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A Novel RNA Genetic Algorithm for the Parameter Estimation of the Ordinary Differential Equations with Multiple Solutions

***Abstract**. In fluid mechanics, to obtain the multiple solutions in ordinary differential equations is always a concerned and difficult problem. In this paper, a novel RNA genetic algorithm (NRNA-GA) inspired by RNA molecular structure and operators is proposed to solve the parameter estimation problems of the multiple solutions in fluid mechanics. This algorithm has improved greatly in precision and the success rate. Multiple solutions can be found through changing accuracy and search coverage and multi-iterations of computer. At last, parameter estimation of the ordinary differential equations with multiple solutions is calculated. We found that the result has great accuracy and this method is practical.*

***Streszczenie.** W artykule zaproponowano nowy algorytm genetyczny NRNA-GA inspirowany strukturą molekularną RNA przeznaczony do rozwiązywania równań z wieloma rozwiązaniami w mechanice cieczy. (**Nowy algorytm genetyczny do ustalania parametrów w równaniach różniczkowych o wielu rozwiązaniach**)*

Keywords: fluid mechanics, the ordinary differential equations, the multiple solutions, genetic algorithm, parameter estimation
Słowa kluczowe: mechanika cieczy, równania różniczkowe

1. Introduction

In fluid mechanics, the multiple solutions of the boundary value problems are important topics and have received considerable attentions. Many researchers had put forward some methods to solve such problems [1]. In general, a shooting method based on the fourth order Runge–Kutta scheme is effective numerically, which have been used to solve a lot of problems. In order to solve such problems, the boundary value problems are transformed into initial value problems by introducing new unknown parameters, which are decided by the boundary conditions. However, it is difficult to find all the unknown parameters which satisfy the boundary conditions, especially for the problems that the equation has multiple solutions. Later, the Homotopy Analysis Method is proposed by Liao SJ[2-3], which also is an efficient analytical method and has been used to solve many problems with multiple solutions[4-6]. However, the HAM suffers from a number of restrictive measures, such as the requirement that the solution sought ought to conform to the so-called rule of solution expression and the rule of coefficient ergodicity. By great search ability of NRNA-GA, many problems could be solved directly.

For traditional genetic algorithm, it's easy to fall into local optimum in multi-peak value. To avoid premature, Wang KT and Wang N[7] put forward the NRNA-GA and achieved better accuracy and computational stability. As the computational stability improves, The GA has been applied to many related fields by many scholars[8-14]. In this paper, combining this NRNA-GA with Runge–Kutta method, two problems with multiple solutions in fluid mechanics have been solved directly.

2. Runge–Kutta method

The procedure of this NRNA-GA and Runge–Kutta method can be summarized as follows:

Step 1: Initialize a population with individuals, individuals contains $s_1, s_2, \cdots\cdots, s_n$ and $t_1, t_2, \cdots\cdots, t_n$,that are the parameters.

Step 2: Having known s and t ,we use the Runge–Kutta method to calculate $f(s_1,t_1), f(s_2,t_2), \cdots\cdots, f(s_n,t_n)$, Choose the best half of individuals and the worst half of individuals composing population. Sort the population into two groups, neural individuals and deleterious individuals.

Step 3: Use the second selection criterion to reselect the individuals and sort the new population according to fitness value. Find the best half of individuals again.

Step 4: Take the permutation operator and the stem-loop operator in the new neural individuals.

Step 5: Carry out mutation operator with adaptive mutation probability.

Step 6: Carry out direct search, the best searching result will be saved to the next generation.

Step 7: Repeat step 2 to step 6 until the termination criteria are met, and the solution is found.

Step 8: If the absolute value of this solution and previous solution is bigger than ξ (ξ denotes the smallest distance of the two solutions, in this paper, we suppose that it is 0.001), then this solution would be saved, the number of solution gets three, iteration would be ended or go to step 1 with changing the search coverage and precision of s and t .

3. Parameter estimation of the multiple solutions of ordinary differential equations

This paper introduces two examples about the multiple solutions of ordinary differential equations. Compared with former results, this method is practical and could be a new method for solving multiple solutions of ordinary differential equations.

Example 1 Standard form of example one

(1) $$F'''' + a(xF''' + 3F'') + R_e(FF''' - F'F'') = 0$$

(2) $$F(0) = 0, F''(0) = 0, F(1) = 1, F'(1) = 0$$

This problem was proposed by Dauenhauer EC and Majdalani J[15].The problem is described as the flow of the fluid through a porous channel with expanding or contracting walls. The multiple solutions about this problem had been verified by HAM.

