Selection of linear filter elements parameters for measuring of voltage and currents components of direct and inverse order

Abstract. In the paper is presented the review of control functions of linear filters of symmetric components, in the systems for measuring of symmetric components of voltages and currents in electric networks, composed of impedances with passive R, L, C parameters.

The characteristics of process and procedure for determination of parameters of filters elements based on mathematical model of three-phase filter equivalent circuit have been emphasized. The results on behaviour of measuring system with linear filter behaviour obtained in the analysis with application of MATLAB simulation have also been presented.

Streszczenie. W artykule przedstawiono przegląd filtrów linioowych składowych symetrycznych, w zastosowaniu do systemów pomiarowych tych składowych prądu i napięcia sieci elektroenergetycznej, zbudowanych z elementów RLC. Opisano wynik działania pomiarów, wykonanych przez podany system, modelu symulacyjnym w programie Matlab. (Dobór parametrów filtru linioowego do pomiaru składowej zgodnej i przeciwnej napięcia i prądu w sieci elektroenergetycznej).

Keywords: measuring, parameter, filter, symmetric components.

Słowa kluczowe: pomiary, parametry, filtr, składowe symetryczne

Introduction

Economic parameters and reliability of power networks exploitation depend on quality parameters of electric energy, and one of the most important and most influential quality parameters which influences the efficiency of three-phase power network is symmetry of network voltage.

In secondary circuits of power networks (measuring, protection and control) filters with two, three of more branches have been used. In determination of values and adjustment of filter parameters, filters have been mainly treated as independent electric circuits, because the values of source impedances are considered lesser in regard to impedances of filter elements [1, 2, 3, 4].

With this assumption, the calculations in this paper have been also performed – it is considered that parameters of filter electric branches elements, connected to appropriate linear voltage values, are independent.

Possible asymmetries mainly are determined, measured or calculated by approximate methods [1, 3, 5, 6]. In the case of measuring, the current or voltage filters, which phase values of voltages/currents convert to symmetric values of components of voltages/currents, are used. All approximate measuring methods include high measuring uncertainties and the biggest disturbance appears in the case of filters failure, which is usually derived as three-phase circuit.

The most used three-phase filter of symmetric components of direct and inverse order is composed of three electric branches and has been presented on Fig. 1 [1, 2].

![Fig.1. Three-phase filter of symmetric components of direct and inverse order](image)

Disintegration of filter leads to impedance value discrepancy from nominal values. This can occur because errors during the manufacturing of filter elements as well as because of changes of impedances values due to changes of frequency, voltage or temperature.

Frequency change leads to changes of reactance value, since inductive reactance is directly and capacity reactance is inversely proportional to frequency.

Voltage discrepancies from nominal values cause changes of coils inductivity with ferromagnetic sheet metal in magnetic circuit. Temperature changes entail changes of active resistances and capacitances [7, 8, 9].

Determination of filter parameters values

Value changes of filter elements impedances are caused by discrepancy of parameters value R, f, L and C from nominal values. Inequality of impedances values and nominal values affects active as well as reactive part of filter impedance. Changed impedance value can be mathematically defined as a sum of nominal Zn value and impedance change ΔZ, with expression:

\[ Z_h = Z_n + \Delta Z \]

where: nominal filter impedance Zn is a sum of active and reactive part, and change of impedance value is included by introduction of coefficient h:

\[ Z_n = R_n + jX_n, \quad \Delta Z = R_n \cdot h_R + jX_n \cdot h_X \]

Coefficients h_R and h_X of change of active and reactive impedance part are determined for each filter respectively, depending on its construction.

Frequency changes the most often, and with the assumption that it changes for value Δf in regard to nominal value (f_n), \( f = f_n + \Delta f \) the value of complex impedance of ohm-inductive (RL) character also changes:

\[ Z_h = R_n + j2\pi f_n + \Delta f \cdot L = Z_n + jX_n \cdot h_X \]

In the case of frequency change, the value of coefficient h, for R,L circuit is:

\[ h_{RL} = \frac{\Delta f}{f_n} \]

Value of impedance of ohm-capacitive RC character is:

\[ Z_h = R_n - \frac{1}{2\pi f_n + \Delta f} \cdot C = Z_n + jX_n \cdot h_{RC} \]

In the case of frequency change in the circuit RC the value of coefficient h is:

\[ h_{RC} = \frac{\Delta f}{f_n + \Delta f} \]

