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Non-linear load model for the local stability studies

Abstract. *This paper presents the non-linear load model which can be used to study the local stability of the power system node. There have been shown the initial assumptions and implemented functions. Model based on Thevenin theory allows to present set part of the power system as voltage source and series impedance. This approach lets perform simple analysis of the influence of the voltage regulation on the local stability margin. In addition prepared program can also plot PU, QU, SU and load curves for the graphical representation of the node condition.*

Streszczenie. *Artykuł prezentuje nieliniowy model obciążenia wykorzystany do badania stabilności lokalnej węzłów systemu elektroenergetycznego. Przedstawiono założenia wstępne oraz realizowane funkcje. Model bazując na teorii Thevenina pozwala zadany fragment systemu, zamodelować jako źródło oraz impedancję systemową. Podejście takie umożliwia przeprowadzenie prostej analizy wpływu regulacji napięcia na wartość zapasu stabilności. Program umożliwia również wykreślanie krzywych nosowych i krzywych typu obciążenia w celu graficznego przedstawienia stanu węzła.* (***Nieliniowy model obciążenia wykorzystany do badania stabilności lokalnej***).

Keywords: non-linear load model, local stability, Thevenin equivalent, voltage regulation.
Słowa kluczowe: nieliniowy model obciążenia, stabilność lokalna, ekwiwalent Thevenina, regulacja napięcia.

Introduction

In the analysis of the power system safety, there are two basic factors: global and local stability. Local stability is also called voltage stability. Global stability is associated with a pair of values: active power P – frequency f, while the local stability: reactive power Q – node voltage U [1]. Studies of these factors can be determined using power flow analysis. To perform this there should be prepared entire power system model. This approach is not suitable for using in protection and automation devices installed in substations.

Global stability relates to the balance of active power generation and consumption. Disturbances of this balance cause frequency changes. Determination of global stability using local measurements seems to be impossible. It is different with local stability. It can be analyzed using measurements and information available on the receiving node.

There are many factors that may cause voltage instability of a node. The most common as well as having the greatest impact on the stability margin are: modification of power system configuration, changes of load impedance parameters and deficit in reactive power. Switching operations that transform system configuration affect the value of series impedance and voltage source of Thevenin model. Load parameters changes cause variations of active and reactive power delivered to the node. This has an influence on voltage value [2]. When such changes occur, incorrect operation of protection automation, voltage regulation systems or system operator can lead to imbalance of stability or in extreme situation to blackout. Therefore it is important to maintain the working point of the node in safety range of voltage stability margin.

To examine safety of the node with using local measurements many algorithms can be used [3, 4, 5]. There has been developed non-linear load model to study the accuracy and properties of the local stability margin determination algorithms during power system configuration changes and the influence of the voltage regulation such as transformer tap changing and reactive power compensation on the accuracy of methods.

There have been the following requirements assumed:

- examination of the stability for freely configurable types of ZIP load such as: constant impedance, constant current, constant power and others
- Thevenin equivalent parameters determination (value of ideal voltage source and series system impedance) of the analyzed part of the power system
- influence of the voltage control (reactive power compensation and/or changing position of tap changer) on the parameters of Thevenin model consideration
- performing changes of modelled power system configuration

Realization

In the initial stage of model development, digital modelling environment has been chosen. Currently, the best known and most commonly used are: MATLAB/Simulink, EMTP (Electromagnetic Transients Program) and PSCAD (Power System Computer Aided Design). There has been paid special attention to simplicity of programming, presentation of the obtained results, the possibility of easy operation and further development of the model, during a choice of environment. Among the above-mentioned programs, MATLAB/Simulink was selected. To facilitate operation, the model has been created in Graphical User Interface (GUI).