In order to obtain numerical solutions, we transfer the problem Eq.1 and Eq.2 into a system of first-order equations by denoting the F, F', F'', F''', F'''' using variables F, U, V and W , respectively

(3) $$F' = U$$

(4) $$U' = V$$

(5) $$V' = W$$

(6) $$W' + a(xW + 3V) + R_e(FW - UV) = 0$$

The corresponding boundary conditions are:

(7) $$F(0) = 0, V(0) = 1$$

(8) $$F(1) = 1, U(1) = 0$$

We introduce the parameters s and t as:

(9) $$U(0) = s, W(0) = t$$

Then, the problem is to find the parameters s, t and the solution of the Eqs.(3)~(7),(9) to satisfy the boundary conditions Eq.(8), which equals to solve the minimum of the Eq (10).

(10) $$\min f(s,t) = \left|F_{s,t}(1) - F(1)\right| + \left|U_{s,t}(1) - U(1)\right|$$

In this paper, a and R_e are constant, F,U,V,W are functions of x, the s and t could be calculated by IRNA-GA combining Runge-Kutta method, which are shown in Table 1 and Fig.1

Table 1. Results of example 1

s	t
$a=1, R_e=-15$	
-3.07073666351716	49.47642075453300
0.90299839780270	1.78086800119114
1.99589227130541	-7.67095271848505
$a=1, R_e=-20$	
-3.82265152902399	61.0776179413526
1.03702993746147	0.29587895119010
2.0447720197766	-7.99985122623911
$a=1, R_e=-25$	
-4.22781458974682	67.8332309623498
1.04706972911731	0.05165295758529
2.12878614480812	-8.38525618100756
$a=1.5, R_e=-11$	
-1.02592176353465	24.3534811945844
0.16967476425616	10.2593624746419
2.7342087588808	-15.4218567423479
$a=1.5, R_e=-15$	
-2.97302052785924	50.3904454468257
0.96194263643516	1.39321598844909
3.04894593132952	-17.7416719225856
$a=1.5, R_e=-20$	
-3.73830353823756	62.2668826917839
1.04798035429035	0.25064712778261
3.14237813863722	-18.4764310889657

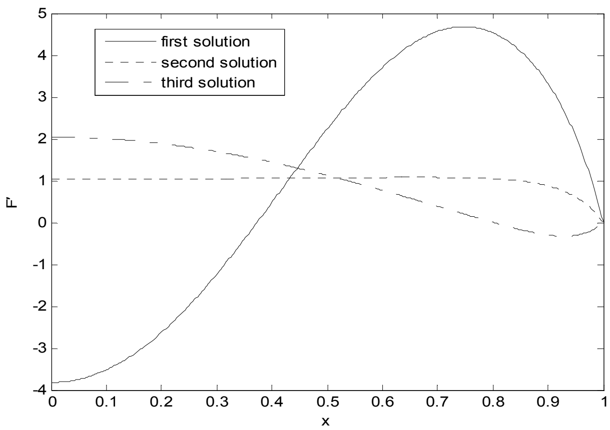

Fig. 1. Three solutions for F' when $a=1, R_e=-20$

Example 2 Standard form of example two

This case is proposed based on the first one, which considers slip boundary condition mainly.

(11) $$F'''' + a(xF''' + 3F'') + R_e(FF''' - F'F'') = 0$$

(12) $$F(0)=0, F''(0)=0, F'(1)=-\phi F''(1), F(1)=1$$

In order to obtain numerical solutions, we transfer the problem Eq.(11) and Eq.(12) into a system of first-order equations by denoting the F, F', F'', F''', F'''' using variables F,U,V and W, respectively

(13) $$F' = U$$

(14) $$U' = V$$

(15) $$V' = W$$

(16) $$W' + a(xW + 3V) + R_e(FW - UV) = 0$$

The corresponding boundary conditions are:

(17) $$F(0)=0, V(0)=1$$

(18) $$F(1)=1, U(1)=-\phi V(1)$$

We introduce the parameters s and t as:

(19) $$U(0)=s, W(0)=t$$

Then, the problem is to find the parameters s, t and the Eqs.(13)~(17), (19) to satisfy the boundary conditions the Eq.18, which equals to solve the minimum of the Eq.20.

(20) $$\min f(s,t) = \left|F_{s,t}(1) - F(1)\right| + \left|U_{s,t}(1) + \phi V_{s,t}(1)\right|$$

In this paper, a, R_e and ϕ are constant, F,U,V,W are functions of x, the s and t could be calculated by NRNA-GA combining Runge-Kutta method, which are shown in Table 2 and Fig.2.

