the total restoring force of rubber bearings under random excitation can be separated into two parts of non-linear elastic force F_K and damping force F_C. Meanwhile the characteristic of random excitation is considered and it is believed that the force - displacement relationship is related to statistic parameters of the random process. The main statistical parameters of random process include mean value, variance, standard deviation, probability density function, correlation function and power spectral density function, *etc.* After attempt and analysis it is determined that the total restoring force is not only the function of displacement and speed but also the function of mean and standard deviation of the random process. So the model of restoring force can be expressed as follows:

$$F(x,\dot{x},E(x),\delta(x)) = F_K(x,E(x),\delta(x)) + F_C(x,\dot{x},E(x),\delta(x)) \tag{6}$$

According to the dynamic property test under random excitation, referencing the model of dynamic stiffness of metal rubber bearings under sine excitation and considering the random characteristics the model of elastic force is expressed as follows:

$$F_K = \sum_{i=1}^{(n+1)/2} K_{2i-1}(E(x),\delta(x))x^{2i-1} \tag{7}$$

From *e.q.* (7) we can see that the non-linear strength of elastic restoring force can be adjusted by the parameter n. Fig.2 show that the non-linearity of elastic restoring force F_K is not very strong. So the model of non-linear elastic force F_K only considers the former two orders of the polynomial and is expressed as follows:

$$F_K(x,E(x),\delta(x)) = K_1(E(x),\delta(x))x + K_3(E(x),\delta(x))x^3 \tag{8}$$

Where: $F_{2i-1}(E(x),\delta(x))$ – the dynamic stiffness coefficient, which is the non-linear function of mean and standard deviation of exciting displacement and can be available by fitting with polynomial of second order power series under the condition of assuring the fitting accuracy:

$$K_1(E(x),\delta(x)) = k_{10} + k_{11}E(x) + k_{12}\delta(x) + k_{13}E(x)^2 + k_{14}\delta(x)^2 + k_{15}E(x)\delta(x) \tag{9}$$

$$K_3(E(x),\delta(x)) = k_{30} + k_{31}E(x) + k_{32}\delta(x) + k_{33}E(x)^2 + k_{34}\delta(x)^2 + k_{35}E(x)\delta(x) \tag{10}$$

In the same way with the reference of modeling under sine excitation and consideration of random characteristics mathematic model of the damping force can be expressed as follows:

$$F_C(x,\dot{x},E(x),\delta(x)) = C_1(E(x),\delta(x))\dot{x} + C_3(E(x),\delta(x))\dot{x}^3 \tag{11}$$

Where: $C_{2i-1}(E(x),\delta(x))$ – the dynamic damping coefficient, which is also the non-linear function of mean and standard deviation of exciting displacement. Similarly, the fitting can be done as follows:

$$C_1(E(x),\delta(x)) = c_{10} + c_{11}E(x) + c_{12}\delta(x) + c_{13}E(x)^2 + c_{14}\delta(x)^2 + c_{15}E(x)\delta(x) \tag{12}$$

$$C_3(E(x),\delta(x)) = c_{30} + c_{31}E(x) + c_{32}\delta(x) + c_{33}E(x)^2 + c_{34}\delta(x)^2 + c_{35}E(x)\delta(x) \tag{13}$$

After all the restoring force model of metal rubber bearings under random excitation can be expressed as follows:

$$F(x,\dot{x},E(x),\delta(x)) = K_1(E(x),\delta(x))x + K_3(E(x),\delta(x))x^3 + C_1(E(x),\delta(x))\dot{x} + C_3(E(x),\delta(x))\dot{x}^3 \tag{14}$$

2.3 Parameter Identification of the Model under Random Excitation

With sampling signal data of displacement and restoring force from the dynamic test under random excitation surface fitting of force - displacement and speed under various working conditions are done by MATLAB. Stiffness and damping coefficients of different working conditions can be obtained. Varying trend of these coefficients changing with mean and standard deviation of the displacement is analyzed and the non-linear theory is used here for parameter identification. The results of parameter identification are as follows:

$$K_1 = -3.555 - 0.5294E(x) - 0.5369\delta(x) - 0.02261E(x)^2 - 0.2128E(x)\delta(x) - 5.206\delta(x)^2 \tag{15}$$

$$K_3 = 0.04245 + 0.006169E(x) - 0.02467\delta(x) + 0.0002456E(x)^2 - 0.0003348E(x)\delta(x) + 0.1111\delta(x)^2 \tag{16}$$

$$C_1 = 0.61 + 0.1027E(x) + 0.8177\delta(x) + 0.004896E(x)^2 - 0.005322E(x)\delta(x) - 7.736\delta(x)^2 \tag{17}$$

$$C_3 = -89.7 - 17.88E(x) - 297.9\delta(x) - 0.7124E(x)^2 + 32.2E(x)\delta(x) + 5444\delta(x)^2 \tag{18}$$

Substituting these dynamic stiffness coefficients and dynamic damping coefficients in *e.q.*(14) and then the restoring force of different working conditions can be obtained.

