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Mathematical model for failure cause analysis of electrical systems with load-sharing redundancy of component

Abstract. *In the paper, a mathematical reliability model for an electrical repairable system with parallel load-sharing redundancy of components is proposed. Such a model adequately accounts for the impact of load-sharing redundancy on cut set probability indexes. For reliability modelling, both dynamic fault tree and Markov analysis are used.*

Streszczenie. *W artykule zaprezentowano model matematyczny system elektrycznego z równoległym obciążeniem i redundancją składowych obciążenia. Zastosowano analizę Markova i drzewo błędu dynamicznego. (**Model matematyczny błędu w systemie elektrycznym powodowanego redundancją składowych obciążenia**)*

Keywords: cut set, load-sharing redundancy, reliability model, dynamical fault tree, Markov model.
Słowa kluczowe: model matematyczny, model Markova, drzewo błędu dynamicznego

Introduction

Recommendations to improve the reliability of technical systems are developed based on failure cause analysis. Each failure cause corresponds to a unique set of downstate components, which is called a cut. Accordingly, the purpose of reliability analysis is to determine all cuts, the content of each cut and the cut probability indexes. During such an analysis of a system with load-sharing redundancy of components, it is necessary to consider the impact of component failures on the upstate component load. Such an impact results in a change of the cut probability indexes. For consideration of these changes, it is necessary to suggest appropriate mathematical reliability models. This problem arises in providing reliability for electrical and power systems, which are used in high-risk applications.

Literature review

There are two different methods to determine the cut probability indexes: a logic and probabilistic approach and Markov analysis. The logic and probabilistic approach is based on the construction of logical conditions, which correspond to cuts. Furthermore, these logical conditions are transformed to appropriate probabilistic expressions [1, 2]. Such an approach is simple to apply but it cannot be used for taking into account load-sharing effects that are caused by component failures, as well as different sequences of component failures and renewals. Markov analysis does bit have these restrictions [3]. However, such an approach is concerned with high complexity modelling and the restriction of time to failure and time to repair by exponential distribution. To reduce the complexity of the Markov analysis, it is necessary to improve the methods for the automatic construction of the Markov models [4]. Such a construction is performed on a fault tree basis. For load sharing, the consideration of the fault tree must be supplemented by logical equations, which should formalise reliability load behaviour [5]. To eliminate the limitations by exponential distribution, it is necessary to use methods for state space splitting [6, 7]. Markov model splitting should provide adequate treatment of non-exponential distributions and the memorising of the load history of components [8-10].

In the paper, such problem solving is presented:

- Dynamic fault-tree-based reliability formalising of repairable system with load-sharing redundancy of components.
- State and event model of system construction.
- Markov model of system creation.
- Cut probability indexes of system determination.

Reliability model and cut set indexes determination and analysis.

Description and dynamic fault tree. The results are shown for the example of a system, the functional diagram of which is shown in Fig. 1*a*. The system parameters are chosen in such a manner as to reflect specifically those that are regarded as having an impact on load sharing. The system structure is composed of three components: the generator G and converters VD1 and VD2. The system function is to provide electricity to consumers connected to its output.

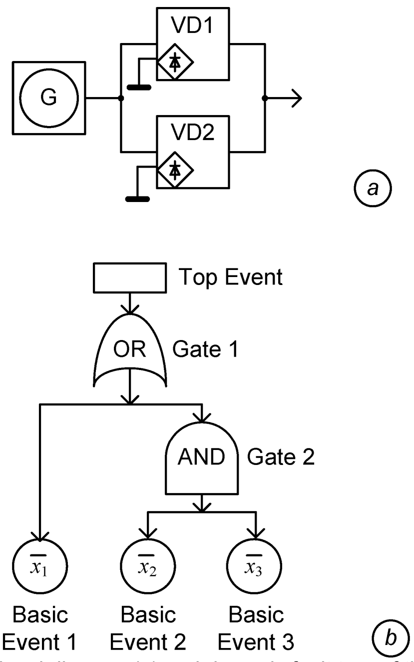

Fig.1. Functional diagram (*a*) and dynamic fault tree of the system (*b*)

The reliability of the system is formalised by a dynamic fault tree, the structure of which is shown in Fig. 1*b*. The dynamic fault tree is a mathematical model that describes the downstate conditions of the load-sharing reliability behaviour based on gate blocks that represent logical and relational operations. The downstate condition is defined by a deductive method starting from the top events across the gates to the basic events.

