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Application of the fuzzy controller in the speed control system of an induction motor

***Abstract**. The paper presents a model of an asynchronous motor powered from an IGBT converter. The control system of the converter includes a conventional current controller and a fuzzy angular speed controller. The tuning procedure of the fuzzy controller is carried out by means of the sequential quadratic programming. The results of simulation tests are presented as time curves of phase-to-phase voltage, phase current, angular velocity and electromechanical moment.*

***Streszczenie.** W artykule przedstawiono model silnika asynchronicznego zasilanego z przekształtnika IBGT. W układzie sterowania przekształtnika zastosowano konwencjonalny regulator prądu oraz rozmyty regulator prędkości kątowej. Procedurę strojenia regulatora rozmytego przeprowadzono z zastosowaniem metody sekwencyjnego programowania kwadratowego. Rezultaty badań symulacyjnych zaprezentowano w postaci przebiegów czasowych napięcia międzyfazowego, prądu fazowego, prędkości kątowej oraz momentu elektromechanicznego. (**Zastosowanie regulatora rozmytego w układzie sterowania prędkości silnika indukcyjnego**).*

Keywords: induction motor, fuzzy controller, sequential quadratic programming method.
Słowa kluczowe: silnik indukcyjny, regulator rozmyty, metoda sekwencyjnego programowania kwadratowego.

Introduction

Induction motors have many advantages such as a simple structure, high reliability and low maintenance costs. However, considering the requirements of modern industrial applications, the control of these engines is a significant challenge. This is due to non-linearities and a large number of parameters that vary depending on the operating conditions.

There are many methods of controlling speed of an induction motor, such as the scalar control, the field-oriented control, the flux control and the adaptive control. The field-oriented or vector method is based on the transformation of currents from a rotating reference frame to a stationary frame, and vice versa. This method provides an optimum transient response and a high steady-state performance of the drive [4, 5].

There are problems with selecting an accurate mathematical model of the unknown load changes, the temperature changes and the impact of non-linearities on the motor parameters. In this case, a good solution will be to use a fuzzy controller in the speed control system.

The controller tuning procedure was carried out using the sequential quadratic programming method. This method belongs to the group of nonlinear programming methods with constraints. The method works in an iterative way and consists in determining the improvement direction and in calculating the optimum of the objective function for the direction. The conditions for ending the iteration are specified on the basis of the step length or on the basis of the value of the objective function gradient. That is why the calculations stop when the step performed turns out to be sufficiently short, or when the gradient is sufficiently small.

The model of an alternating current motor

The AC induction motor model is given by the space vector form of the voltage equations. The system model defined in the stationary α,β-coordinate system attached to the stator is expressed by the differential equations. The following assumptions were used: the motor model is symmetrical and the magnetic circuit has a linear characteristic. The stator voltage differential equations can be expressed in the following form [1, 4]:

$$u_{s\alpha} = R_s i_{s\alpha} + \frac{d}{dt}\Psi_{s\alpha} \tag{1}$$

$$u_{s\beta} = R_s i_{s\beta} + \frac{d}{dt}\Psi_{s\beta} \tag{2}$$

However, the rotor voltage differential equations can be written as:

$$u_{r\alpha} = R_r i_{r\alpha} + \frac{d}{dt}\Psi_{r\alpha} + \omega\Psi_{r\beta} \tag{3}$$

$$u_{r\beta} = R_r i_{r\beta} + \frac{d}{dt}\Psi_{r\beta} + \omega\Psi_{r\alpha} \tag{4}$$

The electromagnetic torque expressed by utilizing space vector quantities [1, 4]:

$$T_e = \frac{3}{2} p\left(\Psi_{s\alpha} i_{s\beta} - \Psi_{s\beta} i_{s\alpha}\right) \tag{5}$$

where: α, β – stator orthogonal coordinate system, $u_{s\alpha}$, $u_{s\beta}$ – stator voltages, $i_{s\alpha}$, $i_{s\beta}$ – stator currents, R_s – stator phase resistance, $\Psi_{s\alpha}$, $\Psi_{s\beta}$ – stator magnetic fluxes, $u_{r\alpha}$, $u_{r\beta}$ – rotor voltages, $i_{r\alpha}$, $i_{r\beta}$ – rotor currents, R_r – rotor phase resistance, ω – rotor speed, p – number of pole pairs, T_e – electromagnetic torque,

