Design of Static var Compensators using a General Reactive Energy Definition

Abstract. This paper describes a methodology to design Static var Compensators (SVC) using the reactive energy definition from the Conservative Power Theory. The application of this methodology is presented through simulations of local and distributed compensation scenarios. Results show effective compensation of reactive current and load unbalance. Moreover, the harmonic distortion caused by thyristor controlled reactor has been suitable attenuated by passive filters designed in combination with the SVC.

Streszczenie. Opisano metodologię projektowania kompensatora mocy biernej SVC wykorzystującą definicję energii biernej z Teorii Konserwacji Mocy. Symulacje wykazały że układ kompensuje prąd bierny i nierówność obciążenia. Tłumione są także harmoniczne. (Projektowanie kompensatora mocy biernej na podstawie definicji energii biernej)

Keywords: Arc Furnace, Conservative Power Theory, Reactive Compensation, Static var Compensator, Unbalance Compensation.

Słowa kluczowe: kompensator mocy biernej, teoria konserwacji mocy.

Introduction

Power conditioning plays an important role in recent researches related to smart grids [1]. Different approaches have been proposed to control compensators that are distributed along the network, in order to reach a cooperative operation, the sharing of compensation demands and the avoidance of detrimental interaction among distinct compensation architectures [2-8].

However, these approaches are mainly focused on the use of inverter-based compensators [2-4,7,10], while traditional passive technologies still may contribute to grid optimization in the smart grid context, as initially proposed in [5,6], when different architectures of compensators were used and controlled cooperatively. Nevertheless, the design and control of traditional compensators should be reviewed and adapted.

Thus, this paper presents a designing methodology for Static var Compensators (SVC) based on the reactive energy definition from the Conservative Power Theory (CPT) [8], which is a conservative term and remains valid under unbalanced, nonsinusoidal or variable frequency conditions. The goals of the designed SVC are reactive and load unbalance compensation, and due to the use of conservative quantities in its control, it may be applied to cooperative compensation approaches for smart grids.

Conservative Power Theory - Background

The Conservative Power Theory (CPT) is a recent mathematical formulation that associates characteristics of electrical circuits to current and power components [8]. Their definitions have been applied to design and control of compensators [6,7,9,10], as well as to accountability and revenue metering proposals [11]. In this paper, CPT will be briefly presented, and additional details of these definitions can be found in [8].

A. Conservative quantities

The two conservative quantities defined by CPT are instantaneous active power $p(t)$ and instantaneous reactive power $w(t)$. For a M-phase circuit on which the voltages $u(t)$ and currents $i(t)$ respect the Kirchhoff laws, that quantities for each phase may be expressed by:

\begin{align}
(1.a) & & p_m(t) = u_m(t) i_m(t), & m = 1 \ldots M \\
(1.b) & & w_m(t) = u_m(t) i_m(t), & m = 1 \ldots M
\end{align}

such that $u$ is the unbiased voltage integrals, i. e., the time integral of voltage minus the average value of such integral function. If the sum of the values of $p_m$ (or $w_m$) from all phases results zero at every instant, this quantity is conservative for any waveform of voltages and currents [8].

The same conclusions arise if one considers different nodes in a particular power grid.

The average values of (1.a) and (1.b) results in active power and reactive energy for each phase, as expressed by (2.a) and (2.b). It is important to mention that reactive energy may result in positive or negative values, depending if load has an inductive or capacitive characteristic.

\begin{align}
(2.a) & & P_m = \frac{1}{M} \int_0^M p_m(t) dt, & m = 1 \ldots M \\
(2.b) & & W_m = \frac{1}{M} \int_0^M w_m(t) dt, & m = 1 \ldots M
\end{align}

B. Current and power decomposition

According to CPT, the electrical current of a polyphase system may be decomposed into five orthogonal components and each of them represents a specific characteristic of the electrical circuits. Furthermore, each current component is associated to a power component, and they may be described as follows.

Active balanced current ($i_a^b$): it is the minimum current responsible for transfer active power to the load. The product of collective values of voltages and active balanced currents is the active power $(P = UI_a^b)$, which results in the same value if calculated by the sum of the $P_m$ defined in (2.a) from the $M$-phases of that circuit. This definition of active power matches with the traditional definitions of active power [12].

Reactive balanced current ($i_q^b$): represents the stored energy in the circuit (phase displacement between voltage and current). This component is associated to reactive power $Q$ by the product of collective values $(Q = UI_q^b)$. The reactive power may be associated to reactive energy only in a determined frequency.

Active unbalanced current ($i_a^u$): this current component appears due to unbalanced equivalent conductances in the phases of the circuit.