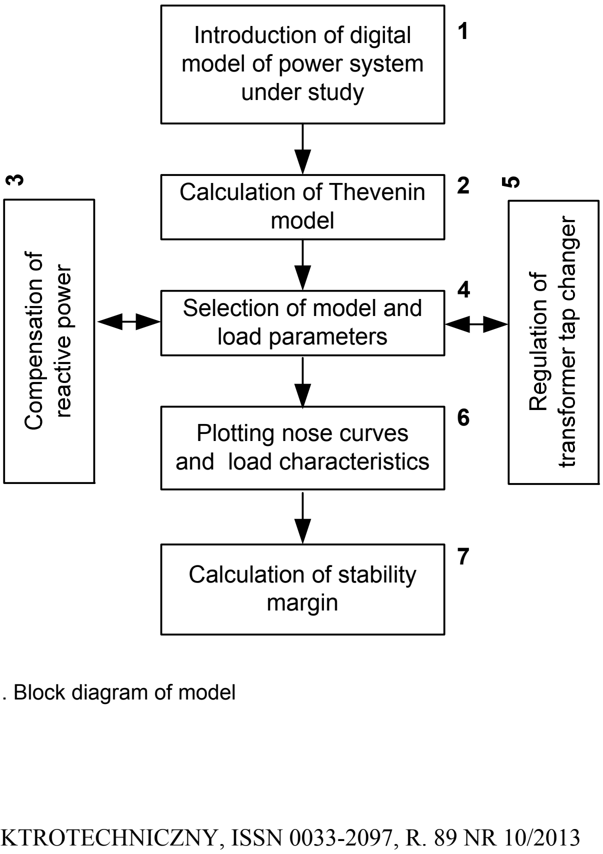
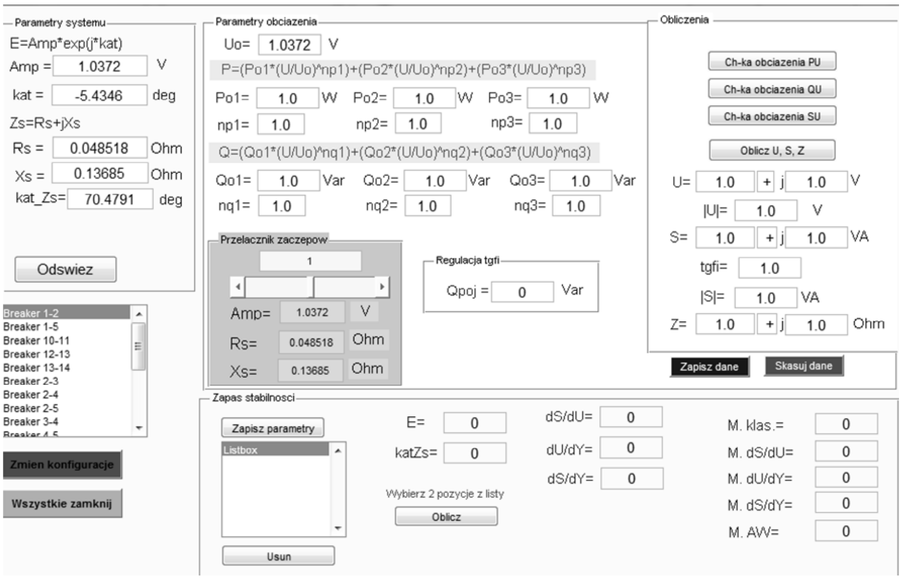


Fig. 1. Block diagram of model



E=Amp*exp(j*kat)

Amp = 1.0372 V

kat = -5.4346 deg

Zs=Rs+jXs

Rs = 0.048518 Ohm

Xs = 0.13685 Ohm

kat_Zs= 70.4791 deg

Uo= 1.0372 V

P=(Po1*(U/Uo)^np1)+(Po2*(U/Uo)^np2)+(Po3*(U/Uo)^np3)

Po1= 1.0 W Po2= 1.0 W Po3= 1.0 W

np1= 1.0 np2= 1.0 np3= 1.0

Q=(Qo1*(U/Uo)^nq1)+(Qo2*(U/Uo)^nq2)+(Qo3*(U/Uo)^nq3)

Qo1= 1.0 Var Qo2= 1.0 Var Qo3= 1.0 Var

nq1= 1.0 nq2= 1.0 nq3= 1.0

1

Amp= 1.0372 V

Rs= 0.048518 Ohm

Xs= 0.13685 Ohm

Qpoj = 0 Var

U= 1.0 + j 1.0 V

|U|= 1.0 V

S= 1.0 + j 1.0 VA

tgfi= 1.0

|S|= 1.0 VA

Z= 1.0 + j 1.0 Ohm

E= 0

katZs= 0

dS/dU= 0

dU/dY= 0

dS/dY= 0

M. klas.= 0

M. dS/dU= 0

M. dU/dY= 0

M. dS/dY= 0

M. AW= 0

Fig. 2. Implementation of the model in the GUI

Figure 1 shows general block diagram of model for the local stability studies. The calculation algorithm of model has been implemented in MATLAB/Simulink. User interface has been prepared in GUI (Fig. 2). The specific functions of block diagram (Fig. 1) correspond to the user interface sections (Fig. 2). User interface is divided into following components:

- Thevenin model parameters calculation,
- load model specification setting,
- voltage control,
- operation point determination and nose curves plotting,
- local stability margin calculation.