In changes of values of inductivity for ΔL regarding the nominal value, the following expression has been used (3).
\[ h_x = \frac{AL}{L_n}, \] and in discrepancy of capacitance values
\[ \Delta C \text{ from nominal value, expression (5) } \Rightarrow h_x = \frac{\Delta C}{C_n + \Delta C}. \]

When the value of active component changes for \( \Delta R \) regarding the given value
\[ Z_n = R_n + \Delta R + jX_n = Z_n + R_n, h_x R_n, \Rightarrow h_x R_n = \frac{\Delta R}{R_n}. \]

three-phase circuit of arbitrary selected filter can be equivalent to single-phase scheme of quadrupole which equation is [1]:
\[ (7) \quad U_1 = Z_{11} I_1 + Z_{12} I_2 , \quad U_2 = Z_{21} I_1 + Z_{22} I_2 \]

Expressions that connect parameters of single-phase equivalent scheme \( Z_{11} = Z_{22} \) to values of impedances \( Z_1 \) and \( Z_2 \) on Fig. 1, are listed hereafter [1].

In feeding of the filter, which scheme is presented on Fig. 1, from the direct order voltage system, equations that connect values of those impedances are:
\[ (8) \quad Z_{11}^F = Z_{22}^F = \frac{Z_1 + Z_2}{3}, \quad Z_{12}^F = \frac{Z_1 + aZ_2}{3} \]
\[ Z_{21}^F = \frac{Z_2 + aZ_1}{3} \]

In the case of filter feeding, with scheme on Fig. 1, from the inverse order voltage system, the following is obtained:
\[ (9) \quad Z_{11}^F = Z_{22}^F = \frac{Z_1 + Z_2}{3}, \quad Z_{12}^F = \frac{Z_2 + aZ_1}{3} \]
\[ Z_{21}^F = \frac{Z_2 + aZ_1}{3} \]

Complex operators \( a \) and \( a^2 \) have values \( a = e^{i\frac{120}{3}} \)

Then, for filters fed from the direct order voltage system by expressions (1) and (8) at symmetric discrepancy of impedance values from nominal, the following relations are valid:
\[ (10) \quad Z_{11h}^F = Z_{22h}^F = \frac{Z_{1h} + Z_{2h}}{3} = \frac{Z_{1n} + a\Delta Z_1}{3} + \frac{\Delta Z_2}{3} \]
\[ Z_{12h}^F = \frac{Z_{2h} + aZ_{1h}}{3} = \frac{Z_{12n} + \Delta Z_2}{3} + \frac{a\Delta Z_1}{3} \]
\[ Z_{21h}^F = \frac{Z_{2h} + aZ_{1h}}{3} = \frac{Z_{21n} + \Delta Z_2}{3} + \frac{a\Delta Z_1}{3} \]

For filters fed from the inverse order voltage system from (1) and (9) is:
\[ (13) \quad Z_{11h}^F = Z_{22h}^F = \frac{Z_{1h} + Z_{2h}}{3} = \frac{Z_{1n} + \Delta Z_1}{3} + \frac{\Delta Z_2}{3} \]
\[ Z_{12h}^F = \frac{Z_{2h} + aZ_{1h}}{3} = \frac{Z_{12n} + \Delta Z_1}{3} + \frac{a\Delta Z_2}{3} \]
\[ Z_{21h}^F = \frac{Z_{2h} + aZ_{1h}}{3} = \frac{Z_{21n} + \Delta Z_1}{3} + \frac{a\Delta Z_2}{3} \]

and values \( \Delta Z_1 \) and \( \Delta Z_2 \) are determined by relation (2).

Equivalent \( T \) schemes can be used as electronic circuits of linear filters symmetric components of direct and inverse order. Equation (7) corresponds to two equivalent \( T \) schemes on Fig. 2.a and Fig. 2.b.

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**Fig. 2. Equivalent schemes of symmetric components filter with passive elements**

In feeding from the direct order voltage system, according to Fig. 2a, from expressions (10) – (12) is obtained:
\[ (16) \quad Z_{11h}^F = Z_{22h}^F = Z_{11h}^F = Z_{12h}^F = \frac{Z_{1n} + \Delta Z_1}{3} \]
\[ Z_{12h}^F = \frac{Z_{2h} + aZ_{1h}}{3} = \frac{\Delta Z_2}{3} + \frac{a\Delta Z_1}{a} \]

If scheme is fed from the inverse order voltage system, according to scheme on Fig. 2a, from expressions (13) – (15) is obtained:
\[ (19) \quad Z_{11h}^F = Z_{22h}^F = Z_{11h}^F = Z_{12h}^F = \frac{Z_{1n} + \Delta Z_1}{3} \]
\[ Z_{12h}^F = \frac{Z_{2h} + aZ_{1h}}{3} = \frac{\Delta Z_2}{3} + \frac{a\Delta Z_1}{a} \]
\[ Z_{21h}^F = \frac{Z_{2h} + aZ_{1h}}{3} = \frac{Z_{21n} + \Delta Z_1}{3} \]

With analogous procedure for scheme on Fig. 2.b can also be obtained expressions similar to group of expressions (16) – (21).

