Dielectrophoresis, arising from spatially nonuniform electric fields, has become one of the more promising tools for particle manipulation in particles on or between special electrodes—among others electric field to create electric forces that will change induced multipole with a nonuniform steady or alternating electric field. Dielectrophoretic (DEP) traps use the force acting on an electrorotational manipulation of living and death cells, and of particles techniques, dielectrophoretic and imposition of electric fields [3], electrically driven separation enhancement of heat and mass transfer in emulsions by the acting on droplets exiting electrospray nozzles, the engineering applications [3] are the determination of forces placed in electric fields. Among different the chemical particles and fluid globules immersed fluid suspension and There are many reasons for studying a behavior of separations and biological analyses. During the past years dielectrophoresis has proved to be of very important in many applications such as, for example, virus, and bacteria. With the appropriate electrode geometry design and careful control of the potentials conditions, single particle trapping can be attained [4].

The Finite Element Method (FEM) is useful method for analyzing electromagnetic fields in devices, because these can model complicated geometries and non-linear electric properties with relatively short computing time. In spite of these advantages, in many papers have been proved that obtaining an accurate force or torque from FEM computation can be inaccurate, particularly when geometry is enough complex, such as in the case of dielectrophoretic traps with multiple particles [5]. Unfortunately, force and torque calculations are influenced by the approximate nature of the discretisation used in FEM meshes. Analytical calculation of forces acting on particle is compared with numerical values computed with finite element method.

In practical applications of dielectrophoresis, inhomogeneous electric filed with sufficient gradient is obtained throughout adequate configuration of electrode geometry of the channel, where particles in dielectric fluid are moving and by property little electrode dimensions. Important role plays here sharp edges of electrodes, where in vicinity of them, as it is commonly known, electric field attains high values and with high gradients. In order to orientate about size of errors, which introduce equivalent dipole method and numerical method used to solve adequate Laplace’s equation on size of occurring here errors, it is necessary to know exact solution of the electromagnetic field and its gradients in computational domain [6]. When geometrical shapes of electrodes and fluid flow channel have complicated shapes, it is very difficult to solve analytically Laplace’s equation together with, frequently, complicated boundary conditions [7]. Inhomogeneous field with arbitrary shape, can be relatively easy obtained in inhomogeneous medium, with very simple geometrical shape of computational domain.

Let us assume that geometrical form of channel has a cuboid shape with following dimensions: \( L \) in direction of \( x \)-axis, \( h \) in direction of \( y \)-axes and \( L_P \) in \( z \)-axis. Two dimensional cross section of the channel with dielectric

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**Abstract**: Dielectrophoresis is the translational motion of neutral matter caused by polarization effects in a non-uniform electric field. Dielectrophoresis, arising from spatially nonuniform electric fields, has become one of the more promising tools for particle manipulation in microfluidics and nanofluidics. Because numerical accuracy of computations of force acting on particle is of great importance, the author has been investigating in this paper numerical error analysis in two-dimensional DC dielectrophoresis. Nonuniform field with high gradient has been obtained by assuming nonlinear permittivity what allowed analytical calculation of potential and electric strength and its gradient distribution. Analytical calculation of forces acting on particle is compared with numerical values computed with finite element method.

**Keywords**: dielectrophoresis, dipole force calculations, error calculation

**Słowa kluczowe**: dielektroforeza, siły działających na dipol, obliczanie błędów.
fluid, but without a particle, is presented in Fig. 1. On the Fig.9 we have the same medium, but with a particle with the same permittivity coefficient, as dielectric fluid. From physical point of view, both cases are entirely equivalent, but from point of view finite element method, which will be used to solve Laplace's equation, they are not equivalent, because on the fluid-particle border electric field continuity conditions have to be fulfilled, what as we shall see, will cause arising substantial errors by calculation forces acting on particle.

Now we assume following computational data: segment AB = CD = \( h = 40 \mu m \), BD = AC = \( L = 100 \mu m \), length \( L_p = 10 \mu m \), the particle radius \( r_p = 3 \mu m \) and is placed in point \( x_p = 40 \mu m \) and \( y_p = 20 \mu m \). On side AB there are zero boundary conditions and on side CD potential is constant and is equal \( V_z = 10V \). On segments BD and AC normal derivative of the potential \( V \) perpendicular to boundary amounts zero. It was assumed that relative permittivity of the particle \( \varepsilon_2 = 1.5 \). On the Fig.1 two segments KL and MN are distinguished along which we shall observe analyzed values. S permittivity following function was assumed:

\[
\varepsilon(x) = \varepsilon_a \varepsilon_0 = \left( a + b e^{cx} \right) \varepsilon_0
\]

where \( a \) and \( c \) are parameters describing permittivity shape, \( x \) is coordinate in rectangular coordinate system, which origin is placed in point B. One has to notice that relative dielectric permittivity depends only on coordinate \( x \). Calculations will be made for \( a = 2 \), \( b = 10 \) and for two parameter values \( c = -0.1/[m] \) \( c = -1.0/[m] \), as in Fig. 2. Let the exact value of potential be \( V_x \). This potential, because of symmetry, depends only on \( x \)-coordinate. The field is described by following Laplace's equation [8]:

\[
\frac{d}{dx} \left( \varepsilon_a(x) \varepsilon_0 \frac{d}{dx} V_a(x) \right) = 0
\]

