Generalisation of transfer functions of inter-digital transducer and filter on basic of surface acoustic waves

Abstract: In the report we deal with the generalisation of transfer function which is derived from the three-gate model of the inter-digital transducer. Stated generalisation of transfer function includes the influence of adapter circuits attached to the transducer and allows its application for the description of any model’s functioning.

Streszczenie: Analizę przetwornik używany do zasilania i detekcji elementu SAW – z powierzchniową falą akustyczną. W modelu uwzględniono także układ adaptera. (Ogólnienie funkcji transferu przetwornika i filtra bazującego na elemencie z akustyczną falą powierzchniową SAW)

Keywords: inter-digital transducer, surface acoustic waves, complex transfer function, diffusion coefficients, diffusion loss.

Słowa kluczowe: powierzchniowa fala akustyczna SAW.

Introduction

Analyzing the activity of inter-digital transducer (IDT) which is used for excitation and detection of surface acoustic waves (SAW) at the moment of realisation of acoustic-electronic components we stem most commonly from the three-gate circuit model (1). Depicted model allows us to state all three-gate transducer transfer functions with the presence of so called secondary features like acoustic reflections created at the edges of electrodes as a consequence of weighted electric load on the surface of the pad (2), (3), which is at the same time often applied to make calculations of delay circuits (DC) or filters with double IDT and various matching circuits. To calculate the transfer functions taking the secondary features into consideration it is convenient to use three-gate matrices. Calculated transfer functions based on the models of “weak binding” does not include the influence of matching circuits and this method does not allow us to calculate reflection acoustic coefficient of transducer. Reflections from the electrodes of the transducer are often little, eventually they can be strongly choked down by the use of doubled electrodes. In the depicted case it is possible to use three-gate model to derive the generalisation of transfer function in which there is included the influence of matching circuits and allows us to derive the nexus between the tree-gate activity model and various geometrical models of “weak binding”.

Complex transfer function, diffusion coefficients and diffusion loss

In the Fig.1 there is a filter with SAW with linear phase characteristics which is most often composed of two IDT.

![Fig.1. Most common filter arrangement with PAV](Image)

The left IDT (apodized) changes electric signal on the SAW and the second transducer, most commonly homogeneous integrates the energy of incoming wave and converts it to the electric signal. This process is linear, reciprocal and in the ideal case we can mathematically express it using a convolution, or equivalently in the following form (4):

\[ H(j\omega) = \frac{U_2}{U_1} = H_1(j\omega)H_2(j\omega)e^{-j\omega\tau}, \]

where: \( \tau = \frac{l}{v} \) is a SAW delay between IDT and \( v \) is a SAW velocity.

The filter transfer function as it results from the equation (1) is fully determined by the features of IDT because the pad material does not have dispersal features. The transducers can find themselves in a different distance from each other; they can be “weighted” and can have a different geometry of electrodes. The right IDT (homogeneous) is most commonly broadband. Then we can approximately express the transfer function with the apozited transducer \( H_1(j\omega)H_2(j\omega) = H_1(j\omega) \).

Based on the circuit theory it is possible to describe IDT in a complex way by using the matrix coefficients \( Y_{ji} \), which are defined by the formula

\[ I_i = \sum_{j=1}^3 Y_{ji} U_j, \quad (i=1,2,3). \]

To simplify we will further assume IDT to be lossless (e.g. we do not consider the creation of volume waves, electrode resistance, diffraction and loss by the SAW diffusion), then coefficients \( X_{ji} \) are often imaginary.

It is convenient besides the admittance coefficients to define the system of complex transfer functions

\[ T_{ji}(j\omega) = 2 \frac{G_i}{G_j} \frac{U_j}{U_i}, \]

where \( U_i \) is a voltage of transfer or reflection SAW on the load \( G_i \), \( j \)-level gate if a voltage \( U_j \) is attached to the \( j \)-level gate with conductance \( G_j \).

Then it is possible to identify the diffusion coefficients \( p_{ij} \) with the nexus after equation adjustment (3), applies the following:

\[ U_i^2 G_i = \frac{1}{2} T_{ij}^2 U_j^2 G_j \quad \text{and} \]

\[ P = \frac{1}{2} T_{ij}^2 P_j, \quad p_{ij} = \frac{P}{P_{\text{ideal}} j} = \left| P_{ij} \right|^2, \]

where \( P_i \) is transferred or reflected power of \( i \)-level gate and \( P_{\text{ideal}} \) is a utilizable from adjusted generator on the \( j \)-level gate. Then \( P_i \) is a part of power reflected by \( i \)-level gate and \( p_{ij} \) (\#) is a part of transferred power \( j \)-level gate to \( i \)-level gate (e.g. characterizes the conversion of electric energy to acoustic energy and vice-versa).
Based on the IDT reciprocity and symmetry results that $P_{11} = P_{22}, P_{13} = P_{23}$ and $P_{ij} = P_{ji}$, respectively $T_i = T_j$.