The structure of the fuzzy controller

The controller is characterized by the structure of the Mamdani type, as well as by the direct operation based on error signals and error integrals. Owing to that, the general base of the rules can be written down in as follows [7]:

$$\begin{aligned}
&R^{(1)}: IF\ (e\ is\ LE^{(1)})\ AND\ (ie\ is\ LIE^{(1)}) \\
&\qquad THEN\ (u\ is\ LU^{(1)}) \\
&R^{(2)}: IF\ (e\ is\ LE^{(2)})\ AND\ (ie\ is\ LIE^{(2)}) \\
&\qquad THEN\ (u\ is\ LU^{(2)}) \\
&\qquad \ldots\ldots\ldots\ldots\ldots\ldots \\
&R^{(k)}: IF\ (e\ is\ LE^{(k)})\ AND (ie\ is\ LIE^{(k)}) \\
&\qquad THEN\ (u\ is\ LU^{(k)})
\end{aligned} \tag{6}$$

where: *e*, *ie* - are the linguistic input variables: of the error and of the error integral, *u* - is the linguistic output variable, $LE^{(1)}$,..., $LE^{(k)}$, $LIE^{(1)}$,..., $LIE^{(k)}$ - are the linguistic values of the error and the error integral, $LU^{(1)},...,LU^{(k)}$ - are the linguistic values of the output variable.

For the sake of the present research, a fuzzy controller has been designed with the use of Fuzzy Logic Toolbox. The controller has two modules of linguistic input data *e* and *ie*, and one module of linguistic output data *u*. The linguistic input data include five triangular membership functions defining the error *e* and the error integral *ie*. The functions are labelled as follows: *NM* – negative medium, *NS* – negative small, *ZO* - zero, *PS* – positive small, *PM* – positive medium.

For the linguistic output variable *u*, membership Gaussian functions were applied. The functions are distributed evenly in the standardised output interval. There are seven functions altogether, with *NB* – negative big and *PB* – positive big introduced additionally.

The rule base for the fuzzy controller was initially defined by means of the standard Mac Vicar-Whelan table, which was subsequently modified taking into account the input and output parameters of the membership functions defining the linguistic variables. The rule base is presented in Table 1.

Table 1. The rule base of the fuzzy controller

Designations	Error integral *ie*				
Error *e*	*NM*	*NS*	*ZO*	*PS*	*PM*
NM	*NB*	*NB*	*NM*	*NS*	*ZO*
NS	*NB*	*NM*	*NS*	*ZO*	*PS*
ZO	*NM*	*NS*	*ZO*	*PS*	*PM*
PS	*NS*	*ZO*	*PS*	*PM*	*PB*
PM	*ZO*	*PS*	*PM*	*PB*	*PB*

When the rule base is entered, the defuzzification process starts. It consists in selecting a mathematical procedure which transforms the fuzzy set, defined by the membership function, into a scalar value. When the Centre of Area (COA) method is applied, the scalar value of the controller output signal for the discrete case is obtained from the following formula [2]:

$$u_N = \frac{\sum_{i=1}^{m} u_i \cdot \mu_U(u_i)}{\sum_{i=1}^{m} \mu_U(u_i)} = \frac{\sum_{i=1}^{m} u_i \cdot \max_k \mu_{CLU(k)}(u_i)}{\sum_{i=1}^{m} \max_k \mu_{CLU(k)}(u_i)} \tag{7}$$

where: u_N – is the normalized output linguistic variable, μ_U - is the membership function of output fuzzy set, $\mu_{CLU(k)}$ - is the compressed membership function for each *k*-th rule.