Reactive unbalanced current ($i_q^u$): this current component appears due to unbalanced equivalent reactivities in the phases of the circuit.

The unbalance currents components can be associated to a unique unbalance power component $N$ that quantifies the effect of unbalanced loads connected to the network. Thus, the unbalance power may be obtained by

\[ N = U \left( I_{a^2} + I_{q^2} \right). \]

1 The collective value of voltages or currents from a three-phase circuit is defined as: $X = \sqrt{X_a^2 + X_b^2 + X_c^2}$, where $X_a, X_b, X_c$ are RMS values of the three phases.
**Void current** \((i_v)\): reflects the non-linearity between voltage and current, which are caused mainly by electronic loads. This current component does not transfer active power or reactive energy through the network, and a particular power component is associated to it, the distortion (or void) power \((D = Ui_v)\).

The total power in a determined point of the circuit, e.g. the Point of Common Coupling (PCC), is the apparent power \(A\), and it is calculated by the product between the collective values of voltage and the total current at PCC \((A = UI)\).

**Power factor**

The power factor \((\lambda)\) defined by CPT is the ratio between the active power and the apparent power, as shown in (3).

\[
\lambda = \frac{P}{A} = \frac{I^P}{I} 
\]

Although (3) is similar to the conventional definition of power factor, \(\lambda\) remain valid even under non-ideal voltage conditions, because it is not related to faroal characteristics and it is unity only in case of pure balanced resistive load. Thus, this definition of power factor may represent a global efficiency factor.

**Static var Compensators**

**A. Design of SVC components**

The proposed architecture of SVC is represented in Fig. 1. It is a three phase delta connected compensator composed by thyristor switched capacitors (TSC) and thyristor controlled reactors (TCR).

Fig. 1. Architecture of the SVC.

All the variables used in the design of SVC are measured or calculated considering the voltages and currents at the point that is the goal of compensation, i.e., the PCC of the network.

The SVC goal is to compensate both reactive and unbalance currents defined by CPT. Thus, the design of the SVC may consider two conditions: if the currents at PCC are balanced but contain balanced reactive currents, only TSC is used. Thus, \(I_{SVC,m} = I_{r,m},\ m = a, b, c\); and the minimum capacitance to compensate \(I_{SVC,m}\) is obtained by (4), in which \(U_i\) is the RMS value of line voltages at PCC and \(\omega\) is the fundamental angular frequency.

\[
\frac{C_{SVC}}{\omega U_i L^2} = \frac{\theta_m}{\omega U_i L^2} \quad (4) 
\]

Another condition is: in the presence of balanced reactive current and unbalance currents at PCC, both TSC and TCR are used, and ever since the currents are different in each phase due to the load unbalance, the maximum value of \(I_{SVC,m}\) must be considered, i.e., \(I_{SVCmax} = \max (|I_{r,m}|, |I_{a,m}|, |I_{m,0,b,c}|)\). In this case, TSC should compensate not only the reactive current from the load, but also the balanced reactive current from TCR. Considering that the current drained by TCR equals to \(I_{SVCm\text{ax}}\), the relation between TCR and TSC reactances at fundamental frequency is expressed by:

\[
X_{TSC} = 2X_{TCR} \quad (5) 
\]

Thereby, the capacitance of TSC and inductance of TCR may be obtained from (5):

\[
C_{TSC} = \frac{I_{SVCmax}}{\omega U_i L^2}, \ L_{TCR} = \frac{U_i^2}{2\omega I_{SVCmax}} \quad (6) 
\]

One may observe that the term \(\omega\) would limit the compensation only to fundamental frequency, but the efficiency is not severely reduced if the SVC is designed considering the RMS values obtained from measured voltages and currents under unbalanced and/or distorted conditions, and due to orthogonality among the current components, it is possible to extract the correct value of each component to be compensated in any condition.

**B. SVC operation**

The TSC banks must be switched only in the zero crossing of line voltages, while the thyristors of the TCR must be fired with an angle \(\alpha\) in every cycle of the line voltages. The angle \(\alpha\) may be determined by the equivalent average reactive energy that would flow through the SVC, which is expressed in (7):

\[
W_{ab} = -W_{ab} + W_{TSC}, \\
W_{bc} = -W_{bc} + W_{TSC}, \ W_{ca} = -W_{ca} + W_{TSC} 
\]

such that:

\[
W_{ab, b, c}, W_{bc, c, b} \ are \ the \ equivalent \ reactive \ energies \ of \ each \ branch \ of \ the \ SVC; \\
W_{ab, b, c, b}, W_{bc, c, b, c} \ are \ the \ equivalent \ reactive \ energies \ of \ each \ TCR, \ when \ it \ is \ switched \ with \ a \ conduction \ angle \ \alpha \ in \ relation \ to \ line \ voltages. \ They \ may \ be \ calculated \ by \ the \ average \ reactive \ energies \ of \ the \ three \ phases \ at \ PCC:\n\]