Then we can define the diffusive (or transfer) loss as following:

$$b_i = -10 \log p_{ij}, \quad [\text{dB}]$$

where $b_i$ is a loss caused by a reflection i-level gate and $b_i$ (i#) is transfer (insertion) loss between the i-level j-level gates.

**Three-gate model of inter-digital transducer**

We derive the IDT transfer function while coming from a single-electrode IDT which we will model by using one-dimensional cross field model, depicted in the fig. 2a and 2b. We can also use an alternative longitudinal field model which is the most convenient one for some pad materials with low coefficients of electro-mechanical binding $K^2$ [1].

Using the equivalent currents and voltage we can express the particles velocity and mechanic power on acoustic gates. The dependence between these currents and voltages (i.e. $I_1 = Y_1 U_1$) can be expressed by the elements of admittance matrix. For the k-level IDT leg applies:

$$Y_{ij} = \begin{pmatrix} -j \cot \theta_j, j \cot \theta_j, -j \cot \theta_j, \frac{2}{\theta_j} \\ j \cot \theta_j, -j \cot \theta_j, -j \cot \theta_j, \frac{2}{\theta_j} \\ -j \cot \theta_j, -j \cot \theta_j, j \omega C_{pk}^2 + 2 \pi f_j \theta_j, \frac{2}{\theta_j} \\ \end{pmatrix}$$

where $\theta_j = 2 \pi f / f_j$ is a phase shift emerging on the k-level leg, $f_j = \nu / 2 D_k$ is a synchronised frequency k-level leg and $D_k$ is k-level leg length.

Transformation rate is given by the nexus:

$$P_k = \left( -1 \right)^n \sqrt{2 f_k} C_{pk} K^2 Z_0, \quad \left[ K \left( \sqrt{\frac{f_j}{f}} \right) K(q_k) \right]$$

where $C_{pk}$ is k-level leg capacity, (as characteristic impedance we chose the unit one, $Z_0=1$). The factors $K \left( \sqrt{\frac{f_j}{f}} \right)$ and $K(q_k)$ are totally elliptic first-order integrals with $q_k = \sin \left( \frac{\theta_j}{2 D_k} \right)$ module, where $\theta_j$ is a width of k-level integral. These integrals can be found in the equation (7) as a consequence of capacity dependence on the rate of electrode and gap width. The k-level leg capacity is given by the formula:

$$C_{pk} = \left( \frac{w_k}{L} \right) \sum_c e^{c_j} \left[ K(q_k) / K(1 - q_k) \right]$$

where $w_k$ is k-level leg aperture. The permittivities $\epsilon_1$ and $\epsilon_2$ express anisotropy features of pad material in direction of diffusion to the pad. The value $C_{pk}$ like it stems from the nexus (8) increases with the growing rate $l_k / D_k$. Next, to simplify, we will suppose that the width of the electrode and the gap is equal ($l_k = D_k / 2$). Then the elliptic integrals in the equation (8) equal one.

**IDT Transfer function**

IDT transfer function at generating the SAW at certain stated conditions can be expressed by the following formula:

$$H_1(j \omega) \equiv T_{01}(j \omega),$$

and IDT transfer function at SAW detection by the nexus:

$$H_2(j \omega) \equiv T_{12}(j \omega).$$

Let make the following deduction to simplify the previous equation by the nexus $e^{-j / 2 \theta_j}$ expressing SAW diffusion towards the end of transducer (e.g. to the gates...
n.1 or 2) and the sum of particular electrodes contributions, while the following applies:

\[
U_i = jU_1 \sum_{k=1}^{\infty} p_k \sin \left( \frac{\pi}{2} f_k \right) e^{-j2\pi f_i}.
\]

In case of IDT connection to the real source \((G_v \neq \infty)\) applies between the voltages \(U_v\) and \(U_3\) the following:

\[
U_3 = U_1 (G_v + Y_{in}(j\omega)) / G_v
\]

where \(Y_{in}(j\omega)\) is IDT input admittance and \(G_v\) is an anode slope conductance of the source (fig.3a).