The sequential quadratic programming method

The fuzzy controller is characterized by an original method of tuning based on the sequential quadratic programming. In this method the objective function $f(x)$ is approximated by a square function $f_{AP}(x)$, which can be described by means of [3]:

$$f_{AP}(\mathbf{x}) = f(\mathbf{x}_0) + \nabla f(\mathbf{x}_0)^T(\mathbf{x}-\mathbf{x}_0) + \\ + \frac{1}{2}(\mathbf{x}-\mathbf{x}_0)^T \mathbf{H}(\mathbf{x}-\mathbf{x}_0) \tag{8}$$

where: x_0 - is the point at which the objective function is examined, $\nabla f(x_0)$ - is the objective function gradient at the point x_0, H - is the square matrix known as the Hesse matrix.

The Hesse matrix consists of the values of the second order derivatives of the function $f(x)$ calculated for the point x_0 [3]:

$$\mathbf{H} = \begin{bmatrix} \frac{\partial^2 f(\mathbf{x}_0)}{\partial x_1 \partial x_1} & \frac{\partial^2 f(\mathbf{x}_0)}{\partial x_1 \partial x_2} & \cdots & \frac{\partial^2 f(\mathbf{x}_0)}{\partial x_1 \partial x_n} \\ \frac{\partial^2 f(\mathbf{x}_0)}{\partial x_2 \partial x_1} & \frac{\partial^2 f(\mathbf{x}_0)}{\partial x_2 \partial x_2} & \cdots & \frac{\partial^2 f(\mathbf{x}_0)}{\partial x_2 \partial x_n} \\ \cdots & \cdots & \cdots & \cdots \\ \frac{\partial^2 f(\mathbf{x}_0)}{\partial x_n \partial x_1} & \frac{\partial^2 f(\mathbf{x}_0)}{\partial x_n \partial x_2} & \cdots & \frac{\partial^2 f(\mathbf{x}_0)}{\partial x_n \partial x_n} \end{bmatrix} \tag{9}$$

where: *n* – the dimension of the space in which the objective function $f(x)$ is described.

Knowing the Hesse matrix of the objective function, one can determine the improvement direction d_f on the basis of the following [3]:

$$\mathbf{d}_f = -\mathbf{H}^{-1} \nabla f(\mathbf{x}_0) \tag{10}$$

It is advisable to apply the variable metric methods, which employ the approximation of the Hesse matrix and not its reverse in order to obtain the improvement direction d_f. The matrix approximating the Hesse matrix reverse is updated in every step of the iteration by means on a mathematical method [8].

Currently, the most popular and effective method of this kind is considered to be the Broyden-Fletcher-Goldfarb-Shanno (BFGS) method. It makes it possible to update the Hesse matrix value in every next iteration (H_{k+1}) on the basis of the current value (H_k) and the objective function gradient.

The tuning procedure of the fuzzy controller

The numerical calculations employing the nonlinear programming method with constraints are the substantial component of the tuning procedure, which comprises the following stages [6]:

- Division of the error input space and of the integral error input space of the angular speed,
- Selection of variables to be optimized, i.e. the amplification factor and the time of doubling the nonlinear angular speed controller,
- Determining the constraints referring to the time characteristics of the angular speed,
- Performing the calculations for the particular fragments of the input space by means of the sequential quadratic programming method,
- Preparing the matrix consisting of the amplification factors and of the doubling times obtained for the particular fragments of the error input spaces and for the error integrals.
- Applying the results obtained in the form of the matrix for determining the parameters of the fuzzy controller.

With the right computational tool, such as the SQP method, it is possible to tune the Mamdani-type fuzzy controller. For this purpose, a universal simulation model of an AC drive was constructed, as shown in Fig. 1.

With respect to drive systems, an indicator is often applied which is calculated as an integral of the product of time and the absolute value of the control deviation. The indicator can be represented as [1]:

$$\mathrm{I}_{ITAE} = \int_0^{\infty} t \cdot |e(t)|\, dt = \int_0^{t_k} t \cdot |e(t)|\, dt \tag{11}$$

where: $e(t)$ - is the control deviation, t_k - is the time of ending the control process, *t* - is the time.

