Fuel Cell Analytical Modeling: solving the trade-off between accuracy and complexity

Abstract. In this paper, a 5.5 kW Proton Exchange Membrane fuel cell is modeled. The proposed analytical model is described and the parameters identification procedure is further discussed. Simulation results of all three sub-models are compared to test the accuracy of each one. A comparison between simulation and experimental results is provided as well validating the modeling approach.

Introduction
In the last few years, a growing attention to efficiency in power conversion, energy saving and renewable energy sources have been highlighted. Several innovative applications appear promoting complete mobility for electric vehicles and handheld devices [1-3]. Yet, state-of-art is not so mature to spread innovative technologies on the market. Wireless power transfer is the key representative of such applications. Research is actually focused on efficiency-related issues [4-8].

Among renewable sources, fuel cells are the most promising source of energy due to their extremely low environmental impact. In the literature, fuel cell-based power supplies are widely presented. Household appliances, automotive and handheld devices are only a few applications [9-16]. In spite of the great advantages brought by fuel cells, hydrogen production and storage related problems must be solved for a successful marketing of fuel cell-based systems. Innovation on fuel cell technology is therefore highly supported by several research programs within and beyond the European Union. Even concerning quite different scientific research areas, fuel cell modelling is a common requirement. A detailed analysis of the electro-chemical and physical behaviour of the fuel cell stack is required by chemical engineers for manufacturing purposes, thus preferring an analytical modelling approach [17-26]. System engineers need to simulate and test the overall system as closely as possible to its effective working conditions, thus preferring an empirical modelling approach [27-32]. Behavioural simulations are commonly used to validate power supply systems to shorten the overall design process, often taking advantage from several co-simulation options [33-36]. Therefore, choosing an analytical or empirical approach is a critical design step.

This paper proposes a steady-state analytical model of a 5.5 kW Nuvera Proton Exchange Membrane (PEM) fuel cell. Several degrees of accuracy are here proposed. Three versions are proposed, each one accounting for a different number of independent parameters: ten-parameters, six-parameters and four-parameters model. The designer can solve the trade-off between the model complexity and accuracy by himself and according to the specific application, taking all the advantages brought by a unique model architecture. A comparison of the accuracy of each sub-model is provided. Simulation and experimental results are compared to test the high performances and reliability of the proposed fuel cell model. The ten-parameters model is the most accurate but the model complexity is quite high.

The six-parameters model is the best trade-off between the ten- and four-parameters models. Instead, the four-parameters sub-model features very low complexity and the lowest accuracy.

The ten-parameters model
The ten-parameters model is the most complete of all sub-models. This model features the highest accuracy, as it will be shown by experimental results. The following assumptions are made:
- Instantaneous chemical reactions in the polymeric membrane;
- Ideal reactive gases.

The fuel cell voltage is given by:

$$v_{fc} = E_{Nernst} - v_{act} - v_{ohm} - v_{con}$$

where \(E_{Nernst}\), also-called the Nernst-voltage, is the ideal electro-motive force of the electro-chemical cell. The Nernst voltage which is approximately 1.2V for low-temperature cells is never truly obtained even if an open-circuit condition is considered. As described by (1), irreversible losses at both the cathode and the anode should be accounted for. Multiple phenomena contribute to the irreversible losses. Three losses can be distinguished: activation polarization \(v_{act}\), ohmic polarization \(v_{ohm}\), concentration polarization \(v_{con}\). These losses result in a cell voltage \(V\) that is less than its ideal potential, \(E_{Nernst}\).

The Nernst voltage can be expressed as:

$$E_{Nernst} = 1.229 - 0.85 \cdot 10^{-3} (T - 298.15) +$$

$$+ 4.3 \cdot 10^{-5} T \cdot \left[ \ln(p_{H_2}) + \frac{1}{2} \ln(p_{O_2}) \right]$$

where \(T\) is the absolute temperature, \(p_{O_2}\) is the oxygen pressure, \(p_{H_2}\) is the hydrogen pressure.

Activation losses are mainly related to the activation energy which is the required amount to start the chemical reaction. The activation voltage drop is given by:

$$v_{act} = -\xi_1 + \xi_2 T + \xi_3 \ln(C_{O_2}) + \xi_4 T \ln(i_{fc})$$

where \(i_{fc}\) is the fuel cell current, \(T\) the absolute temperature and \(C_{O_2}\) is the oxygen concentration at the cathode electrode which is given by:
related to the conductivity while the electrodes should be designed. Further, thin electrodes minimize the model error. A parameters identification procedure has been carried out to minimize the model error. The ten-parameters model includes the cell dynamic behavior. The dynamic response of the fuel cell is modeled by introducing the capacitor $C$. In PEM fuel cells, the transient response is mainly affected by the charge double-layer phenomenon.