The IDT transfer function can be calculated by instituting the equations (13) and (14) to the equation (11) in the following form:

\[
T_{13}(j\omega) = \frac{2G_v}{\sqrt{Z_0 G_v}} jU_1 \sum_{k=1}^{\infty} p_k \sin \left( \frac{\pi}{2} f_k \right) e^{-j2\pi f_i}.
\]

Generalised IDT transfer function

Using the cross field model it is convenient to divide the electric input admittance \(T_{13}(j\omega)\) which is a very important constant to calculate \(Y_{in}\) into capacity susceptance which is parallely connected to emitting admittance \(Y_r(j\omega)\):

\[
Y_{in}(j\omega) = Y_r(j\omega) + j\omega C T
\]

where \(C_T\) is IDT static capacity. Reference frequency \(f_0\) can be defined like non-dimensional emitting admittance given by the nexus

\[
y_r(j\omega) = \frac{Y_r(j\omega)}{\omega_0 C T} = \frac{Q_r}{\omega_0 C T},
\]

where

\[
Q_r = \frac{\omega_0 C T}{G_v}.
\]

and introduce non-dimensional transformation rate defined by the formula

\[
p_i = p_{k1} \sqrt{\frac{2G_v}{\omega_0 C T}} = (-1)^i \sqrt{\frac{2K^2 Q_r f_j C_{in}}{\pi f_j C T}}
\]

after adjustment we can define the transfer function \(T_{13}(j\omega)\) as following (Z_2=1):

\[
T_{13}(j\omega) = \frac{2G_v}{\sqrt{2} G_v} \sum_{k=1}^{\infty} p_i \sin \left( \frac{\pi}{2} f_k \right) e^{-j2\pi f_i}.
\]

In the previous nexus \(Y_r\) and the sum in the dividend of the fraction approximately equal 1, function \(T_{13}(j\omega)\) is normed in a way that \(T_{13}^2 = p_{13}\). The coefficient \(Q_r\) depends on transducer’s geometry, is indirectly commensurable to \(K^2\) and can be in a simple case expressed by the nexus

\[
Q_r = \frac{\pi K^2}{4K^2 f_0}.
\]

It is efficient to generalise the equation (21) at the further analysis. We suppose that a matching circuit is connected to the IDT electric gate according to the fig.4. In such a case the transfer function \(T_{13}\) and the reflection coefficient \(T_{11}\) can be expressed by the following generalisation:

\[
T_{13}(j\omega) = \frac{Q_v}{Q_r} [A(j\omega)]^2
\]

\[
T_{11}(j\omega) = \frac{Q_v}{Q_r} B(j\omega)
\]

The scheme coefficient \(A(j\omega)\) is the scheme coefficient and is dependant only on the IDT geometry. Circuit coefficients \(C(j\omega)\) and \(B(j\omega)\) are dependent on the input admittance \(Y_{in}(j\omega)\) of the transducer, source impedance or load and on the features of matching circuit which are expressed by the elements of gradual cascade matrix \(A_{12}\), \(A_{12}\), \(A_{22}\).

The scheme coefficient \(A(j\omega)\) is in the case of cross-field model (as well as in the case of function \(\delta\) model) a Fourier transformation of conveniently located SAW generating sources. Can be expressed by the nexus:

\[
A(j\omega) = \frac{2}{\sqrt{2} G_v (\omega_0)} \sum_{k=1}^{\infty} e_{k1}(j\omega) e^{-j2\pi f_k},
\]

where \(e_{k1}(j\omega)\) is called element coefficient of k-level electrode and exponential factor expresses the delay and space location of electrodes. Normalization coefficient \(\frac{2}{\sqrt{2} G_v (\omega_0)}\) is used for circuit models containing dimensional constants. As a consequence, \(A(j\omega)\) equals one and insertion loss is initially given by the constants \(Q_v\) and \(Q_r\) in the equation (23).

Element coefficients for two weak binding models and two different three-gate models are included in the Table n.1. Model of cross field and model of functions \(\delta\) have similar element coefficients because the geometrical shape
of exciter functions is almost identical. At the same time the element coefficient of circuit model generalisation resembles the element coefficient of weak binding model that stems from the solution of electric field because all the stated models use real IDT electric field as an exciter function.