Utilizing Eq. (11), one can represent the integral quality indicator of the angular speed as:

$$I_{\Omega} = \int_0^{\infty} t \cdot \left| e_{\omega}(t) \right| dt = \int_0^{t_k} t \cdot \left| e_{\omega}(t) \right| dt = \int_0^{t_k} t \cdot \left| \omega_R - \omega(t) \right| dt \qquad (12)$$

where: $\omega(t)$ - is the angular speed, ω_R - is the prescribed angular speed, $e_{\omega}(t)$ - is the angular speed error.

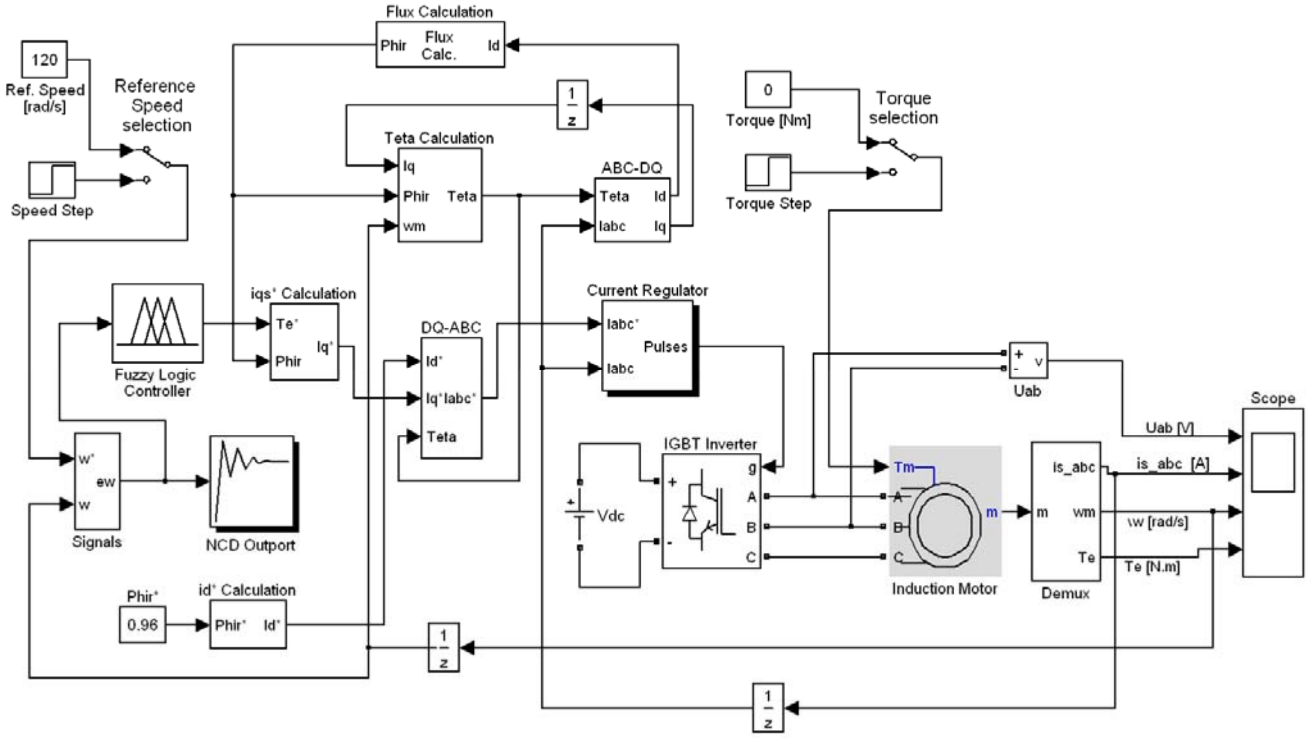

Fig. 1. Simulation model of an AC drive

Results of simulations

Simulations on the AC converter drive model with the fuzzy angular speed controller were performed at the start-up with constant angular speed (ω_R=120 rad/s) and with various load torques. The selected results in the form of time characteristics of the phase-to-phase voltage, phase current, angular speed, and electromechanical torque are presented in Fig. 2-7.