The following paragraphs describe individual blocks and functions performed by them.

Thevenin equivalent and load model

To calculate the Thevenin equivalent, digital model of power system section should be prepared. To develop such model, Simulink (SimPowerSystem library) should be used (Fig. 1 block 1). In the voltage stability studies single-phase circuits and established conditions are considered. To the construction of any digital model of power system the basic blocks can be used: connected in series *R*, *L*, *C* components, *PI* type line, voltage sources and switches. Then model to local stability studies developed in GUI system should be connected to the selected receiving node (in place of the load impedance).

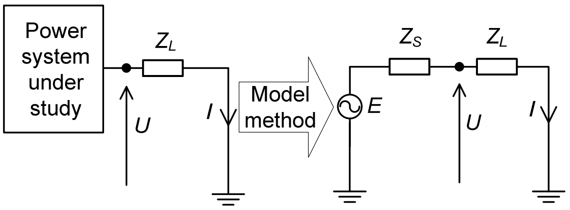


Fig. 3. Thevenin equivalent

When the program starts, Thevenin model parameters (magnitude and angle of series system impedance Z_S and ideal voltage source E (Fig. 3)) are determined automatically (Fig. 1 block 2). Calculation of series impedance parameters is being done by short-circuit all voltage sources and connecting to the considered node unit AC voltage source. Phasor of current measured in such circuit is being used to determine the magnitude and angle of series impedance. Ideal voltage source E parameters are calculated basing on phasor of voltage measured at idle node. To measure the voltage and current phasors Fourier transform with one period window was used [6, 7].

When the Thevenin model parameters are determined then active, reactive power and the shape of voltage characteristic of load should be assumed (Fig. 1 block 4). In the presented model the characteristic of load can be arbitrarily formed. This is very important in local stability studies. There has been applied load model, where the value of active and reactive power depends on the exponential voltage value. Additionally, the polynomial model has been used, where the value of active and reactive power is a weighted sum of each component. It can be written as follows [1, 2]:

$$
\begin{aligned}
P &= P_1\left(\frac{U}{U_0}\right)^{np1} + P_2\left(\frac{U}{U_0}\right)^{np2} + P_3\left(\frac{U}{U_0}\right)^{np3} \\
Q &= Q_1\left(\frac{U}{U_0}\right)^{nq1} + Q_2\left(\frac{U}{U_0}\right)^{nq2} + Q_3\left(\frac{U}{U_0}\right)^{nq3}
\end{aligned} \qquad (1)
$$

where: $np1$, $np2$, $np3$ – active power exponent, $nq1$, $nq2$, $nq3$ – reactive power exponent, P_1, P_2, P_3 – active power components, Q_1, Q_2, Q_3 – reactive power components.

In equation (1) exponents determine the shape of load voltage characteristic and type of load. Table 1 shows examples of the values of exponents and corresponding types of load [2]. Assuming that the exponents are equal to: 2 – impedance does not depend on voltage (constant impedance model – power is proportional to the square of the voltage value), 1 – impedance is proportional to the voltage (constant current model – power is proportional to the voltage), 0 – impedance is proportional to the square of the voltage value (constant power model – power does not depend on the voltage).

Table 1. Typical load model parameters [8]

Type of load	n_p	n_q
Residential	0,9 – 1,7	2,4 – 3,1
Commercial	0,5 – 0,8	2,4 – 2,5
Industrial	0,1 – 1,8	0,6 – 2,2

The presented load model allows shaping load voltage characteristic, therefore different types of load can be analyzed. As a result, this helps to study accuracy of voltage stability margin determination algorithms more precisely.