3 Model Validation of Rubber Bearings System

Because the test time course of displacement - time and force - time history is long enough these coefficients K_1、K_3、C_1、C_3 are identified by surface fitting of this long enough sample section. Then with different sample section under the same working condition the force - displacement and force - time relationship are reconstructed. Fig.6 shows the result of simulation in contrast with test for different sampling segment under the condition of 0.5kN preloading and amplitude within 0.2mm. This graph shows that the simulation results are in good agreement with the test data and the biggest relative error is 3.58% and the average relative error is 1.023%. In a word the results show that the model is credible and the identification accuracy satisfies the requirements of engineering application. Besides, the model can be used to reconstruct the restoring force under excitation with same mean and standard deviation, which provides reference for sample sizes collecting from the test data.

a)

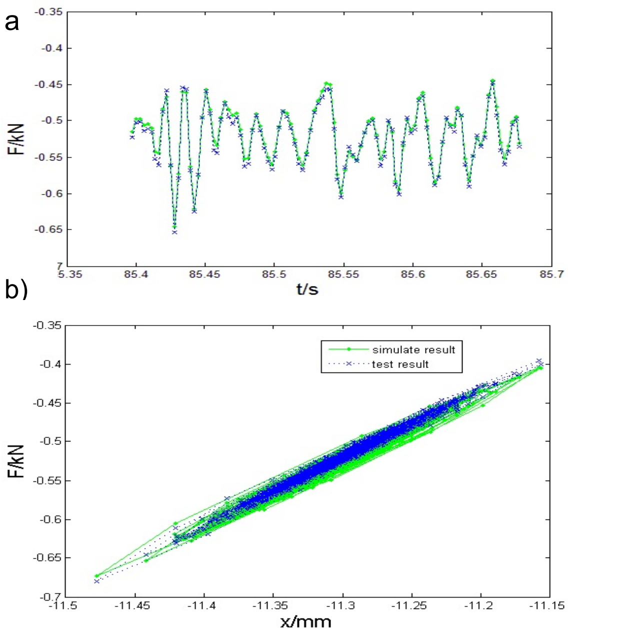

Fig.6. contrast of simulation and test results with different sampling segment

In order to verify the applicability of models to different preloading and different amplitude and the algorithm accuracy of parameter identification the means and standard deviations of the random exciting displacement are calculated with the data of displacement and restoring force under different conditions retained during dynamic test. And the coefficients of K_1、K_3、C_1、C_3 are calculated by *e.q.* (15) to *e.q.*(18) and then substituted into *e.q.* (14). In the end the force - displacement and force - time relationship are reconstructed. Fig.7 shows the simulating result of force - time relationship in contrast with the test when the preloading is 0.75kN and amplitude is within 0.2mm. From the fig.7 we can see that the simulating result has good agreement with the test result and the biggest relative error is 5.136% and the average relative error is 2.969%, which shows that the model can satisfy the requirements of engineering application.

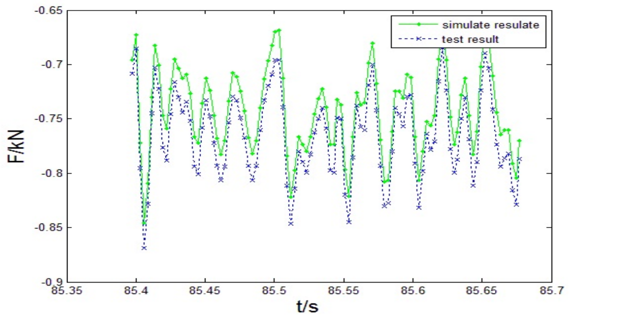

Fig.7. contrast of simulating and test result and the relative error under random excitation

4 Conclusion

(1)Through analysis for dynamic property test of rubber bearings a mathematical model with hysteretic non-linearity. Is built. Under random excitation the hysteretic restoring force of rubber bearings is not only relevant to the displacement and the speed but also relevant to the mean value and the standard deviation of the random displacement.

(2) The force - displacement and force - time relationship are reconstructed by the model built in this paper,which is under the condition of different sampling sections with same preloading and same displacement amplitude and the condition of different working conditions with different preloading and different displacement amplitude. Simulating and test results match well, which verifies the crediblity of this model and the accuracy of parameter identification meets the requirements of engineering application.

(3) Research of modeling and parameter identification under sine excitation is used for reference here and the total restoring force of rubber bearings under random excitation is resolved into two parts of the non-linear elastic force F_K and the damping force F_C. The two models have consistency in structure form but have essential difference in concrete expression of function. Models under random excitation have relationship with statistical parameters of random process.

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