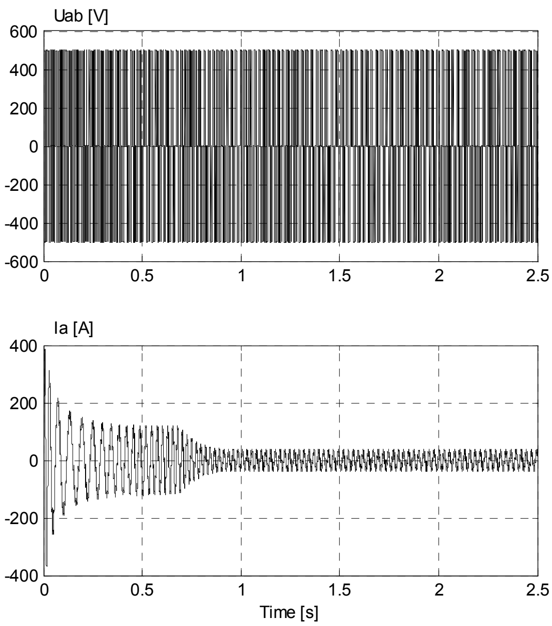

Fig. 2. Time characteristics of the phase-to-phase voltage and the phase current (T_o=0 Nm)

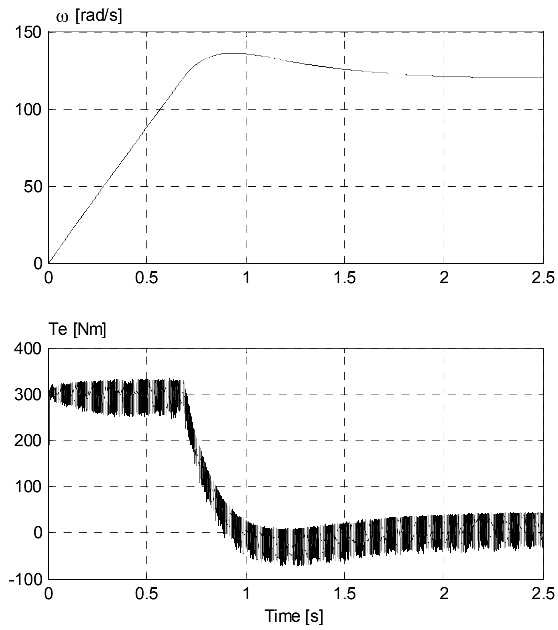

Fig. 3. Time characteristics of the angular speed and the electromechanical torque (T_o=0 Nm)

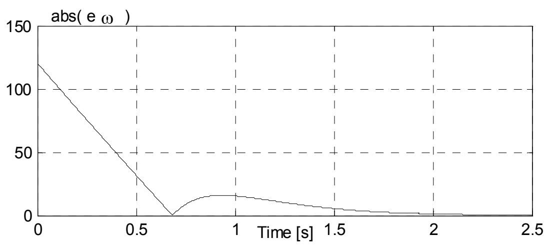

Fig. 4. Time characteristics of the angular speed error (T_o=0 Nm)

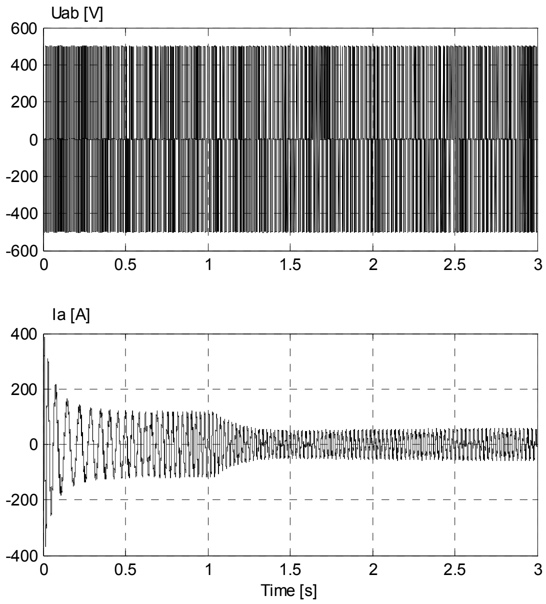

Fig. 5. Time characteristics of the phase-to-phase voltage and phase current (T_o=100 Nm)