Voltage magnitude control

In addition this program allows regulate the voltage of considered node using the following methods: reactive power compensation and changing of transformer tap.

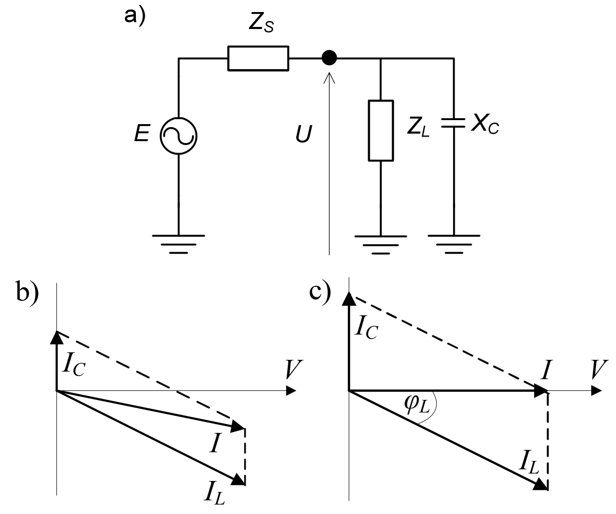


Fig. 4. Thevenin model with compensation of reactive power a) and vector plots of under compensation b) and full compensation c)

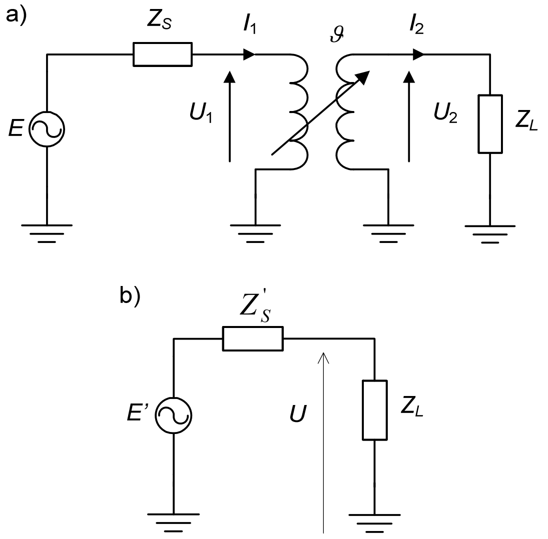


Fig. 5. Thevenin model with transformer tap changer

Reactive power compensation in receiving node (Fig 1 block 3) is represented by static compensator X_C parallel connected to the load impedance (Fig. 4a). These results in a decrease of received reactive power and consequently decrease of load phase angle φ_L. As a result, at constant value of reactive power the voltage level increases. To include compensation in equation (1) value of capacitive reactive power should be subtracted. Load angle tangent $tg\varphi_L$ (Fig. 4c) is changed (reduced) for reactive power Q_C>0, while for Q_C=0 there is no compensation and $tg\varphi_L$ depends only on given active and reactive power of load.

Another way of voltage regulation at the receiving node is transformer tap changing. Transformer tap operation, thereby changing of transformer ratio ϑ causes decrease or increase of the secondary voltage U_2 (Fig. 5a). Analysis of circuit (Fig. 5) can be simplified if series and parallel transformer parameters are omitted. Then only changes of transformer ratio ϑ should be considered. The ratio value influences on Thevenin model parameters. It is assumed that the parameters of the system (Fig. 5b) are converted into secondary terminals of transformer. For such assumption value of voltage source E' and series impedance Z'_S should be calculated as follows:

$$E' = E \cdot \vartheta$$
$$Z'_S = Z_S \cdot (\vartheta)^2 \tag{2}$$

Equation (2) shows that the E' and Z'_S parameters are not changed only for ratio ϑ = 1. Such case occurs when tap changer is in the neutral position. For other values of ratio (different from 1) the E' and Z'_S parameters are changed. It causes variation of local stability margin. In the model it is possible to change ratio ϑ in the range of 0,6 to 1,4.

