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Lightning induced effects on lossy multiconductor power lines with ground wires and non-linear loads - Part I: model

***Abstract**. In the paper, which is a companion paper of part II: simulation results and experimental validation, we will present a model for the calculation of induced voltages produced by indirect lightning on multiconductor power lines. In particular, the case of power lines with ground wires terminated on non-linear loads is studied. The power line is represented by an equivalent time domain m-port, and the effects of the lightning excitation are represented through equivalent independent sources. This equivalent time-domain circuit allows treating easily non-linear terminations such as surge arresters.*

Streszczenie.** W artykule zaprezentowano model obliczeń napięć indukowanych podczas nie bezpośredniego wyładowania na wieloprzewodową linię zasilania. Uwzględniono przewód uziemiony podłączony do nieliniowego obciążenia. (**Efekty powodowane wyładowaniem w wieloprzewodowej linii zasilającej z przewodem uziemionym obciążonym nieliniowo – część I-model)

Keywords: lightning, power transmission lines, surge arresters.
Słowa kluczowe:wyładowania, linia zasiilająca.

Introduction

The annual cost of lightning damage to power lines, transformers and other utility equipment is very high. In some instances lightning may be the cause for 45 percent of all power outages and disturbances on equipments, e.g. [1-6]. Furthermore, overvoltages induced on power lines by lightning affect significantly power quality. Recent progress in the area of lightning induced voltages calculation is significant, both from numerical and analytical point of view [e.g 7-14] . This paper, which is a companion paper of *Part II: simulation results and experimental validation*, aims to analyse a model for the calculation of induced voltages produced by indirect lightning on multiconductor lines with ground wires and non-linear terminations such as surge-arresters: Fig.1 shows the geometry of the system. The lightning produced fields are usually calculated considering the stroke channel as a vertical antenna above a ground plane [7,8]; we have made this assumption too, although lightning tortuosity can play a role [e.g 15-18]. Starting from the lightning current at the channel base [19], we have used the MTLL model [20] to specify the space-time distribution of the current along the channel; however, the procedure developed allows the use of different return stroke models [21,22]. As to the electromagnetic coupling of the lightning field with the multiconductor line, we will use a time-domain convolution method to analyse the case of a lossy line terminated on non-linear loads. In this way the boundary-initial value problem describing the overall system is recast into a system of non-linear Volterra integral equations of the second kind. The line is represented as a time-domain dynamic m-ports through the convolution between the terminal currents and voltages with the impulse responses of the line, and the effects of the exciting fields are taken into account through equivalent sources at the two terminations. As the time-domain equivalent circuit is obtained starting from a frequency domain model, the proposed method can be easily applied to the general case of lossy lines with frequency dependent parameters. The procedure considered allows the exact evaluation of the irregular parts of the line impulse responses, namely the parts describing the propagation.

In the paper the model describing the lightning phenolmenon is first presented, then the calculation of the produced electromagnetic fields is carried out; finally the field-to-line coupling model and the time-domain equivalent circuit of the line are presented. Conclusions will close the paper. Experimental validation and numerical results will be left to the companion paper Part II.

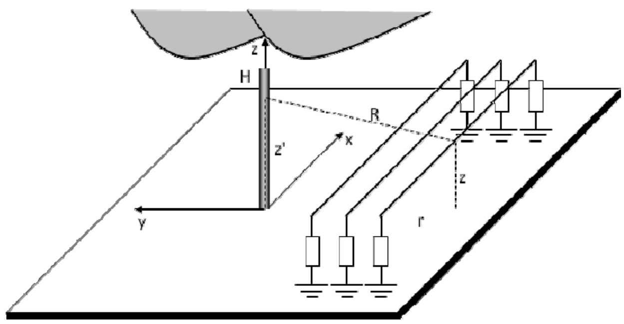

Fig.1. Geometry of the system

Lightning model

The model used in the present study for the propagation of the lightning current along the channel is the Modified Transmission Line Linear (MTLL) model [20]. According to the this model the current at the time instant t and at the abscissa z' of the stroke channel is given by