System downstate, labelled as "Top Event 1" block, is when the system cannot provide energy to the consumers connected to its output. It is considered that such a system downstate is critical, or in other words, after its appearance, repair of the system is not taken into account. This state of the system occurs if generator G is in downstate or if converters group VD1-VD2 are in the downstate labelled "Gate 1" block, which is set as a logical OR operation. Generator G downstate, labelled "Basic Event 1" block, occurs after its failure. The time to failure of the generator is given by Weibull with parameters α = 120 000 h and β = 1.1. The converter block downstate occurs if both converters are in the downstate labelled "Gate 2" block, which is set as a logical AND operation. The downstates of converters VD1 and VD2, labelled "Basic Event 2" and "Basic Event 3" blocks, occur after they failure. The time to failure of both converters is given by Weibull with parameters α = 10 000 h and β = 1.3.

System reliability behaviour by load sharing is set by the scale functions for the wearing processes of the components. This function $f(\mathbf{x})$ defines the load-scaling ratio for the process depending on the logical states (downstate or upstate) of all components of the system.

For the considered reliability model, scale functions must describe three phenomena:

- Generator G turns off if both converters VD1 and VD2 are in downstate.
- Convertors VD1 and VD2 turn off if generator G is in downstate.
- Upstate converter load changes if the other converter is in downstate.

The scale function for the wearing process of generator G ("Basic Event 1") is defined by the logical expression:

$$f_1(\mathbf{x}) = \begin{cases} 1, & if \quad x_1(x_2 \vee x_3), \\ 0, & else. \end{cases}$$

If generator G is upstate and at least one convertor VD1 or VD2 is upstate, then the scale function for this process is 1, whereas in all other cases it is 0.

The scale function for the wearing process of converter VD1 ("Basic Event 2") is defined such as:

$$f_2(\mathbf{x}) = \begin{cases} 1, & if \quad x_1 x_2 x_3, \\ k_2, & if \quad x_1 x_2 \overline{x_3}, \\ 0, & else. \end{cases}$$

If all components of the system are in upstate, then the scale function of this process is 1 and if converter VD2 is in downstate, then the function takes the value k_2 = 5. In all other cases, the scale function is 0.

The scaling function for the wearing process of converter VD2 ("Basic Event 3") is formed in the same way:

$$f_3(\mathbf{x}) = \begin{cases} 1, & if \quad x_1 x_2 x_3, \\ k_3, & if \quad x_1 \overline{x_2} x_3, \\ 0, & else. \end{cases}$$

The difference between these expressions is that if converter VD1 is in downstate, then the scale function for the wearing process of VD2 is taken as k_3 = 6.

General repair of the system is distributed exponentially with parameter μ = 0,025 1/h. This process restores any downstate component. It is not necessary for the scale function $f_4(\mathbf{x})$ of such a process to be set explicitly, because the information presented is sufficient.

State and event model. Based on the dynamic fault tree of the system with load-sharing redundancy, according to the formalised rules [10, p. 67], the state and event model is formed. This model is a mathematical description of the states in which the system may be and of the events that may occur. States and events are obtained from the projection to the processes. The state and transition diagram of this model is shown in Fig. 2.

