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Back EMF Computation of BLDC Motor

Streszczenie. *Artykuł przedstawia metodę modelowania i wyznaczania numerycznego indukowanej siły elektromotorycznej w uzwojeniach trójfazowego silnika BLDC przy wykorzystaniu metody elementów skończonych. W prezentowanym modelu równania pola elektromagnetycznego uwzględniające oddziaływanie magnesów trwałych są sprzężone z równaniami ruchu i rozwiązywane wspólnie w dyskretnych chwilach czasowych. Siła elektromotoryczna indukowana w obwodzie elektrycznym jest obliczana dla zadanej prędkości obrotowej silnika. Wyniki obliczeń numerycznych porównano z pomiarami.*

Abstract. *In this paper a method of modeling and numerical calculation of the back electromotive force in the three phase BLDC motor using the finite element technique is presented. In analyzed model the electromagnetic field model considering permanent magnets is coupled with motion model and solved together at discrete time steps. The back electromotive force induced in the electric circuit is computed for a given rotor velocity. Numerical calculations are compared with experimental results. (**Metoda obliczania siły elektromotorycznej w silniku BLDC**).*

Słowa kluczowe: silnik BLDC, siła przeciwelektromotoryczna, metoda elementów skończonych.
Keywords: BLDC motor, back electromotive force, finite element method.

Introduction

The wide application of electric machines legitimizes the need for constant improvements, which lead to enhanced performance in terms of mechanical and electrical characteristics. This is achieved by modifications in design understood as shape, material properties, electronic inverter and, last but not least, applied control law [2,3,12].

All this is possible when accurate and precise numerical model is applied. This overall model should be a product of three different sub models describing electromagnetic field, electrical circuit and motion. The most crucial thing is the effectiveness of their mutual coupling giving exact problem description at every time instance. There is a vast interest among researchers to develop this numerical technique as a powerful tool for electric machines prototyping [2,4,6,10,13].

This paper continues the research and aims at presenting the accuracy of a time stepping finite element technique in computation of back EMF of the three phase BLDC motor. The numerical results are compared with experimental measurements presented by Kawase in his article [1].

Coupled field-circuit model of the BLDC motor

In the time – stepping finite element technique for a voltage controlled BLDC motor, the inputs are stator phase voltages, motor geometry and material characteristics, whereas all the other variables such as magnetic vector potentials, currents, the rotor position and the rotor speed are calculated. For an analysis of the back EMF of the motor, the problem is inverse. The prescribed rotor speed is an input value and the voltage excitation becomes the unknown variable [11].

This article presents the analysis of the back EMF with an employment of the 3D time - stepping finite element method. Fig. 1 shows configuration of the motor.

The motor field model is represented by the magnetic vector potential. The equation that describes the magnetic field is written in the cylindrical co-ordinate system. In the area of the stator conductor the field equation may be represented as

(1) $$\nabla \times \left(\frac{1}{\mu} \nabla \times \mathbf{A} \right) = \mathbf{j} + \frac{1}{\mu} \nabla \times \mathbf{M} \ .$$

The magnetic vector potential **A** is the magnetic field variable, μ is a permeability, **j** is a current density of the thin winding and **M** denotes the magnetization of magnets. In this model the eddy current problem is ignored, because the stator and rotor are laminated and the iron losses have a very small impact on the motor dynamics in this case [2,3,7,11].