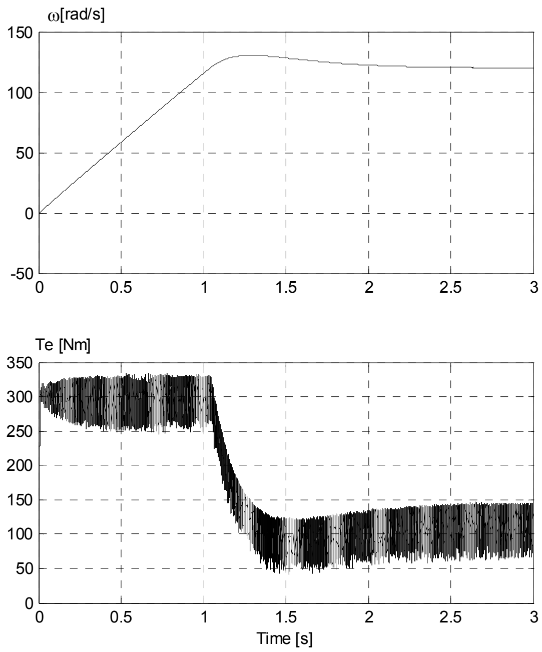

Fig. 6. Time characteristics of the angular speed and the electromechanical torque (T_o=100 Nm)

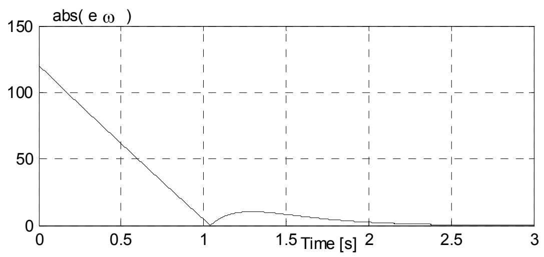

Fig. 7. Time characteristics of the angular speed error (T_o=100 Nm)

To verify the results obtained, a system with a conventional controller has also been tested for identical prescribed parameters, and the comparison of the quality indicators can be seen in Table 2.

Table 2. Quality indicators for the drive model with the fuzzy speed control system and with the conventional speed control system.

Parameters	Fuzzy speed control system		Conventional speed control system	
Load torque T_o *[Nm]*	Setting time t_r [s]	Integral quality indicator I_Ω	Setting time t_r [s]	Integral quality indicator I_Ω
0	2,14	94,6	2,21	99,2
20	2,26	102,5	2,36	108,8
40	2,31	110,9	2,45	117,3
60	2,45	119,5	2,59	125,7
80	2,55	128,3	2,71	135,6
100	2,68	136,1	2,96	143,1

Conclusions

The time characteristics of angular speed, phase current, and electromechanical torque, as well as quality indicators of angular speed confirm the usefulness and validity of the sequential quadratic programming method in the tuning procedure for the fuzzy controller. The method enables creating a knowledge base and determining the mutual position of input and output membership functions of the linguistic variables in the Mamdani controller used in the drive control system of an inductive motor.

The control space of the designed fuzzy logic controllers consists of a number of segments which are not, in the general case, planes. The segments are multilinear spaces which can assume various locations and various degrees of convexity, which makes it possible to shape the time characteristics of the drive system angular speed.

The comparative analysis of the drive system with the fuzzy controller and the one with the conventional controller demonstrates that control time is shorter for the fuzzy controller, especially with higher load torques. Integral quality indicators are also smaller for the fuzzy controller, however the correlation with load torque values is significantly smaller than that for the control time.

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