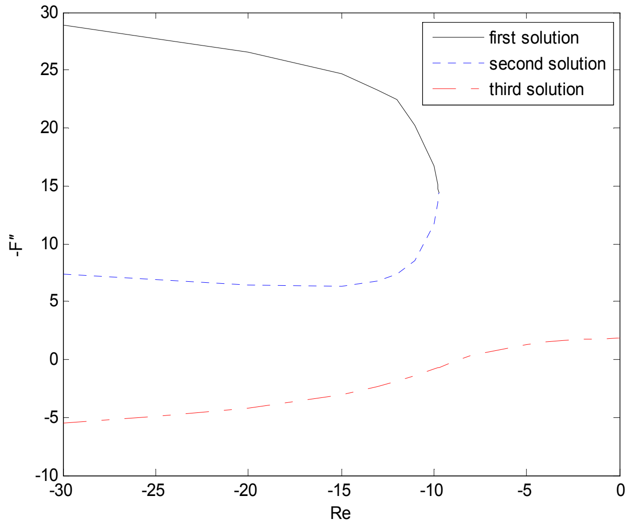

Fig. 2. Variation of $F''(1)$ with R_e when $a=1, \phi=-1$

Table 2. Results of example 2

	t	$F''(1)$
$a=1, R_e=-30, \phi=0.1$		
-4.76529711849621	68.5776158756293	-28.9057
1.01928828663859	0.00288486549493	-7.3478
2.14694062094302	-8.48814539070719	5.4881
$a=1, R_e=-25, \phi=0.1$		
-4.54719175033372	65.0716698340582	-27.7435
1.01464866102083	0.02205372382491	-6.8544
2.12497990129461	-8.37004001576469	4.9166
$a=1, R_e=-20, \phi=0.1$		
-4.22089200517941	59.8687076255578	-26.5179
1.01091020065614	0.13835433443889	-6.3986
2.10347013804449	-8.23864977150653	4.1778
$a=1, R_e=-15, \phi=0.1$		
-3.60491361735191	50.7943512903097	-24.7075
0.94384679942016	0.84593316219643	-6.2855
2.05470359349966	-7.94017504219414	3.0588
$a=1, R_e=-13, \phi=0.1$		
-3.12511995437622	44.3618164925137	-23.2684
0.86207370107576	1.75917190532014	-6.7675
1.98580910963607	-7.54497231125171	2.2968
$a=1, R_e=-12, \phi=0.1$		
-2.83219881825418	40.6888820383267	-22.4503
0.72305219723039	2.92933057053818	-7.3224
1.96405584802014	-7.36181911511876	1.904
$a=1, R_e=-11, \phi=0.1$		

-2.23047786485621	33.3837350329721	-20.2706
0.490547035935	5.06837488850948	-8.4944
1.91274891279469	-7.00938868746488	1.3827
$a=1, Re=-10, \phi=0.1$		
-1.29568607704307	22.9796225976044	-16.7559
-0.16386663614862	11.3364008275034	-11.7346
1.86085297932403	-6.60646119271783	0.8319
$a=1, Re=-9.8, \phi=0.1$		
-0.89065683219827	18.7471886473758	-15.1064
-0.54395807550832	15.1822221744104	-13.561
1.85007700927449	-6.51853883325089	0.7187
$a=1, Re=-9.75, \phi=0.1$		
-0.78337246975242	17.6706726471126	-14.6787
-0.68012279990263	16.6066961168792	-14.2176
1.84659719142646	-6.4926967738412	0.6884
$a=1, Re=-9.74, \phi=0.1$		
-0.72510464201416	17.0778479234031	-14.4293
1.84218401163484	-6.47052566205862	0.6727
$a=1, Re=-9, \phi=0.1$		
1.79843767134007	-6.11290776441621	0.2493
$a=1, Re=-8, \phi=0.1$		
1.7259840311966	-5.55555952919948	-0.311
$a=1, Re=-5, \phi=0.1$		
1.58259906353928	-4.37109574582475	-1.3362
$a=1, Re=-4, \phi=0.1$		
1.55603922749501	-4.14102176213784	-1.5049
$a=1, Re=-3, \phi=0.1$		
1.53610742953001	-3.96568869726389	-1.6243
$a=1, Re=-2, \phi=0.1$		
1.52137315782861	-3.83160205640841	-1.7102
$a=1, Re=-1, \phi=0.1$		
1.50983369584887	-3.72607796813917	-1.7727
$a=1, Re=0, \phi=0.1$		
1.5010122063189	-3.64282218046718	-1.8194

4. Conclusions

In this paper, the IRNA-GA combining with Runge–Kutta method is applied to two examples and the result is satisfying. So the method is practical and could be a new method for solving multiple solutions of ordinary differential equations.

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