After integration we obtain formula for analytical calculation of potential function in computational domain. Knowing nonlinear function of electric permittivity allow us calculate analytical form of solution.

\[
V_a(x) = \frac{C_1}{\varepsilon_a(x)} dx + C_2
\]

where \( C_1 \) and \( C_2 \) are integrating constants. One should notice that \( \varepsilon_0 \) in the above equation reduces. After taking into account equation (1) we get

\[
V_a(x) = \frac{C_1}{ac} \left( cx - \log \left( a + b e^{cx} \right) \right) + C_2
\]

From boundary conditions for \( x = 0 \) and \( x = L \) we have

\[
\begin{align*}
\varepsilon(x) &= \varepsilon_a \varepsilon_0 = \left( a + b e^{cx} \right) \varepsilon_0 \quad \text{at} \quad x = 0 \\
\varepsilon(x) &= \varepsilon_a \varepsilon_0 = \left( a + b e^{cx} \right) \varepsilon_0 \quad \text{at} \quad x = L
\end{align*}
\]

Finally, exact value of the force acting on particle calculated by equivalent dipole method is given by formula [9]:

\[
F_a = \varepsilon_a \pi \varepsilon_0^2 L_a k_{CM} \left( \varepsilon_a, \varepsilon_0 \right) V \left( E_a^2 \right)
\]

And introduction of equation (10) yields final form of analytical force, which depends on parameters of nonlinear dependence of dielectric permittivity from spatial coordinates. Minus sign means that force is acting from right to left side.

\[
\begin{align*}
F_a &= -2\varepsilon_a \pi \varepsilon_0^2 L_a k_{CM} \left( \varepsilon_a, \varepsilon_0 \right) \left( \frac{C_1^2 b c e^{cx}}{\left( a + b e^{cx} \right)^3} \right) a_s
\end{align*}
\]
relative error resulting from numerical solution of the
Laplace’s equation was defined as [10]

\[
\text{error} = \frac{V(x) - V_0(x)}{V_0(x)} \times 100\%
\]

(13)

Fig.3. Relative error of potential \(V(x)\) for parameter \(c = -0.1\)

Because for \(c = -0.1\) dependence of permittivity from coordinate \(x\) is almost linear (Fig.2), then potential in great extend is homogeneous. Relative error calculation of potential is less than \(20 \cdot 10^{-6}\%\) (Fig.3). Relative error of electric field strength is also small and is the same order (Fig.4). Force acting on particle is proportional to derivative of electric field and its error is less than \(30 \cdot 10^{-3}\%\).

Fig.4. Relative error of potential \(E_0(x)\) for parameter \(c = -0.1\)

For \(c = -10\) the shape of the permittivity relative \(x\) is in great extend nonlinear, what causes substantially greater error in computations of \(E(x)\)

The second difference on (Fig.5) potential results in substantially greater error in computations force \(F(x)\) (Fig. 7) to value about 3%.

Fig.5. Relative error of potential \(E(x)\) for parameter \(c = -10\)

This is caused mainly by two factors. First through increasing of field inhomogeneity and what is as a result, increasing electric field gradient, and secondly, by using Lagrange’s polynomial to approximate unknown potential in every finite element. As it is well known these potentials do not assure potential continuation on elements boundaries. Because force is proportional to second derivative of potential, error in computation of these derivatives causes arising great errors in calculation of force \(F\). Application of Hermite’s lub Argyris’s polynomials, which guaranty continuity of derivatives on element boundaries can help to solve this problem.

Comparisons of relative error computation for two values of parameter \(c\) show (Fig. 6 and Fig.7) that for highly inhomogeneous electric field computational error is substantial.

Fig.6. Relative error of potential \(F(x)\) for parameter \(c = -0.1\)

Fig.7. Relative error of potential \(F(x)\) for parameter \(c = -10\)

Fig.8. Force \(F(x)\) for two values of parameter \(c\)
On figure 8 dependence of the force $F(x)$ from coordinate $x$ is shown. For highly inhomogeneous case this dependence is highly nonlinear especially at beginning of the coordinate's origin.

Let us now examine which influence on computational accuracy has placement of particle with the same permittivity as surrounding dielectric (see figure 9).

Differentiation of potential causes yet greater error in electric field (see Fig. 11) and subsequently yet greater error in computation of force $F(x)$. The value of the error attains level 7%. This should be taken into account in computation forces by numerical methods.

Fig.9. Particle placed in symmetry axis in inhomogeneous medium

From mathematical point of view both cases as in Fig 1 and 9 are completely equivalent, so analytical solution is in both cases identical. But in the case of numerical solutions with boundary element method, additional errors arise due to fulfillment of continuity conditions of electric field on particle-dielectric boundary.

Particle is placed at coordinate $x_p = 40 \mu m$. On figure 10 relative error of potential in function coordinate $x$ is depicted. One can see hat on both sides of the coordinate $x_p = 40 \mu m$ we have increasing error. Also at beginning of $x$-axis the error is substantial.

![Diagram](image)

**Fig.10. Potential $V(x)$ for parameter value $c = -1$**

Conclusions

In this article, cylindrical particle in uniform electric field perpendicular to the particle was considered. Error calculation of numerical method of force computation is presented. I was shown which errors and where are cause by finite element method used in solution Laplace's equation.

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