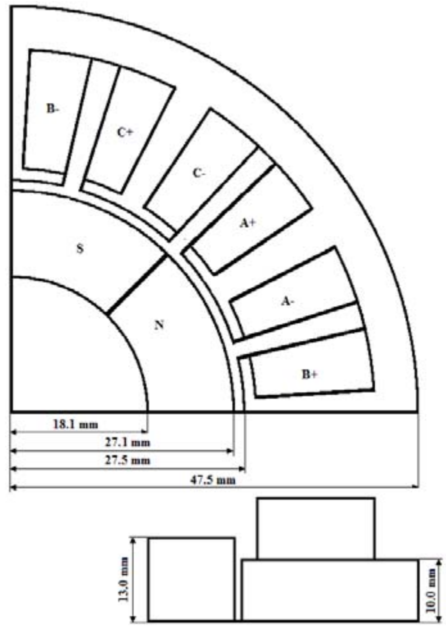

Fig.1. The BLDC motor configuration

The stator phase circuit equation for the described motor is

(2) $$\frac{d}{dt} \oint_{l_s} \mathbf{A} d\mathbf{l} = u_s - R i_s \ ,$$

where *s*={1,..,3} denotes the phase number, *R* is a winding resistance of a one phase, *i* is a phase current and *u* is a supply voltage.

From the approach (1) the matrix equation system is derived, where the weighting functions are the same as the shape functions. The solution of the equation (1) is obtained by minimizing the corresponding energy functional. The minimization is performed by means of the finite element method using 27-node, first order cylindrical elements. The magnetic vector potential may be expressed by

(3) $$\mathbf{A} = \sum_{i=1}^{27} N_i \mathbf{A}_i \ ,$$

where N_i are the element shape functions and the $\mathbf{A}_i$ are the approximations of the vector potentials at the nodes of the elements.

In the time stepping case, the field equation (1) may be written in matrix form as follows:

(4) $$\mathbf{C}\mathbf{A}^{t+\Delta t} + \mathbf{P}\mathbf{I}^{t+\Delta t} = \mathbf{M}(\omega^{t+\Delta t}),$$

where **A** represents the vector of the unknown vector potentials, **I** represents the vector of the unknown phase currents, **C** represents the symmetrical matrix related to the magnetic field, **P** represents the matrix related to the winding currents.

The equation (2) may be also represented in the discrete form:

(5) $$\frac{1}{\Delta t}\mathbf{\Phi}^{t+\Delta t} + \mathbf{R}\mathbf{I}^{t+\Delta t} = \mathbf{U}^{t+\Delta t} + \frac{1}{\Delta t}\mathbf{\Phi}^{t},$$

where $\mathbf{\Phi} = \left[\oint_{l_1} \mathbf{Adl} \ \ldots \ \oint_{l_3} \mathbf{Adl} \right]^T$ represents the vector of the winding flux linkage, **R** represents the diagonal matrix of the winding resistance. If to substitute $\mathbf{\Phi} = \mathbf{QA}$, where **Q** represents matrix related to the winding linkage flux, the equation (5) may be rewritten as

(6) $$\frac{1}{\Delta t}\mathbf{Q}\mathbf{A}^{t+\Delta t} + \mathbf{R}\mathbf{I}^{t+\Delta t} = \mathbf{U}^{t+\Delta t} + \frac{1}{\Delta t}\mathbf{Q}\mathbf{A}^{t}.$$

The field equation (4) and the circuit equation (6) have two common variables: magnetic vector potential **A** and current vector **I**. This way, the following coupled global system of equations is obtained [4,9]:

(7) $$\begin{bmatrix} \mathbf{C} & \mathbf{P} \\ \frac{\mathbf{Q}}{\Delta t} & \mathbf{R} \end{bmatrix} \begin{bmatrix} \mathbf{A}^{t+\Delta t} \\ \mathbf{I}^{t+\Delta t} \end{bmatrix} = \begin{bmatrix} \mathbf{M}(\omega^{t+\Delta t}) \\ \mathbf{U}^{t+\Delta t} + \frac{\mathbf{Q}}{\Delta t}\mathbf{A}^{t} \end{bmatrix}.$$

The equation (7) describes the magnetic system coupled with the electric circuit when voltage excitation is known. In this case winding currents and magnetic vector potentials are computed as unknown variables. However, when back EMF is computed, as in our area of interest, the problem is inverse. Now the input electric circuit is open, which means currents do not flow through the windings and the voltage excitation, originated from rotating permanent magnets, is unknown. In this case the equation (7) may be reduced to the following system [1]:

(8) $$\begin{bmatrix} \mathbf{C} & \mathbf{0} \\ \frac{\mathbf{Q}}{\Delta t} & -\mathbf{Y} \end{bmatrix} \begin{bmatrix} \mathbf{A}^{t+\Delta t} \\ \mathbf{U}^{t+\Delta t} \end{bmatrix} = \begin{bmatrix} \mathbf{M}(\omega^{t+\Delta t}) \\ \frac{\mathbf{Q}}{\Delta t}\mathbf{A}^{t} \end{bmatrix},$$

where **Y** is the identity matrix, **A** and **U** are unknown and **M** may be treated as an excitation. The system of equations (8) describes the field-circuit model exited by movable permanent magnets where the field and the induced electromotive force in the windings are obtained by computation. The presented system of the field equations (8) is nonsymmetric and solved at each iteration step by the preconditioned bi-conjugate gradient algorithm (BiCG) dedicated for the large and sparse linear systems.

Mechanical motion model

In the prescribed speed case, the rotor displacement is evaluated by solution of the rotational motion equation defined for a one degree of freedom:

(9) $$\frac{d\varphi}{dt} = \omega,$$

where φ is the rotor displacement and ω is the rotor angular speed. The discrete equation of motion is derived from the form of (9). When the rotor speed is known, then displacement may be determined using the backward Euler's approximation for the first order equation (9):

(10) $$\varphi^{t+\Delta t} = \varphi^{t} + \Delta t\,\omega^{t+\Delta t}.$$

Each iteration of the motion model solution comprises two steps. The first is to determine rotor's new angular displacement for the time step Δt. The second one is to reflect this position in the discretized space.

The rotor motion in the electromagnetic field is realized with the fixed grid technique. The grid of discretization is independent of the rotor position. It is important to provide an even discretization (Δφ) in direction (φ) along which the movement is realized to keep the same volume of the moving body [3].

The rotor displacement in the grid of discretization is updated at each iteration step, which is depicted in Fig. 2. Thus the time step Δt should be chosen in a way which grants the condition (11):

(11) $$\hat{\varphi}^{t+\Delta t} = \varphi^{t} + \Delta\varphi, \qquad \varphi^{t} + \Delta\varphi > \hat{\varphi}^{t+\Delta t} > \varphi^{t} + \frac{\Delta\varphi}{2},$$

where $\hat{\varphi}$ stands for a position in the node of discretization grid. If the above inequality is not to be fulfilled, the time step is being automatically shortened to $\Delta t = \Delta t / 2$ for the current iteration. This approach enables to overcome the stability loss during the solution of the field equations (8) [5].

The field model (8) and the motion model (10) are coupled and solved together at each iteration step [6,8].

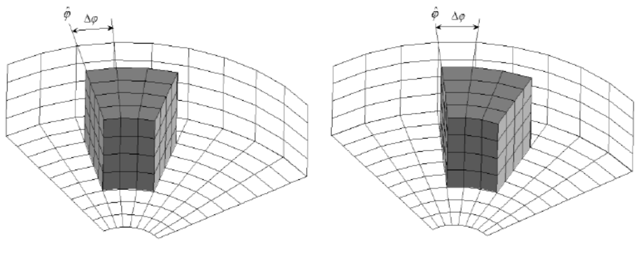

Fig.2. Mechanical motion iterative implementation

Numerical experiment

The numerical technique described in this paper is applied to analysis of the three phase BLDC motor. The motor has 12 stator slots, 4 per each phase and 4 rotor permanent magnets. The half-length 3D model of the BLDC motor in cylindrical coordinate system is employed and shown in Fig. 3. The discrete grid has 71 478 nodes. Each phase includes 200 windings.