$$i(z',t) = i\left(0, t - z'/v\right)\left(1 - z'/H\right)u(t - z'/v) \quad (1)$$

where v is the velocity of the return stroke, H is the height of the channel constant and $u(t)$ is the step function. The expression for the channel base current $i(0,t)$ is the one proposed by Heidler [16]

$$i(0,t) = \frac{I_0}{\eta}\frac{(t/\tau_1)^n}{1+(t/\tau_1)^n}\exp\left(-t/\tau_2\right) \quad (2)$$

where I_0 is the amplitude of the channel base current, τ_1 and τ_2 are respectively the front and decay time constants, n is an integer number between 2 and 10, and η is the amplitude correction factor, given by

$$\eta = \exp\left[-(\tau_1/\tau_2)(n\tau_2/\tau_1)^{(1/n)}\right] \quad (3)$$

A sum of two functions (2) has been chosen in order to better reproduce the overall waveshape of the current as observed in typical experimental results. In Fig. 2 the current $i(0,t)$ is plotted assuming for the above parameters the values given in Table 1 of reference [7].

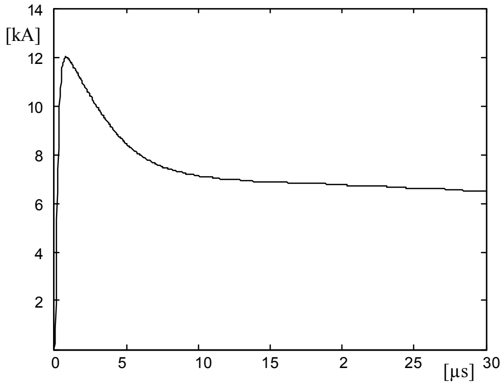

Fig.2. Typical lightning current over 30 μs

The analysis has to be performed in the frequency domain, hence we evaluate the Fourier transform of $i(0,t)$

$$(4) \qquad I(0,\omega)=\frac{I_0}{\eta}\int_0^{+\infty}\frac{(t/\tau_1)^n}{1+(t/\tau_1)^n}\exp(-t/\tau_2-j\omega t)dt$$

To this aim let us put $u=t/\tau_1$. By introducing the complex variable $p=\tau_1/\tau_2+j\omega\tau_1$ we have

$$(5) \qquad \begin{aligned} I(0,\omega)&=\frac{I_0\tau_1}{\eta}\int_0^{+\infty}\frac{u^n}{1+u^n}\exp(-pu)du= \\ &=\frac{I_0\tau_1}{\eta p}-\frac{I_0\tau_1}{\eta}\int_0^{\infty}\frac{\exp(-pu)}{1+u^n}du \end{aligned}$$

which is formally a Laplace transform. Expression (5) has been analytically solved in [23] and the result reads

$$(6) \qquad I(0,\omega)=\frac{I_0}{\eta p}+\frac{I_0}{\eta n p}\sum_{k=0}^{n-1}(-pu_k)\exp(-pu_k)E_1(-pu_k)$$

where u_k are the n roots of unity, and E_1 is the exponential integral function [24]. Using (6), the amplitude spectrum for the channel base current of Fig. 2 has been calculated and plotted in Fig.3. We have also plotted in Fig. 4 the amplitude spectrum of the first contribution of the total current, the most relevant for the peak of the current [25].

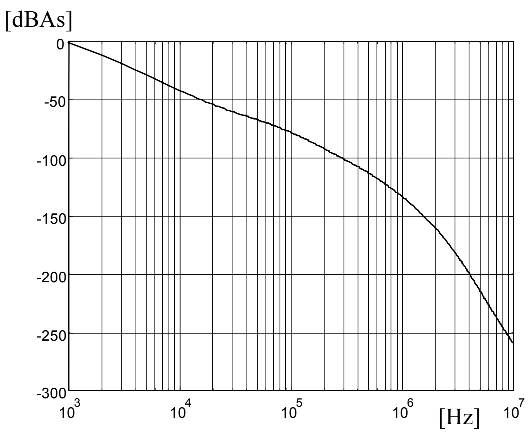

Fig.3. Amplitude spectrum of the total current calculated using (6)