Operating point calculation

Actual voltage of receiving node is calculated basing on the input parameters: voltage source E, series impedance Z_S, transformer ratio ϑ, value of reactive power compensation and active and reactive power load. Value of voltage is used for the calculation of the load impedance and received power. In model actual active and reactive power depend on the voltage at the receiving node. The load impedance can be considered as non-linear. To calculate the node voltage $\underline{U}$ at the receiving node following equation should be solved:

$$\underline{U} = \underline{E}' - \left(\frac{P + jQ}{\underline{U}}\right)^{*} \cdot \underline{Z}'_S \tag{3}$$

where: P and Q are calculated using formula (1)

As one can see that on both sides of equation (3) the unknown voltage $\underline{U}$ is searched. In this case, it is not possible to determine directly the value of the voltage at the receiving node. Therefore, to determine the voltage either one of the iterative or graphical (Fig. 6) methods should be used.

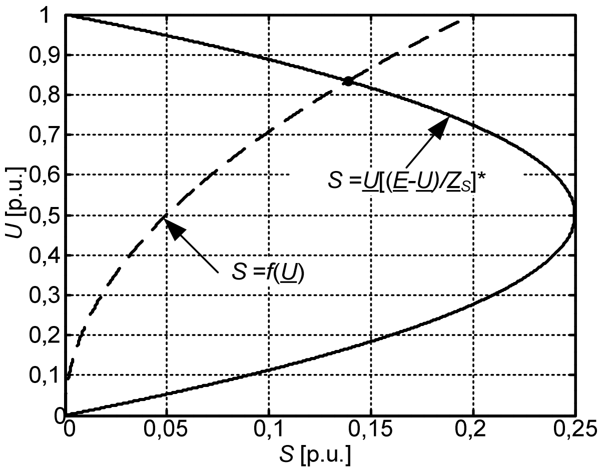


Fig. 6. Operation point calculation

Solution of this non-linear equation using graphical method is not practical, thus iterative methods are generally used. Among many methods of solution non-linear equations Aitken method has been selected [7, 9]. It is multistep algorithm, which has good convergence and simplicity of calculation. Using the Aitken method to solve non-linear equation described by formula (3) following steps should be taken:

1. Assumption of the initial conditions of voltage $\underline{U}$,
2. Taking of two steps of simple iteration:

$$P_{11}=P_1\left(\frac{|\underline{U}|}{U_0}\right)^{np1}+P_2\left(\frac{|\underline{U}|}{U_0}\right)^{np2}+P_3\left(\frac{|\underline{U}|}{U_0}\right)^{np3}$$

(a) $$Q_{11}=Q_1\left(\frac{|\underline{U}|}{U_0}\right)^{nq1}+Q_2\left(\frac{|\underline{U}|}{U_0}\right)^{nq2}+Q_3\left(\frac{|\underline{U}|}{U_0}\right)^{nq3}$$

$$\underline{U}_{wz}=\underline{E}'-\left(\frac{P_{11}+jQ_{11}}{\underline{U}_1}\right)^{*}\cdot\underline{Z}'_S$$

$$P_{22}=P_1\left(\frac{|\underline{U}_{wz}|}{U_0}\right)^{np1}+P_2\left(\frac{|\underline{U}_{wz}|}{U_0}\right)^{np2}+P_3\left(\frac{|\underline{U}_{wz}|}{U_0}\right)^{np3}$$

(b) $$Q_{22}=Q_1\left(\frac{|\underline{U}_{wz}|}{U_0}\right)^{nq1}+Q_2\left(\frac{|\underline{U}_{wz}|}{U_0}\right)^{nq2}+Q_3\left(\frac{|\underline{U}_{wz}|}{U_0}\right)^{nq3}$$

$$\underline{U}_{w}=\underline{E}'-\left(\frac{P_{11}+jQ_{11}}{\underline{U}_2}\right)^{*}\cdot\underline{Z}'_S$$

3. Correction of the results:

$$\Delta=\frac{\left(U_{wz}-U\right)^2}{U_w-2U_{wz}+U}$$

4. Calculation: $U = U - \Delta$
5. When $|\Delta|>1e\text{-}5$ go to step 2

After calculation of voltage of receiving node, active and reactive power, tangent of load angle $tg\varphi$, resistance and reactance of load are calculated according to the equation (1). Below there have been shown calculation of the mixed-type load operating point and an analysis of the influence of the tap changer regulation on the local stability margin.