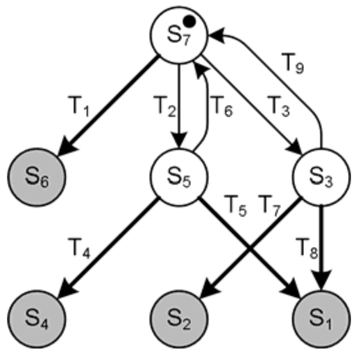

Fig.2. State and transition diagram for state and event model of the system

In the state and event model, the wearing process for generator G is designated as P_1, for converter VD1 — P_2, VD2 — P_3 and for the repair process — P_4. The system can stand in seven states, where three of them are upstate — S_7, S_5 and S_3 and four of them are downstate — S_6, S_4, S_2 and S_1. Nine events can occur in the system, where five of them are failures — T_1, T_4, T_5, T_7 and T_8, two are deterioration — T_2 and T_3 and two are renewal — T_6 and T_9. State parameters (see Table 1) are the scale function values for the P_1–P_4 processes and logical function $y(\mathbf{x})$, which is assigned the value "1" if the system is in upstate, or "0" if the system is in downstate.

Table 1. State parameters of the system

Sate name	Graphical description	Scale function				$y(\mathbf{x})$
		$f_1(\mathbf{x})$	$f_2(\mathbf{x})$	$f_3(\mathbf{x})$	$f_4(\mathbf{x})$	
S_7		1	1	1	0	1
S_6		0	0	0	0	0
S_5		1	0	k_3	1	1
S_4		0	0	0	0	0
S_3		1	k_2	0	1	1
S_2		0	0	0	0	0
S_1		0	0	0	0	0

Event parameters (see Table 2) are source state name, finished process name, destination state name and logical function $z(\mathbf{x})$, which is assigned the value "1" if the event is a failure or "0" if the event is not a failure.

Table 2. Event parameters of the system

Event name	Source state	Finished process	Destination state	$y(\mathbf{x})$
T_1	S_7	P_1	S_6	1
T_2	S_7	P_2	S_5	0

Event name	Source state	Finished process	Destination state	$y(\mathbf{x})$
T_3	S_7	P_3	S_3	0
T_4	S_6	P_1	S_4	1
T_5	S_6	P_3	S_1	1
T_6	S_6	P_4	S_7	0
T_7	S_3	P_1	S_2	1
T_8	S_3	P_2	S_1	1
T_9	S_3	P_4	S_7	0

Markov model. Based on the state and event model of system with load-sharing redundancy, according to the formalised rules [10, p. 78], the split homogeneous Markov model is formed. Under such a model, Chapmen's deferential equation system is suggested:

$$\frac{d}{dt}\mathbf{p}(t) = \mathbf{A}\,\mathbf{p}(t),$$
$$\mathbf{y}(t) = \mathbf{C}\,\mathbf{p}(t).$$

where t — time, $\mathbf{p}(t)$ — vector of phase probability functions and $\mathbf{y}(t)$ — vector of cut probability functions.

The Markov model consists of a phase intensity matrix $\mathbf{A}$, initial phase probability vector $\mathbf{p}(0)$ and matrix $\mathbf{C}$, which sets how the cut probability functions relate to the phase probability functions:

$$\mathbf{A} = \left[\begin{array}{c|c|c|c|c|c|c} & & \mathbf{A}_{\mathrm{T}_8} & & \mathbf{A}_{\mathrm{T}_1} & & \\ \hline & & \mathbf{A}_{\mathrm{T}_7} & & & & \\ \hline & & \mathbf{A}_{\mathrm{S}_3} & & & & \mathbf{A}_{\mathrm{T}_3} \\ \hline & & & & \mathbf{A}_{\mathrm{T}_4} & & \\ \hline & & & & \mathbf{A}_{\mathrm{S}_5} & & \mathbf{A}_{\mathrm{T}_2} \\ \hline & & & & & & \mathbf{A}_{\mathrm{T}_1} \\ \hline & & \mathbf{A}_{\mathrm{T}_9} & & \mathbf{A}_{\mathrm{T}_6} & & \mathbf{A}_{\mathrm{S}_7} \end{array}\right], \mathbf{p}(0) = \left[\begin{array}{c} \\ \hline \\ \hline \\ \hline \\ \hline \\ \hline \\ \hline \mathbf{p}_{\mathrm{S}_7} \end{array}\right],$$

$$\mathbf{C} = \left[\begin{array}{c|c|c|c|c|c|c} \mathbf{C}_{\mathrm{S}_1} & & & & & & \\ \hline & \mathbf{C}_{\mathrm{S}_2} & & & & & \\ \hline & & & \mathbf{C}_{\mathrm{S}_4} & & & \\ \hline & & & & & \mathbf{C}_{\mathrm{S}_6} & \end{array}\right].$$