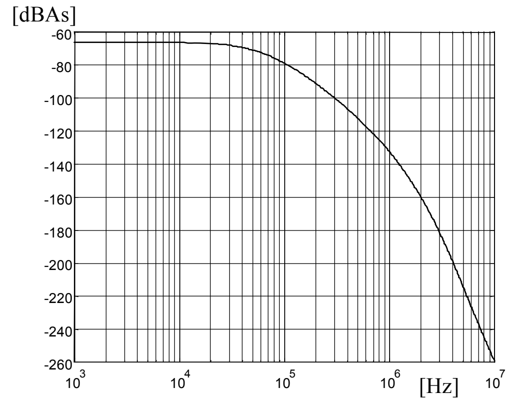

Fig.4. Amplitude spectrum of the first contribution to total current

Lightning return-stroke electromagnetic field

In the calculation of the electromagnetic fields associated to the lightning return stroke we will assume the ground as perfect conductor. This is a reasonable approximation for the vertical component of the electromagnetic fields, but the horizontal field is much more sensible to the ground conductivity [26]. However, we remark that the calculations are made starting from an analysis in the frequency domain, where both the rigorous theory [27] and the proposed approximations [28,29] could be directly implemented.

In the hypothesis of perfect conductive ground the vertical and the horizontal components of the electromagnetic field are given in [7]. In the frequency domain these expressions read

$$(7) \qquad E_z=\frac{1}{4\pi\varepsilon_0}\int_{-H}^{H}\left[\frac{2(z-z')-r^2}{R^5}\frac{1}{j\omega}+\frac{2(z-z')-r^2}{cR^4}-\frac{r^2}{c^2R^3}j\omega\right]\cdot \cdot I(z',\omega)\exp(-j\omega R/c)dz'$$

$$(8) \qquad E_r=\frac{1}{4\pi\varepsilon_0}\int_{-H}^{H}\left[\frac{3r(z-z')}{R^5}\frac{1}{j\omega}+\frac{3r(z-z')}{cR^4}+\frac{r(z-z')}{c^2R^3}j\omega\right]\cdot \cdot I(z',\omega)\exp(-j\omega R/c)dz'$$

where c is the light speed, R, r, H and z are shown in Fig.1 and $I(z',\omega)$ is the current along the channel. These fields can be directly related to the channel-base current:

$$(9) \qquad E_z=\frac{1}{4\pi\varepsilon_0}\int_{-H}^{H}\left[\frac{2(z-z')-r^2}{R^5}\frac{1}{j\omega}+\frac{2(z-z')-r^2}{cR^4}-\frac{r^2}{c^2R^3}j\omega\right]\cdot \cdot I(0,\omega)\exp(-j\omega(R/c+|z'|/v))\exp(-|z'|/\lambda)dz$$

$$(10) \qquad E_r=\frac{1}{4\pi\varepsilon_0}\int_{-H}^{H}\left[\frac{3r(z-z')}{R^5}\frac{1}{j\omega}+\frac{3r(z-z')}{cR^4}+\frac{r(z-z')}{c^2R^3}j\omega\right]\cdot \cdot I(0,\omega)\exp(-j\omega(R/c+|z'|/v))\exp(-|z'|/\lambda)dz$$

In Fig.5 the amplitude spectrum is shown for the vertical field at a distance of 50 *m*, at the ground level, for the base channel current considered and $\lambda=2000 \quad v=1.9\cdot10^8 m/s$.

In Fig.6 a plot of the vertical electric field evaluated by using an IFFT on the spectrum of Fig.5 is shown.