\[
W_{ab} = \frac{1}{\pi} \left(\pi - 2\beta_{ab} + 2\alpha_{ab}\right) \\
W_{bc} = \frac{1}{\pi} \left(2\beta_{bc} - 2\alpha_{bc}\right) + \sin 2\alpha_{bc} \\
W_{ca} = \frac{1}{\pi} \left(2\beta_{ca} - 2\alpha_{ca}\right) + \sin 2\alpha_{ca} \quad (9) 
\]

**C. Design of passive filters**

The switching of TCR causes harmonic distortion in the currents, which has been mitigated by passive filters (PF) tuned at 5th, 7th, 11th and, due to the switching of unbalanced reactors, 3rd harmonic frequencies [13].

Considering that the capacitances of passive filters may over-compensate the reactive current at low frequencies, the additional reactive energy due to these capacitors should be considered in \(W_{TSC}\), which can be re-written as:

\[
W_{TSC} = \left(C_{TSC} + C_{PF}\right) U_i^2 \quad (10) 
\]

such that \(C_{PF}\) is the capacitance of the passive filter.

Consequently, (9) would result in proper angles to avoid reactive over-compensation.

It is important to observe that the additional reactive energy from PF capacitors plus \(W_{TSC}\) should not exceed \(W_{TCR}\), in order to maintain the angles \(\alpha\) into the interval \([\pi/2, \pi]\) rad.

After determining \(C_{PF}\) in relation to TCR, it is converted to wye and divided by the number of harmonic components to be attenuated by PF. Thus, the inductors are tuned into each harmonic component and the corresponding resistances are determined by (11):
such that:

\[ R_b, L_b \] are the resistance and tuned inductance at each harmonic frequency \( f_b \);

\[ MF \] is the merit factor, used to reduce filter selectivity. Its typical values vary from 40 to 200 [5].

D. SVC control strategy

The proposed control strategy to adjust the SVC components in case of load variation is presented in Fig. 2. Essentially, when the average value of load reactive energy varies, \( C_{TSC} \) is re-calculated. Due to the limited number of capacitor banks installed, if \( C_{TSC} \) does not match the available TSC, only TCR conduction angles will be updated. Otherwise, \( W_{TSC} \) is re-calculated according to (10), TSC is switched in the next zero-crossing of each line voltage and the conduction angles are calculated to compensate the change in the load and in TSC. It is important to mention that for a variable load case, the capacitance of the passive filters should be determined considering the maximum value of \( W_{TSC} \), which corresponds to the worst case.

Simulation Results

A. Local compensation of an electric arc furnace

In this application, the SVC is used to increase power factor at the PCC, by compensating the variable reactive and unbalance power components. The electric arc furnace (EAF) model simulated on PSCAD is based on [14,15]. It consists on a variable resistance that is a function of the electrical arc length in every phase of the melting process. Fig.3 represents the topology of the compensators in the simulated circuit. The PCC is located at medium voltage side and the reference for voltages is a virtual star point, as recommended in [8].

Thus, the electric arc length is configured to simulate three stages of loading and melting and the stage of refining at the end of the process [16]. The lengths are different in each phase of the system in order to produce load unbalance. The simulation total time was 11.6 s, which in real furnaces would correspond to at least 80 minutes. Fig.4 presents the CPT power components, power factor and reactive energies at PCC without compensation.

Table 1 presents the designed components of the SVC and PF (passive filter). TSC banks were designed to result in a proper capacitance value for each steady state level of power components from Fig. 4. The additional reactive energy of PF capacitors is 15% of \( W_{TSC} \) in the worst case, when the reactive energies from the load are in their maximum values. The merit factor adopted was 200.

Table 1. Designed parameters for the SVC and the passive filter.

<table>
<thead>
<tr>
<th>TSC values</th>
<th>Capacitor in each PF branch</th>
</tr>
</thead>
<tbody>
<tr>
<td>163.6 ( \mu F ) (worst case); 154.8 ( \mu F ); 146.7 ( \mu F ); 137.5 ( \mu F ); 126.2 ( \mu F )</td>
<td>36.7 ( \mu F )</td>
</tr>
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</table>

Fig.5 presents the CPT power components, power factor and reactive energies at PCC when the compensators are operating. The flow of reactive energies at PCC is closed to zero during all the simulation. Consequently, reactive power and apparent power are reduced too. Unbalance power is limited between 0.926 MVA and 0.578 MVA and distortion power is 2.2 MVA, which indicates that most part of harmonic distortion caused...
by the load is not mitigated, but the main harmonic components caused by TCR are reduced and do not have a big contribution to the PCC harmonic distortion. The power factor varies between 0.9834 and 0.9538.