The Markov model components of the system are formed based on the Markov model of processes. Process Markov model parameters are determined according to such criteria. It is understood that both the mean and variance of the process distribution and the corresponding process Markov model (also called the phase-type distribution) are equivalent.

It is considered that for process P_1, the parameters of the corresponding Markov model are {$\mathbf{A}_1$, $\mathbf{p}_1(0)$, $\mathbf{C}_1$}, for process P_2 — {$\mathbf{A}_2$, $\mathbf{p}_2(0)$, $\mathbf{C}_2$}, for process P_3 — {$\mathbf{A}_3$, $\mathbf{p}_3(0)$, $\mathbf{C}_3$} and for process P_4 — {$\mathbf{A}_4$, $\mathbf{p}_4(0)$, $\mathbf{C}_4$}. Accordingly, for the presented parameters, we can write the following equations for the initial upstate S_7:

$$\mathbf{A}_{\mathrm{S}_7} = \mathbf{A}_1 \otimes \mathbf{E}_2 \otimes \mathbf{E}_3 \otimes \mathbf{E}_4 + \mathbf{E}_1 \otimes \mathbf{A}_2 \otimes \mathbf{E}_3 \otimes \mathbf{E}_4 +$$
$$+ \mathbf{E}_1 \otimes \mathbf{E}_2 \otimes \mathbf{A}_3 \otimes \mathbf{E}_4,$$
$$\mathbf{p}_{\mathrm{S}_7} = \mathbf{p}_1(0) \otimes \mathbf{p}_2(0) \otimes \mathbf{p}_3(0) \otimes \mathbf{p}_4(0).$$

where $\otimes$ — tensor product operator; $\mathbf{E}_1$, $\mathbf{E}_2$, $\mathbf{E}_3$, $\mathbf{E}_4$ — identity matrix, the dimensions of which are equal to the dimensions of the $\mathbf{A}_1$, $\mathbf{A}_2$, $\mathbf{A}_3$, and $\mathbf{A}_4$ matrices.

For upstate S_5:

$$\mathbf{A}_{\mathrm{S}_5} = \mathbf{A}_1 \otimes \mathbf{E}_2 \otimes \mathbf{E}_3 \otimes \mathbf{E}_4 + k_3 \mathbf{E}_1 \otimes \mathbf{E}_2 \otimes \mathbf{A}_3 \otimes \mathbf{E}_4 +$$
$$+ \mathbf{E}_1 \otimes \mathbf{E}_2 \otimes \mathbf{E}_3 \otimes \mathbf{A}_4.$$

For upstate S_3:

$$\mathbf{A}_{\mathrm{S}_4} = \mathbf{A}_1 \otimes \mathbf{E}_2 \otimes \mathbf{E}_3 \otimes \mathbf{E}_4 + k_2 \mathbf{E}_1 \otimes \mathbf{A}_2 \otimes \mathbf{E}_2 \otimes \mathbf{E}_3 +$$
$$+ \mathbf{E}_1 \otimes \mathbf{E}_2 \otimes \mathbf{E}_3 \otimes \mathbf{A}_4.$$

For downstates S_1, S_2, S_4 and S_6:

$$\mathbf{C}_{S_1} = \mathbf{C}_{S_2} = \mathbf{C}_{S_4} = \mathbf{C}_{S_6} = \mathbf{I},$$

where $\mathbf{I}$ — row-vector of ones, the dimension of which is equal to dimension product of the $\mathbf{A}_1$, $\mathbf{A}_2$, $\mathbf{A}_3$, and $\mathbf{A}_4$ matrices.