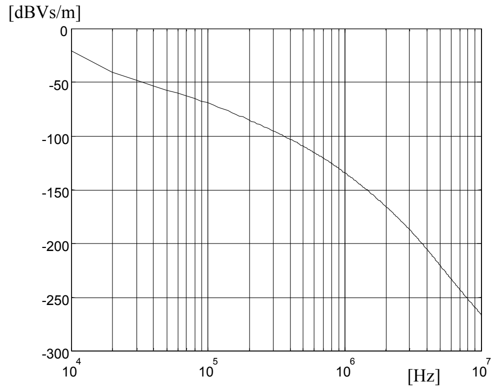

Fig.5. Amplitude spectrum of the electric field for the base-channel in Fig.2 at a distance of 50 *m*

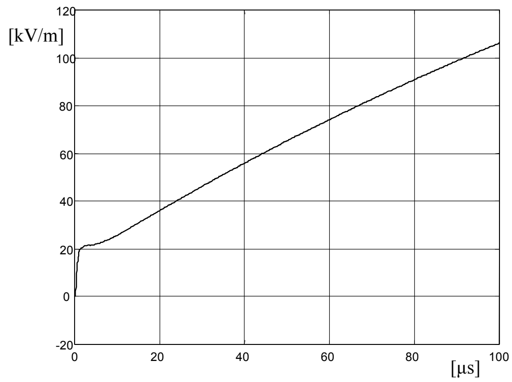

Fig.6. Time-domain vertical electric field for the base-channel in Fig.2, at a distance of 50 *m*

Coupling model and equivalent circuit

The transmission line theory provides many models for describing the coupling of an external EM field to a line [25]. In this paper we adopt the model proposed by Agrawal [30].

In order to describe this model, let us consider a line constituted by n conductors above a ground plane. At the abscissa x and the time instant t, the current flowing in the k^{th} conductor and the voltage of the k^{th} conductor referred to the ground plane are the k^{th} entries of the current and voltage vectors $\boldsymbol{i}(x,t)$ and $\boldsymbol{v}(x,t)$, respectively. For a lossless line, the Agrawal model leads to the following equations

$$\text{(11)} \qquad \begin{cases} \dfrac{\partial \mathbf{v}^s(x,t)}{\partial x} + \mathbf{L}\dfrac{\partial \mathbf{i}(x,t)}{\partial t} = \mathbf{f}(x,t) \\ \dfrac{\partial \mathbf{i}(x,t)}{\partial x} + \mathbf{C}\dfrac{\partial \mathbf{v}^s(x,t)}{\partial t} = \mathbf{0} \end{cases}$$

where $\boldsymbol{L}$ and $\boldsymbol{C}$ are the per unit length inductance and capacitance matrices of the line. The effect of the vertical component of the incident electric field is taken into account as the above equations are written for the *scattered* voltages

$$\text{(12)} \qquad v_k^s(x,t) = v_k(x,t) - v_k^i(x,t)$$

where $v_k^i(x,t)$ is the voltage of the k^{th} conductor due to the vertical incident field $\boldsymbol{E}^i(x,z,t)$ in absence of the line

$$\text{(13)} \qquad v_k^i(x,t) \approx -h_k E_z^i(x,0,t)$$

(h_k is the height of the k^{th} conductor). Besides, (11) are "forced" by the term $\boldsymbol{f}(x,t)$, depending on the horizontal incident field: its k^{th} entry is $f_k(x,t) = E_x^i(x,h_k,t)$.

An effective time-domain model can be achieved by representing the excited line as a dynamic 2n-port. This allows analyzing easily lines terminated on non-linear loads. To achieve such a representation, a preliminary Laplace domain analysis is needed. In this domain, the excited line can be described by the following equations

$$\text{(14)} \qquad \begin{cases} \dfrac{d\boldsymbol{V}^s(x;s)}{dx} + \boldsymbol{Z}(s)\boldsymbol{I}(x;s) = \boldsymbol{F}(x;s) \\ \dfrac{d\boldsymbol{I}(x;s)}{dx} + \boldsymbol{Y}(s)\boldsymbol{V}^s(x;s) = \boldsymbol{0} \end{cases}$$

where the upper case fonts represent the Laplace transforms. Note that equations (14) generalise the model described by (11), as the lossless line is just a particular case [31-33]. By solving (14) we obtain a Laplace domain equivalent representation of the line at the two terminations (Fig.7), indicated with subscripts 0 and d (d is the length of the line) with