Example

Figure 7 presents fragment of power system (with parameters) developed in MATLAB/Simulink. It was used for testing and evaluation of non-linear load model. The test model consists of three lines (Z_{11}, Z_{21}, Z_{31}) and two subsystems represented by ideal voltage sources and system impedances (Z_{S1}, Z_{S2}). To the examined node there is connected developed non-linear model. After starting the program there are automatically determined Thevenin model parameters ($\underline{Z}_S$=0,012634+j0,31364; $\underline{E}$=1,0673+j0,0002).

The first case (Fig. 8) presents the assumed load curves and the corresponding PU, QU, SU curves. Load voltage characteristic has the following parameters: P_1=0,2; $np1$=1; P_2=0,15; $np2$=2; P_3=0,2; $np3$=0,15; Q_1=0,12; $nq1$=0; Q_2=Q_3=0; $nq2$=$nq3$=0 (see equation (1)).

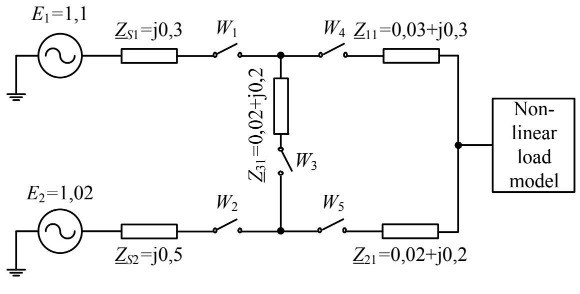


Fig. 7. Example of test model

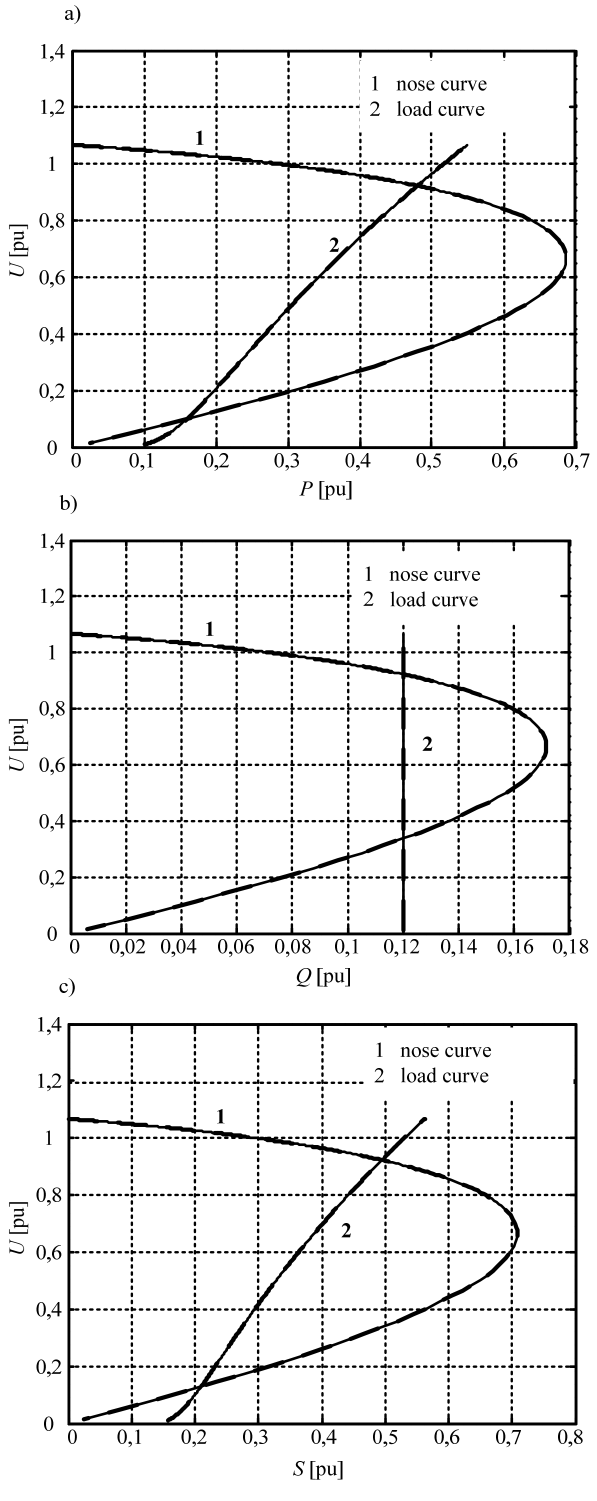


Fig. 8. Load curves for mixed-type load and PU a), QU b), SU c) curves

The load curve of reactive power (Fig. 8b) represents constant power type of load, while the load of active power (Fig. 8a) has an untypical shape. This is caused by the various exponents of active power parameters. Apparent power curve (Fig. 8c) is a composite of two previous curves. For such defined load characteristic, parameters of operation point are: node voltage $\underline{U}$=0,881874-j0,270114; complex apparent power $\underline{S}$=0,480514+j0,12 and complex load impedance $\underline{Z}_L$=0,833197+j0,208076. This example clearly shows that it is possible to obtain any shape of load voltage characteristic using the non-linear model.

Another example is analysis of the influence of the voltage regulation using tap changer on the voltage stability margin.

