An Application of Canonical Decomposition to TDOA Estimation for 3D Wireless Sensor Network Node Localization

Abstract. In this paper, a time difference of arrival (TDOA) estimation algorithm is proposed by extending canonical correlation decomposition (CCD) to measure multipath delays for determining node positions in ultra-wideband wireless sensor networks (WSNs). Multilateral localization based on 3D Chan algorithm is employed to solve the non-linear problems for performance improvement. The method can perform well in the environment with unknown noise. The effectiveness is validated by simulations and the effects of noise and the number of anchor nodes are analyzed.

Keywords: wireless sensor network, canonical correlation decomposition, MUSIC algorithm, time difference of arrival, ultra-wideband, node localization, 3D Chan algorithm.

Introduction

Recently, ranging and localization are becoming key techniques in wireless sensor networks (WSN) [1-3]. There are several methods that can be used to determine the distances, such as time of arrival (TOA), time difference of arrival (TDOA), direction of arrival (DOA) and received signal strength (RSS). RSS methods need little cost, however, the ranging resolution is greatly influenced by the environment. DOA methods need multiple antennas, and is not preferred in low hardware cost and simple system. TOA and TDOA methods measure the distance between nodes using signal propagation time. TDOA methods is more preferred because they are impressively accurate under line-of-sight conditions without an accurate synchronization between the transmitter and receiver clocks. Some range-based localization methods have been discussed. However, most algorithms are only applicable for two-dimensional (2D) localization, leading to the limitation in actual environment. Specifically, in the environment of multipath and unknown noise, these traditional localization algorithms suffer from severe performance degradation.

In this paper, we put forward a novel range-based node localization method for ultra wideband (UWB) based WSN. In the ranging for WSN localization, a multipath time delay estimation algorithm is developed based on the multipath signal classification (MUSIC) algorithm [4,5] employing canonical correlation decomposition (CCD) technology [6-8]. CCD is a high-resolution method originally used in the direction of arrival (DOA) estimation in unknown correlated noise. In the proposed paper, we investigate its application in TDOA measurement for 3D WSN node localization. CCD-based delay estimation algorithm is proposed to measure the TDOA of an unknown node by two anchor nodes. The proposed method can work well in a colored noise situations, and give a quantitatively controllable performance. Furthermore, three-dimensional (3D) Chan algorithm with multilateral localization instead of trilateral localization is employed to determine the physical coordinates of a group of sensor nodes. Combined with multilateral localization, it much enhances the accuracy. Using 3D Chan algorithm [9], the nonlinear equations can be accurately solved to obtain 3D positions of the nodes.

Received Signals from Two Anchors

The fine time-resolution of UWB signals provides potentially accurate ranging for WSN location. Assume that UWB signals are transmitted from an unknown node to two anchor nodes by $L_1$ and $L_2$ paths, respectively. The received signals in the $q$-th ($q=1,2,\ldots,Q$) snapshot can be expressed as

$$
\begin{align*}
\begin{bmatrix}
y_1^{(q)}(t) \\
y_2^{(q)}(t)
\end{bmatrix} &= \begin{bmatrix}
\beta_{11}^{(q)} & \beta_{12}^{(q)} \\
\beta_{21}^{(q)} & \beta_{22}^{(q)}
\end{bmatrix}
\begin{bmatrix}
s(t-	au_1^{(q)}) + v_1^{(q)}(t) \\
s(t-	au_2^{(q)}) + v_2^{(q)}(t)
\end{bmatrix}
\end{align*}
$$

(1)

where $s(t)$ is a typical UWB signal, and here we use a basic Gaussian pulse waveform,

$$
(2) \quad s(t) = A_p e^{-\pi t^2/\tau_p^2}
$$

where $A_p$ is the amplitude of the UWB pulse, $\tau_p$ is a parameter to determine the UWB pulse width. $\beta_{11}, \beta_{22}$ and $\tau_{11}, \tau_{22}$ represent the complex fading amplitudes and multipath time delays of the $l$-th path arriving at the anchor node 1 and 2, respectively, where $\tau_{11}, \tau_{22}$ denote the first path delays, and $\tau_{21} - \tau_{11}$ is the time difference of arrival.

We sample the received signals and assume the sampling number in the $q$-th snapshot to be $N$. The received data models of the two anchor nodes can be written as

$$
\begin{align*}
\begin{bmatrix}
Y_1^{(q)} \\
Y_2^{(q)}
\end{bmatrix} &= \begin{bmatrix}
S_1(\tau) \\
S_2(\tau)
\end{bmatrix}
\begin{bmatrix}
B_1^{(q)} \\
B_2^{(q)}
\end{bmatrix} + \begin{bmatrix}
V_1^{(q)} \\
V_2^{(q)}
\end{bmatrix}
\end{align*}
$$