For events T_1, T_4 and T_7, which occur due to the completion of process P_1:

$$\mathbf{A}_{\mathrm{P}_1} = \mathbf{p}_1 \mathbf{C}_1 \otimes \mathbf{E}_2 \otimes \mathbf{E}_3 \otimes \mathbf{E}_4,$$
$$\mathbf{A}_{\mathrm{T}_1} = \mathbf{A}_{\mathrm{T}_4} = \mathbf{A}_{\mathrm{T}_7} = \mathbf{A}_{\mathrm{P}_1}.$$

For events T_2 and T_8, which occur due to the completion of process P_2:

$$\mathbf{A}_{\mathrm{P}_2} = \mathbf{E}_1 \otimes \mathbf{p}_2 \mathbf{C}_2 \otimes \mathbf{E}_3 \otimes \mathbf{E}_4,$$
$$\mathbf{A}_{\mathrm{T}_2} = \mathbf{A}_{\mathrm{P}_2},$$
$$\mathbf{A}_{\mathrm{T}_8} = k_2\, \mathbf{A}_{\mathrm{P}_2}.$$

For events T_3 and T_5, which occur due to the completion of process P_3:

$$\mathbf{A}_{\mathrm{P}_3} = \mathbf{E}_1 \otimes \mathbf{E}_2 \otimes \mathbf{p}_3 \mathbf{C}_3 \otimes \mathbf{E}_4,$$
$$\mathbf{A}_{\mathrm{T}_3} = \mathbf{A}_{\mathrm{P}_3},$$
$$\mathbf{A}_{\mathrm{T}_5} = k_3\, \mathbf{A}_{\mathrm{P}_3}.$$

For events T_6 and T_9, which occur due to the completion of process P_4:

$$\mathbf{A}_{\mathrm{P}_4} = \mathbf{E}_1 \otimes \mathbf{E}_2 \otimes \mathbf{E}_3 \otimes \mathbf{p}_4 \mathbf{C}_5,$$
$$\mathbf{A}_{\mathrm{T}_6} = \mathbf{A}_{\mathrm{T}_9} = \mathbf{A}_{\mathrm{P}_4}.$$

The resulting Markov model contains 56 phases and 68 transitions, a diagram of which is shown in Fig. 3.

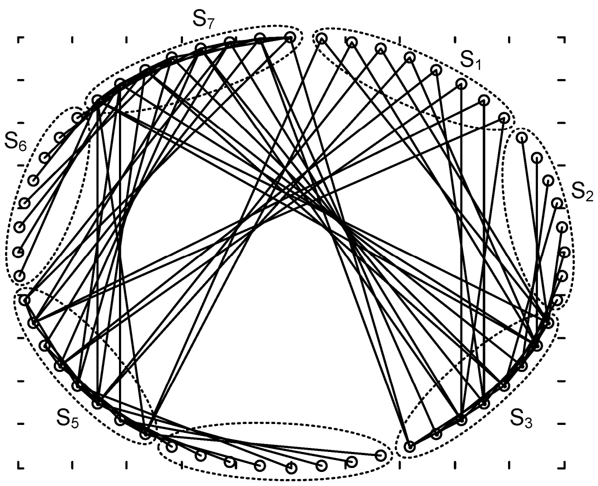

Fig.3. State and transition diagram for splitting homogeneous Markov model of the system

Probability characteristics of the system are determined using the matrix exponent:

$$\mathbf{y}(t) = \mathbf{C}\exp(\mathbf{A}t)\mathbf{p}(0),$$

$$\exp(\mathbf{A}t) = \mathbf{E} + \mathbf{A}t + \frac{\mathbf{A}^2 t^2}{2!} + \frac{\mathbf{A}^3 t^3}{3!} + \ldots$$

where $\mathbf{E}$ — identity matrix, the dimension of which is equal to the dimension of matrix $\mathbf{A}$.