Figure 9 and 10 show *SU* curves obtained for two different values of the transformer ratio (ϑ = 1 and ϑ = 1,1) and different load type curves. In both cases (Fig. 9 and 10) value of power and angle of specified load is identical, difference occurs in the type of load. The curves in Figure 9 have been plotted for constant impedance model and Figure 10 for constant power model. Changing transformer ratio not only increases node voltage but also changes the voltage stability margin.

In the case of study of constant impedance load model (Fig. 9) value of voltage increases however at the same time local stability margin decreases. This is caused by changing of series system impedance according to the equation (2), whereas there is no change of load impedance. If the load is constant power type, the voltage changes with unchanged stability margin. Current load impedance depends on the square of voltage which means that increasing the voltage causes the raising both impedances (system and load). Relation of these impedances, which have influence on the stability margin, remains the same. Influence of the transformer tap changing on the local stability margin is described in detail in [10].

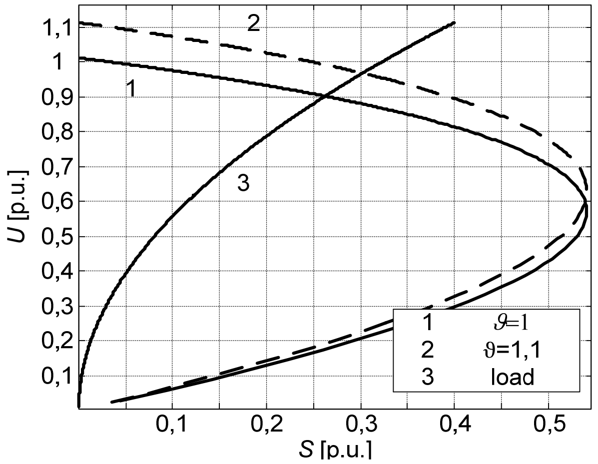


Fig. 9. Regulation of tap changer for constant impedance type of load

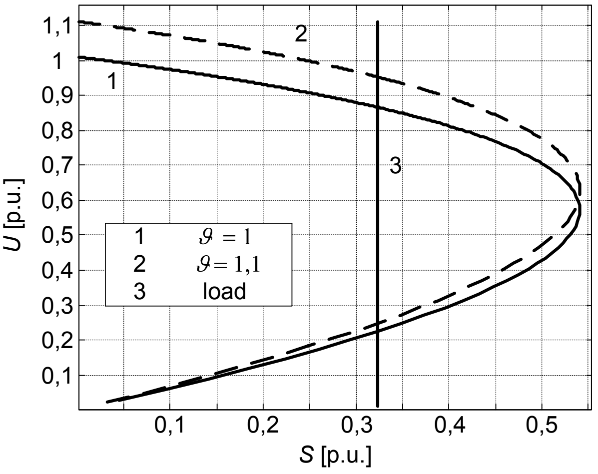


Fig. 10. Regulation of tap changer for constant power type of load

Conclusions

Presented non-linear load model allows to:

- determine Thevenin model parameters for specific power system configuration
- change analyzed system configuration
- shape any voltage characteristic of the load
- study the influence of the voltage control processes on the local stability margin
- test voltage stability margin determination algorithms and consequently analyze safety of receiving node

Model will be used to perform further work: studying the influence of node voltage control processes on the current stability margin, forecasting an effects of switching operations and testing new local stability margin determination methods based on local measurements and global information.

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