(3)

where,

$$
S_1(\tau) = \begin{bmatrix}
s(T_1 - \tau_{11}),s(T_1 - \tau_{12}),\ldots,s(T_1 - \tau_{1L_1}) \\
s(2T_1 - \tau_{11}),s(2T_1 - \tau_{12}),\ldots,s(2T_1 - \tau_{1L_1}) \\
\vdots \\
s(NT_1 - \tau_{11}),s(NT_1 - \tau_{12}),\ldots,s(NT_1 - \tau_{1L_1}) \\
s(T_2 - \tau_{21}),s(T_2 - \tau_{22}),\ldots,s(T_2 - \tau_{2L_2}) \\
\vdots \\
s(NT_2 - \tau_{21}),s(NT_2 - \tau_{22}),\ldots,s(NT_2 - \tau_{2L_2})
\end{bmatrix},
$$

$$
S_2(\tau) = \begin{bmatrix}
s(T_1 - \tau_{21}),s(T_1 - \tau_{22}),\ldots,s(T_1 - \tau_{2L_2}) \\
\vdots \\
s(NT_1 - \tau_{21}),s(NT_1 - \tau_{22}),\ldots,s(NT_1 - \tau_{2L_2})
\end{bmatrix}
$$

are two matrices of multipath delay signals.
$$B_1^{(q)} = [\beta_1^{(q)}, \cdots, \beta_k^{(q)}]^T$$ and $$B_2 = [\beta_1^{(q)}, \cdots, \beta_k^{(q)}]^T$$ are two matrices of complex fading. $$V_1^{(q)}$$ and $$V_2^{(q)}$$ are $$N \times 1$$ matrices of noise. $$T_j$$ is the sampling period.

**CCD-based Multipath Delay Estimation Algorithm**

In this section, MUSIC with CCD algorithm is employed for multipath delay estimation. The process is as follows: Firstly, a composite matrix $$Y^{(q)}$$ is defined as

$$Y^{(q)} = \begin{bmatrix} Y_1^{(q)} \\ Y_2^{(q)} \end{bmatrix}$$

Then its covariance matrix can be obtained as

$$R_y = E\left\{ Y^{(q)}Y^{(q)H} \right\} = \begin{bmatrix} R_{Y_1^{(q)}} & R_{Y_1^{(q)}Y_2^{(q)}} \\ R_{Y_1^{(q)}Y_2^{(q)}}^T & R_{Y_2^{(q)}} \end{bmatrix}$$

where $$R_{Y_1^{(q)}}, R_{Y_2^{(q)}}, R_{Y_1^{(q)}Y_2^{(q)}}, R_{Y_1^{(q)}Y_2^{(q)}}^T$$ are four $$N \times N$$ submatrices of $$R_y$$. Since the two anchor nodes are separated, the noise $$V_1^{(q)}$$ and $$V_2^{(q)}$$ are uncorrelated. Therefore, the noise items in $$R_{Y_2^{(q)}}$$ and $$R_{Y_1^{(q)}Y_2^{(q)}}^T$$ can be eliminated. Then, from the singular value decomposition (SVD) of the matrix $$R_{Y_1^{(q)}Y_2^{(q)}}^T$$, we can obtain

$$R_{Y_1^{(q)}Y_2^{(q)}}^T = U_1 \Sigma U_2^T$$

Define

$$L_1 = R_{Y_1^{(q)}Y_2^{(q)}}^T U_1, \quad L_2 = R_{Y_1^{(q)}Y_2^{(q)}}^T U_2$$

Here $$L_1^H R_1 = L_2^H R_2 = I$$. Decompose $$L_1 = [L_{1_1}, \cdots, L_{1_1}]$$, $$L_2 = [L_{2_2}, \cdots, L_{2_2}]$$, $$R_1 = [R_{1_1}, \cdots, R_{1_1}]$$, $$R_2 = [R_{2_2}, \cdots, R_{2_2}]$$, and we define

$$P_{1v} = L_{1v} R_{1v}^H, \quad P_{2v} = L_{2v} R_{2v}^H$$

Refer to the derivation process in literature [8], we can obtain

$$P_{1v}^H S_1(\tau) = 0, \quad P_{2v}^H S_2(\tau) = 0$$

Therefore, when the MUSIC pseudo spectrum achieves peak values, the corresponding abscissas are the estimated multipath time delays. The curve function of the MUSIC pseudo spectrum is

$$P_{\text{peak1}} = \frac{1}{s(\tau)^H P_{1v}^H P_{1v} s(\tau)}$$

$$P_{\text{peak2}} = \frac{1}{s(\tau)^H P_{2v}^H P_{2v} s(\tau)}$$

where $$s(\tau) = [s(T_1 - \tau), s(2T_1 - \tau), \cdots, s(NT_1 - \tau)]^T$$.