The numerical model has 170 640 unknown variables, where 55 440 unknown A_r, 50 400 unknown A_φ, 64 800 unknown A_z. The BiCG accuracy is set to $1{,}0 \cdot 10^{-5}$.

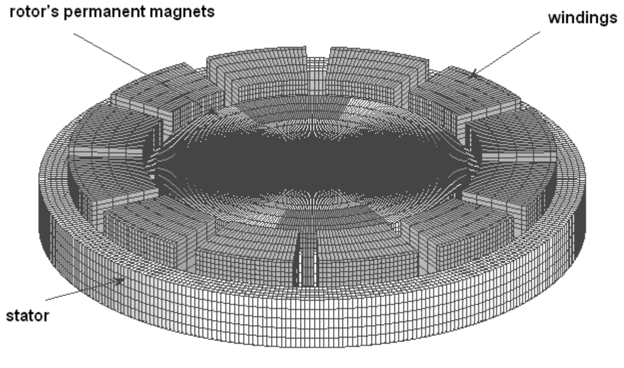

Fig.3. The BLDC motor model

The model complies the end-coil effect. The back electromotive forces induced in stator windings are investigated for prescribed rotor speed equal to 600 rpm. Fig. 4. presents comparison of experimental measurements [1] with numerical computations.

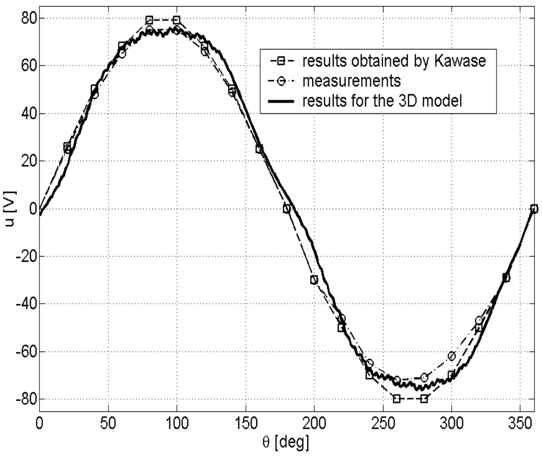

Fig.4. The back EMF course in calculation and measurement

The above figure depicts relation between induced voltage in one phase and electrical degree. This proves that back EMF value is dependent on rotor position. Due to PM poles number 360^o electrical corresponds to 90^o mechanical.

Fig. 5 presents back EMF characteristics in all three BLDC phases. Voltage courses are shifted 30^o mechanical due to number of stator poles.

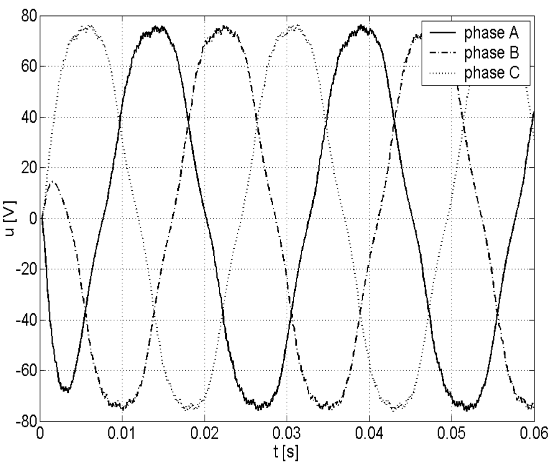

Fig.5. The back EMF course in three BLDC phases

Presented results correspond with measurements and other researchers' calculations.

Conclusions

In this work, a method of a time - stepping finite element method is presented. The proposed model tightly couples field - circuit - motion phenomena and enables their simultaneous computing at every iteration step. The model considers also the permanent magnet effect.

The model usefulness is depicted on the example of a three phase BLDC motor. The stress is put on the back EMF calculation, induced in the motor windings by a rotating magnetic field from the PM of the rotor moving with a prescribed speed. The numerical results show full compliance with the experimental results obtained by other authors.

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