In a PEM fuel cell, the two electrodes are separated by a solid membrane which only allows the H+ ions to pass through, blocking the electron flow. The electrons will flow from the anode through the external load and gather at the surface of the cathode, to which the protons of hydrogen will be attracted at the same time. Thus, two charged layers of opposite polarity are formed across the boundary between the porous cathode and the membrane. The layers, known as electrochemical double layer, can store electrical energy and behave like a supercapacitor.

For example, if the load current suddenly drops to zero level the charge double layer will take some time to disperse and so will the associated overvoltage. However, the ohmic losses which are proportional to the drawn current will immediately reduce to zero. Therefore, the fuel cell response will exhibit an instantaneous drop followed by an exponential decay towards the new steady-state value. The capacitive effect is not affected by ohmic losses. The time-constant is therefore modeled by a capacitor $C$ and equivalent resistances of concentration and activation losses, but not ohmic losses. The time constant is given by:

$$
\tau = C(R_{act} + R_{conc})
$$

The 5.5 kW Nuvera Proton Exchange Membrane Fuel Cell Stack characteristic parameters are listed in Table 1.

The ten parameters are listed in Table 2. The model is implemented and tested in MATLAB/Simulink environment. Simulation and experimental results will be compared to test the high-accuracy of the ten-parameters model.
be expressed as:
\[ \sigma_b = b_1 \exp \left( b_2 \left( \frac{1}{303} - \frac{1}{T_{fc}} \right) \right) \]
where \( b_1 \) and \( b_2 \) are empirical parameters and \( \lambda \) is the stoichiometric speed. According to (23), (24) and (25), the internal resistance is given by:
\[ R_{ohm} = \frac{t_m}{b_1 \exp \left( b_2 \left( \frac{1}{303} - \frac{1}{T_{fc}} \right) \right)} \]

The concentration drop is modelled by:
\[ v_{con} = i \left( c_2 \frac{i}{i_{max}} \right)^{c_3} \]
where \( c_2 \) and \( c_3 \) parameters depend on temperature and pressure operating conditions, \( i \) is the current density and \( i_{max} \) the maximum allowable current density value. The parameter \( c_2 \) is obtained by the following relationship:
\[ c_2 = (7.16 \times 10^{-4} T_{st} - 0.622) \left( \frac{P_{O_2} - 0.1173}{P_{sat} + p_{sat}} \right) + (1.68 - 1.45 \times 10^{-3} T_{st}) \]
if \( \frac{P_{O_2}}{0.1173} + p_{sat} < 2 \text{ atm} \)
or by:
\[ c_2 = (8.66 \times 10^{-5} T_{st} - 0.068) \left( \frac{P_{O_2} - 0.1173}{P_{sat} + p_{sat}} \right) + (0.54 - 1.6 \times 10^{-4} T_{st}) \]
if \( \frac{P_{O_2}}{0.1173} + p_{sat} > 2 \text{ atm} \).

The saturation pressure is evaluated by:
\[ P_{sat} = 1.508 \times 10^{-6} T_{st}^4 - 0.00187 T_{st}^3 - 179.6 T_{st}^2 + 1.428 \times 10^4 \]

All parameters of the six-parameters sub-model are listed in Table 3.

### Table 2. Six-parameters model coefficients

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Tuned value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \xi_1 )</td>
<td>-0.9152</td>
</tr>
<tr>
<td>( \xi_2 )</td>
<td>1.79 \times 10^{-4}</td>
</tr>
<tr>
<td>( \xi_3 )</td>
<td>8.692 \times 10^{-3}</td>
</tr>
<tr>
<td>( c_0 )</td>
<td>0.0173</td>
</tr>
<tr>
<td>( c_1 )</td>
<td>0.00845</td>
</tr>
<tr>
<td>( c_2 )</td>
<td>2.9706</td>
</tr>
<tr>
<td>( c_3 )</td>
<td>0.01983</td>
</tr>
<tr>
<td>( \psi )</td>
<td>20.8218</td>
</tr>
<tr>
<td>( n )</td>
<td>9 \times 10^{-4}</td>
</tr>
</tbody>
</table>

### Six-parameters model

The six-parameters sub-model is based on the following assumption: instantaneous chemical reactions in the polymeric membrane, constant operating temperature, constant pressure of reactants.