Table 1. Element coefficients of some IDT activity models

<table>
<thead>
<tr>
<th>Model</th>
<th>Circuit coefficients ( e_i(j\omega) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Of functions ( \delta )</td>
<td>( 2l_i \cos \left( \frac{\pi \delta}{V} \right) ), ( l_i ) is the distance between functions ( \delta )</td>
</tr>
<tr>
<td>Harmonic (direct solution of electric field)</td>
<td>( F, T, \left( \frac{dE}{dx} \right) ), ( E ) is a part of intensity vector</td>
</tr>
<tr>
<td>Of cross field</td>
<td>( jK \frac{2f_1C}{w_0} \left( \frac{2f_1}{w_0} \sin \left( \frac{\pi f_1}{2} \right) \right) ), ( w_0 ) is the aperture of the through apodized transducer</td>
</tr>
<tr>
<td>Generalised</td>
<td>( \frac{1}{2} j^2 E^2 f^2 (\chi_1, \eta_1) ), ( \chi_2 = \frac{2\pi f}{V} ) is wave number, ( \eta_k ) metallization coefficient</td>
</tr>
</tbody>
</table>

Table 2. Circuit coefficients for simple matching circuits

<table>
<thead>
<tr>
<th>Matching circuit</th>
<th>Gradual cascade matrix</th>
<th>Circuit coefficients ( C(j\omega), B(j\omega) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Without matching the circuit</td>
<td></td>
<td>( C(j\omega) = B(j\omega) = 1 + R_{xy}Y_{xy} )</td>
</tr>
<tr>
<td>Parallel admittance ( Y_{xy}, \text{ e.g. inductor, resistor} )</td>
<td></td>
<td>( C(j\omega) = B(j\omega) = 1 + R_{xy}Y_{xy} + Y_{yp} )</td>
</tr>
<tr>
<td>Serial impedance ( Z_{xy}, \text{ e.g. inductor, resistor} )</td>
<td></td>
<td>( C(j\omega) = 1 + Y_{xy}(R_{xy} + Z_{xy}) ), ( B(j\omega) = R_{xy}Y_{xy} + \left( \frac{1}{Y_{xy} + Z_{xy}} \right) )</td>
</tr>
<tr>
<td>Other than parallel element ( A_{11}, A_{12}, A_{21}, A_{22} )</td>
<td></td>
<td>( C(j\omega) ) and ( B(j\omega) ) can have various shapes and multiple dependence from ( R_{xy} )</td>
</tr>
</tbody>
</table>

Frequency dependences caused by matching circuit enable to include two circuit coefficients \( C(j\omega) \) and \( B(j\omega) \) while the circuit coefficient \( C(j\omega) \) for transfer function \( T_{13} \) and circuit coefficient \( B(j\omega) \) for the reflection coefficient \( T_{11} \) are formulated by the formulae:

\[
C(j\omega) = A_{11} + A_{12}Y_{xy} + R_{xy}(A_{21} + A_{22}Y_{xy}),
\]

\[
B(j\omega) = R_{xy} \left[ Y_{xy} + \frac{A_{11} + A_{22}R_{xy}}{A_{12} + A_{22}R_{xy}} \right].
\]

The elements of gradual cascade matrix for random matching circuit are defined by the formulae:

\[
U_k = \begin{bmatrix} A_{11}, A_{12}, A_{21}, A_{22} \end{bmatrix}, \quad I_k = \begin{bmatrix} U_1, U_2 \end{bmatrix}.
\]

In the table n.2 we mention the values of both circuit coefficients for some frequently used matching circuits. In case of that the circuit coefficients equal \( A_{12} = 0, A_{22} = 1 \) (then \( B(j\omega) = C(j\omega) \)) then the circuit coefficient for coefficient of reflection is given by the following formula:

\[
T_{11}(j\omega) = \frac{1}{2} \left[ T_{13}(j\omega) \right]^2 C(j\omega).
\]

Based on the stated nexus which is valid only for the matching circuits composed of the parallel elements (or doubled conductor) we can assess the rate of insertion loss and three times transferred signal for the parallel tuning circuit.

**Conclusion**

In the report we derived generalisation of transfer function of transducer and reflection acoustic coefficients based on the defined complex transfer function, diffusion coefficients and diffusion loss with the generalisation being achieved by the introduction of system coefficient and circuit coefficient. The significant advantage of installed functions lies at the fact that they allow the application of random model on the process of modelling the IDT activity and at the same time they involve the influence of matching circuits.

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**Authors**

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