The matrix exponent values are calculated following the approach of Golub and van Loan, which is based on the Pade approximation.

Cut probability indexes. By using probability characteristics calculated by splitting the homogeneous Markov model for time moment t = 10 000 h, cut probability indexes can be determined (Table 3 and Fig. 4*a*).

Table 3. Cut probability indexes of the system

State name	Content	Probability	Relative weight
S_1	VD1 and VD2 in downstate	0.082494	54.916 %
S_6	G in downstate	0.066606	44.349 %
S_2	G and VD2 in downstate	0.00056263	0.374 %
S_4	G and VD1 in downstate	0.00055695	0.371 %

As can be seen from Table 3, the system contains four downstates S_1, S_2, S_4 and S_6, which correspond to the cuts. The cuts in the table are sorted by decreasing their relative weight in the overall probability of system failure and their contents list only downstate components. Thus, based on the information about the cuts, we can conclude that in order to reduce the failure probability of the system (Q_1 = 0.1502), it is necessary to improve the reliability of converters VD1 and VD2. The simultaneous downstate of these two components is the most probable cause of system downstate with 54.916% relative weight. In addition, we note that "G and VD1" and "G and VD2" cuts have different relative weights. This feature is due to non-uniform load sharing between the converters defined by the $k_2 \neq k_3$ inequality; however, for this model, such a phenomenon has been neglected.

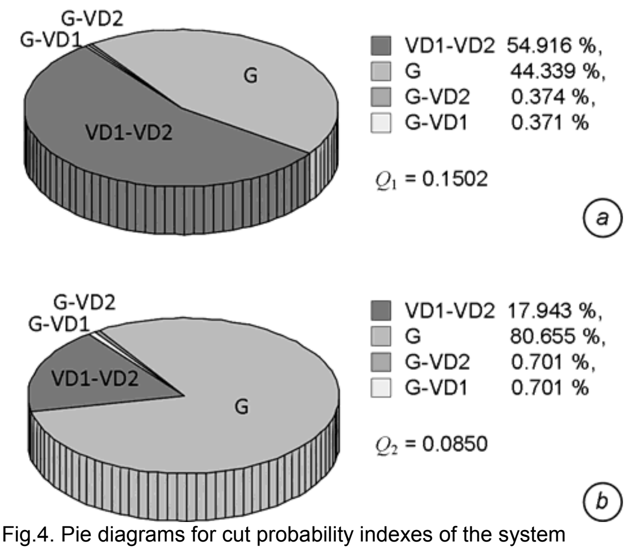

Fig.4. Pie diagrams for cut probability indexes of the system

If we calculate the cut probability indexes of the system for the case where the state of one converter does not affect the load on the other converter, i.e., $k_2 = k_3 = 1$, then we have the following two differences. Firstly, the predicted probability of system failure is reduced to the value Q_2 = 0.0850 (Fig. 4*b*), i.e., it is reduced by almost one-half. Secondly, by using the proposed model, it is quantitatively shown how load sharing effects on the cut probability indexes. If load sharing is neglected, then the most probable cause of downstate with 80.655% relative weight is the downstate of generator G, which for this system is a false statement.

Conclusion

In this paper, a mathematical reliability model of electrical systems with load-sharing redundancy for cut probability indexes determination is suggested. The formalisation of the system reliability is based on a dynamic fault tree. The determination of system probability characteristics is based on a split homogeneous Markov model. Such a model provides adequate load sharing, taking into account the components for which time to failure is distributed by Weibull. By using such a model, cut probability indexes are determined accurately and it shows those components for which it is recommended that reliability be improved in order to reduce the probability of system failure.

Using the proposed approach, the authors have achieved adequate reliability models for the analysis of the cause of failure for repairable systems with standby and sliding redundancy, as well as for two out of the three load-sharing systems.

Further studies are intended on approaches for developing methods of analysis for the cause of failure of multifunctional systems and repairable systems with complex structures.

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