Therefore, the six-parameters sub-model is based on the ten-parameters model neglecting temperature and pressure effects on the fuel cell stack performances. The unavoidable loss in accuracy is therefore counterbalanced by the lower complexity of the proposed sub-model. Following the above mentioned assumption, the activation overvoltage is given by:
\[ v_{act} = A \ln \left( \frac{i}{i_0} \right) \]
where \( A \) is a constant given by:
\[ A = \frac{RT}{2aF} \]
where \( R \) is the universal gas constant, \( F \) the Faraday constant and \( a \) is the charge transmission coefficient whose value depends on the reaction type and electrodes material. From (21) it is worth noting that higher temperatures correspond to higher values of the \( A \) parameter and therefore higher values of the activation drop. The ohmic voltage drop is given by:
\[ v_{ohm} = R_{ohm} i_{fc} \]

With regards to the ohmic resistance, it could be demonstrated that the internal resistance depends on the cell conductivity \( \sigma_m \) according to:
\[ R_{ohm} = \frac{t_m}{\sigma_m} \]
where \( t_m \) is the membrane thickness. The conductivity can be expressed as:
\[ \sigma_m = b_1 \exp \left( b_2 \left( \frac{1}{303} - \frac{1}{T_{fc}} \right) \right) \]
where \( b_2 \) is empirically obtained and \( b_1 \) is given by:
\[ b_1 = b_{11} \lambda - b_{12} \]

### Four-parameters model

The four-parameters sub-model features the lowest complexity. The sub-model is based on the assumption:
- Uniform distribution of reactant gases;
- Constant temperature;
- Constant pressure;
- Resistance is independent of the cell temperature.

The sub-model is based on the equations:
Four parameters are listed in Table 4.

Table 4. Four-parameters model coefficients.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Tuned value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b$</td>
<td>0.0696</td>
</tr>
<tr>
<td>$K_1$</td>
<td>0.0417</td>
</tr>
<tr>
<td>$K_2$</td>
<td>0.1101</td>
</tr>
<tr>
<td>$R_{ohm}$</td>
<td>0.0001275</td>
</tr>
</tbody>
</table>

**Simulation results**

A 5.5kW Nuvera PEM fuel cell stack is modelled. The stack is made up of 40 cells which are connected in series. Assuming a uniform behaviour of all elementary cells, the output of the fuel cell stack is obtained by summing the output of each fuel cell model. As an example, in Fig. 1. the top level model of the ten-parameters model implemented in MATLAB is shown.

Fig. 1. MATLAB model of the ten-parameters model.

The cell voltage is multiplied by 40 which is the number of active cells of the fuel cell stack under test. Experimental waveform of the fuel cell stack current is forced from MATLAB workspace. The simulated stack voltage is sampled and stored in MATLAB workspace to be compared with the corresponding experimental waveforms.

**Model validation**

Simulation and experimental results are compared to test the accuracy of each sub-model. In Figure 2 the averaged iso-themal V-I curve at 30°C and simulated curves of each sub-model are shown. Experimental results are shown by the dotted curve, the ten-parameters model output by the red dashed curve, the six-parameters model by the blue dashed curve and the four-parameters by the pink dashed curve.

Fig. 2. Experimental and simulated steady-state V-I curves under 30°C.

Deviations of each sub-model from experimental results are shown in Fig. 3. If compared with other sub-models, the ten-parameters model features the highest accuracy. Decreasing the number of independent parameters leads to a decreasing accuracy. Indeed, the deviation is maximum for the four-parameters model.

Fig. 3. Standard deviation of proposed analytical models.

The deviation reaches its peak value of 9% in the 0-15A current range, where the activation voltage losses prevail. If the activation polarization region is considered, all deviation curves overlap. If the stack current is higher than 20A, deviation curves split from each other. In the ohmic polarization region, deviation drops to its minimum value. A 2% deviation is achieved by the four-parameters sub-model, a 1% deviation by the six-parameters sub-model while the deviation of the ten-parameters model is really close to a zero value.

In the concentration polarization region, while the ten-parameters model deviation is less than 1%, the six- and four-parameters sub-models features really high deviation, even beyond 3-4%.

As shown by simulation and experimental results, the deviation from experimental results is lower within the ohmic region. Since in the four-parameters sub-model the proton resistance has been assumed constant, the highest deviation is achieved if compared with other sub-models.

**REFERENCES**