Fig. 5. CPT power components, power factor and reactive energies at PCC without compensators.

Distributed compensation results

In order to evaluate the effectiveness of the proposed methodology in a distributed compensation approach, a distribution network based on IEEE 13 node distribution test feeder [17] was simulated. Fig. 9 represents this network with the number of conductors in each branch.

Node 650 is feed by an 115 kV source. The delta-grounded wye 5 MVA substation transformer between nodes 650 and 632 reduces the voltage level to 4.16 kV, and the grounded wye-grounded wye 500 kVA transformer between nodes 633 and 634 reduces the voltage level from 4.16 kV to 480 V.

Transmission lines were modeled using impedance and capacitance matrices. Loads are RL connected in delta or between two phases, so there is no return conductor on this network. Additionally, a single phase 100 kvar capacitor bank was connected to phase \(c\) of node 611 and a three-phase 600 kvar capacitor bank was connected to node 675. Node 632 was adopted as the PCC of this network, so the compensation goals and the design of the compensators were related to this node. The waveforms of voltages and currents were measured at nodes 632 and 671, in order to evaluate the effects of compensation not only at PCC, but also at other points of the network.

The simulation of this 13 node network without compensators results in the values shown in Table 2. Fig. 10 shows voltage and current waveforms at nodes 632 and 671.

To compensate reactive and unbalance power at PCC, two SVCs were designed, one considering 70% of \(I_{\text{SVCmax}}\)
measured at node 632 and other considering 30% of that current. For the passive filters, the maximum additional reactive energy that may be compensated by TCRs was 25% of $W_{TSC}$.

Table 3 presents the values of the two designed compensation units, the one with 70% of compensation demand was connected to node 633 and the other was connected to node 680.

Since the conduction angle $\alpha_{bc}$ results close to $\pi$/2 rad, one may observe that the range of additional reactive energy adopted, which may be compensated by TCR, is the maximum in this application.

The simulation total time was 1.6 s and the compensators were turned on at $t = 0.5$ s. Table 4 shows the voltage, current, power, and total harmonic distortion (THD) of voltages and currents at node 632 after compensation.

The power factor was close to unity and both reactive and unbalance power were considerably reduced at PCC. Moreover, distortion power and harmonic distortion were maintained under suitable values by the passive filters.

Another interesting result is that the instantaneous reactive energy at node 632 remained conservative during all the simulation, which can be observed in Fig. 12 that presents the sum of the instantaneous reactive energies from the three phases of node 632 at each instant. As it always results zero, the instantaneous reactive energy is sinusoidal, while currents from node 671 were not so much affected by the compensation.

The results of this simulation show that compensation demand from a determined node may be effectively shared among compensators with different capacities and distributed along the network.

Table 2. Voltage, current, power, energy and power factor values at node 632 (PCC) without compensation.

Table 3. Designed parameters of the two SVC and passive filters.

Table 4. PCC variables with compensation.

Table 5. Designed parameters of the two SVC and passive filters.

Fig. 9. IEEE 13 node test feeder-based network [17].

Fig. 10. Voltages (left) and currents (right) at PCC (node 632) and node 671 without compensation.

Fig. 11 presents the waveforms of the voltages, currents and its fundamental component at node 632 and currents at node 671 after compensation. Both voltages and currents from node 632 were balanced and may be considered sinusoidal, while currents from node 671 were not so much affected by the compensation.

Another interesting result is that the instantaneous reactive energy at node 632 remained conservative during all the simulation, which can be observed in Fig.12 that presents the sum of the instantaneous reactive energies from the three phases of node 632 at each instant. As it always results zero, the instantaneous reactive energy is sinusoidal, while currents from node 671 were not so much affected by the compensation.
conservative and may be used to control distributed compensators in case of variable loads.

Fig. 12. Sum of instantaneous reactive energies from node 632 at each instant of simulation.

Conclusion
This paper presented a new methodology to design SVC and its passive filters by using the recently defined reactive energy concept.

The results show suitable compensation of reactive and unbalance current considering an electrical arc furnace (EAF) as a load. The harmonic distortion caused by TCR was also mitigated and the power factor ended close to unity.

The application of such methodology in distributed compensation was also effective, and the reactive energy from PCC was shared between two compensation units connected to different points in the grid.

The simulations presented here, likewise the discussions in [5,6,9], indicate that reactive energy may be a fundamental variable to insert traditional passive and/or quasi-stationary compensation technologies in the modern power system scenario.

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