**Three-dimensional Chan Algorithm for Localization**

We assume that the coordinate of anchor nodes is $$(X_i, Y_i, Z_i)^T$$, $$i = 0, 1, 2, 3$$, where $$i = 0$$ represents the main anchor node, and $$i = 1, 2, 3$$ represent the other three anchor nodes. $$r_i$$ denotes the distance between the unknown node and the $$i$$-th anchor node, and $$\Delta r_i$$ denotes the difference of the distances from the unknown node to $$i$$-th anchor node and the main anchor node.

$$r_i^2 = (x - X_i)^2 + (y - Y_i)^2 + (z - Z_i)^2$$

$$r_i^2 = (x - X_0)^2 + (y - Y_0)^2 + (z - Z_0)^2$$

From equation (13), we have

$$\Delta r_i = r_i - r_0 = c\Delta t_i$$

where $$c$$ is velocity of light, $$\Delta t_i = r_i - r_0$$ is the TDOA from the unknown node to the $$i$$-th anchor node and to the main anchor node.

To solve the non-linear equations (14), the solutions $$r_0$$ are

$$r_0 = -b \pm \sqrt{b^2 - ac},$$

where

$$a = n_i^2 + n_2^2 + n_3^2 - 1$$

$$b = (m_1 - x_0)n_1 + (m_2 - y_0)n_2 + (m_3 - z_0)n_3$$

$$c = (m_1 - x_0)^2 + (m_2 - y_0)^2 + (m_3 - z_0)^2$$

**Multilateral 3D Localization Process**

According to the estimation of multiple time delay for an node, the TDOA and distances can be determined by multiple anchor nodes. An example of the geometry relationship of the cross points of spheres using 4 anchor nodes is shown in fig.1. We use multilateral localization to calculate the 3D positions of an unknown node.

![Fig. 1: The geometric relationship in 3D localization](image-url)
Simulations

CCD-based multipath delay estimation

Due to the multipath effect, we assume that the propagation multipath delays of the unknown node to two anchor nodes are \([0.5, 0.6, 0.7]\) ns and \([0.2, 0.25]\) ns, respectively. The sampling period is \(T_s=0.01\) ns. The sampling number is \(N=100\) in one snapshot and the number of snapshots is \(Q=200\) to ensure high-resolution. The background noises are colored noise of AR model. Fig. 3 (a) shows the waveform of transmitted signal and fig. 3 (b)(c) shows the estimated time delays arriving at two anchor nodes, SNR=10dB.

a) Transmitted UWB pulses.

![Transmitted UWB pulses](image1)

b) Multipath delay estimation at anchor node 1.

![Multipath delay estimation at anchor node 1](image2)

c) Multipath delay estimation at anchor node 2.

The simulations confirm good resolution performance of the proposed algorithm in a multipath environment.

Position Computation using 3D Chan Algorithm

In fig. 4, the number of anchor nodes is 8, and \(R\) (the coverage radius of anchor nodes) is 10m. In a 40m\(\times\)40m\(\times\)40m space, we randomly generate 10 and 100 unknown nodes in fig. 4 (a) and (b), respectively. The nodes are located using 3D Chan algorithm by the TDOA estimation. Fig. 4 shows the results under colored Gaussian noise, SNR=10dB. It shows that the coordinates of unknown nodes can be estimated with high accuracy in complex noise environment even with low density of anchor nodes.

a) 10 nodes

![10 nodes](image3)

b) 100 nodes

![100 nodes](image4)
Accuracy of the Proposed Algorithm

We define Relativeerror as follows,

$$\text{Relativeerror} = \frac{1}{N_{\text{node}}} \sum_{n=1}^{N_{\text{node}}} \left( (x_n - \hat{x}_n)^2 + (y_n - \hat{y}_n)^2 + (z_n - \hat{z}_n)^2 \right)$$

where Relativeerror denotes the location relative error. \((x_m, y_m, z_m)\) and \((\hat{x}_m, \hat{y}_m, \hat{z}_m)\) denotes the coordinates of the real and the estimated positions for the \(m\)-th unknown node, respectively. \(N_{\text{node}}\) denotes the number of unknown nodes. We generate 100 unknown nodes randomly in the environment added with colored noise of AR model, then localize them and calculate Relativeerror. In fig. 5 (a), SNR is 10 dB and the number of anchor nodes varies from 5 to 8. In fig. 5 (b), SNR ranges from -10 dB to 15 dB and the number of anchor nodes is 8. We generate randomly 100 unknown nodes and localize them using traditional MUSIC-based estimation algorithm and the proposed algorithm in colored noise environment.

Summary

In this paper, we design a novel 3D localization scheme for UWB wireless sensor network. It mainly utilizes CCD-based TDOA estimation, multilateral localization and 3D Chan algorithms. As we known, accurate node localization in multipath and complex noise is an existing problem in most localization algorithms of WSN. Our algorithm has some key superiorities, such as high accuracy, robust inhibition on multipath effect and colored noise. The CCD-based TDOA measurement algorithm is effective in multipath and colored noise environment, and by the use of multilateral localization instead of trilateral localization, the accuracy of the proposed method is improved.

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REFERENCES


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