$$\text{(15)} \qquad \begin{cases} \mathbf{I}_0(s) = \hat{\mathbf{Y}}_c(s)\mathbf{V}_0^S(s) + \mathbf{J}_0(s) + \mathbf{J}_0^*(s) \\ \mathbf{I}_d(s) = \hat{\mathbf{Y}}_c(s)\mathbf{V}_d^S(s) + \mathbf{J}_d(s) + \mathbf{J}_d^*(s) \end{cases}$$

$$\text{(16)} \qquad \begin{cases} \mathbf{J}_0(s) = \hat{\mathbf{P}}(s)\left[-2\mathbf{I}_d(s) + \mathbf{J}_d(s) + \mathbf{J}_d^*(s)\right] \\ \mathbf{J}_d(s) = \hat{\mathbf{P}}(s)\left[-2\mathbf{I}_0(s) + \mathbf{J}_0(s) + \mathbf{J}_0^*(s)\right] \end{cases}$$

where the *characteristic admittance* matrix $\hat{\boldsymbol{Y}}_C(s)$ and the *propagation function* matrix $\hat{\boldsymbol{P}}(s)$ are given by

$$\text{(17)} \qquad \begin{aligned} &\hat{\boldsymbol{Y}}_C(s) = \sqrt{\boldsymbol{Z}^{-1}(s)\boldsymbol{Y}^{-1}(s)}\,\boldsymbol{Y}(s), \\ &\hat{\boldsymbol{P}}(s) = \exp\left(-d\sqrt{\boldsymbol{Y}(s)\boldsymbol{Z}(s)}\right). \end{aligned}$$

Finally, the independent current sources $\boldsymbol{J}_0^*(s)$ and $\boldsymbol{J}_d^*(s)$ take into account the horizontal electric incident field through

$$\text{(18)} \qquad \begin{cases} \mathbf{J}_0^*(s) = \displaystyle\int_0^d e^{-x\sqrt{\mathbf{Y}(s)\mathbf{Z}(s)}}\mathbf{Y}_C(s)\mathbf{F}(x;s)dx \\ \mathbf{J}_d^*(s) = -\displaystyle\int_0^d e^{-(x-d)\sqrt{\mathbf{Y}(s)\mathbf{Z}(s)}}\mathbf{Y}_C(s)\mathbf{F}(x;s)dx \end{cases}$$

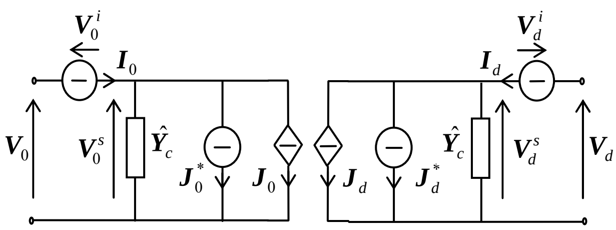

Fig.7. 2*n*-ports equivalent representation of an excited lossy multiconductor line (Laplace domain)

Now, the time-domain *2n*-ports representation of the line can be obtained by computing the inverse Laplace transforms of (15)-(18), and by applying Borel convolution theorem. In the most general case of lossy multiconductor lines, the time-domain impulse responses contain a regular part in addition to the Dirac pulses and other kind of irregular terms. All the irregular terms are evaluated analytically from the asymptotic behaviour of the matrices $\hat{\boldsymbol{Y}}_C(s)$ and $\hat{\boldsymbol{P}}(s)$ as $s \to \infty$, and the regular remainders are easily calculated numerically. The only difference in the resulting time-domain model is the presence of convolution products, due to the regular parts of the impulse responses. The solution can be achieved by means of a recursive approach as in the lossless case.

Conclusions

In this paper, which is a companion paper of *part II: simulation results and experimental validation*, a model for the calculation of induced voltages produced by indirect lightning has been analysed. To perform this analysis, a time-domain equivalent circuit has been considered, in which the transmission line effects are taken into account through the impulse responses of the line, while the effects of the external excitation are reproduced by equivalent independent sources.

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