Characteristics of filters symmetric components of direct and inverse order depend on filter type, that is, on the fact whether the filter is of direct \( FDR \) or inverse order \( FIR \), and also on the fact whether scheme is fed from the voltage system or direct order \( U^d \) or voltage system of inverse order \( U^i \) and, finally, on whether the values are \( Z_{12n} = 0 \) or \( Z_{21n} = 0 \).

In order to have final expression with simpler form, in the case of \( Z_{21n} = 0 \), the parameters of equivalent scheme on Fig. 2.a can be used, and when \( Z_{21n} = 0 \) the parameters of equivalent scheme on Fig. 2.b should be used.

Values determined according to this procedure for the filter fed from the inlet side are presented in Table 1. Similar expressions can be obtained if filter is fed from the discharge side.

If it possible for any filter impedance to determine discrepancies from nominal values, the real question is whether by regulating the parameters of other elements in a different way is possible to achieve better accuracy, that is, is the filter adjustment possible.

Better accuracy can be simply achieved if the condition that \( Z_{3h}^F = 0 \) is introduced (which can be seen of Fig. 2).

For direct order filter than is obtained:
\[ (22) \quad Z_{3h}^F = \frac{\Delta Z_2}{3} + \frac{a\Delta Z_1}{3} = 0 \]
Table 1. Characteristic values of filter parameters of equivalent quadripole when fed from the inlet side

<table>
<thead>
<tr>
<th>Filter Type</th>
<th>Order of phases of feeding voltage</th>
<th>Equivalent scheme on Fig. 2</th>
<th>$Z_{1h}^F$</th>
<th>$Z_{3h}^F$</th>
<th>$Z_{vh}^F$</th>
</tr>
</thead>
<tbody>
<tr>
<td>FDR</td>
<td>$U^d$</td>
<td>$a$</td>
<td>$Z_{1h}^F = Z_{3h}^F$</td>
<td>$Z_{3h}^F$</td>
<td>$Z_{vh}^F$</td>
</tr>
<tr>
<td></td>
<td>$U^i$</td>
<td>$a$</td>
<td>$Z_{1h}^F + \frac{\Delta Z_1(1-a)}{3}$</td>
<td>$\frac{\Delta Z_2 + a\Delta Z_1}{3}$</td>
<td>$\frac{\Delta Z_1(a^2 - a)}{3}$</td>
</tr>
<tr>
<td>FIR</td>
<td>$U^d$</td>
<td>$a$</td>
<td>$Z_{1h}^F + \frac{\Delta Z_1(1-a)}{3}$</td>
<td>$\frac{\Delta Z_2 + a^2\Delta Z_1}{3}$</td>
<td>$\frac{\Delta Z_1(a^2 - a)}{3}$</td>
</tr>
<tr>
<td></td>
<td>$U^i$</td>
<td>$b$</td>
<td>$Z_{1h}^F + \frac{\Delta Z_1(1-a^2)}{3}$</td>
<td>$\frac{\Delta Z_2 + a^2\Delta Z_1}{3}$</td>
<td>$\frac{\Delta Z_1(a-a^2)}{3}$</td>
</tr>
</tbody>
</table>

where: $\Delta Z_1 = R_{n1}hR_{R1} + jX_{n1}hX_{R1}$, $\Delta Z_2 = R_{n2}hR_{R2} + jX_{n2}hX_{R2}$

And for inverse order filter:

\[
Z_{3h}^F = \frac{\Delta Z_2}{3} + \frac{a^2\Delta Z_1}{3} = 0
\]

Left-hand parts of expressions (22) and (23) contain complex values, so each of those equations divides into two equations. In one of them is real value equal to zero, and in other imaginary.

If expressions (2) are considered, for direct order filter the condition for accuracy of filter adjustment is obtained in the form:

\[
R_{n2}hR_{R2} - \frac{R_{n1}hR_{R1}}{2} - \frac{\sqrt{3}X_{n1}hX_{R1}}{2} = 0
\]

and for inverse order filter (FIR):

\[
R_{n2}hR_{R2} - \frac{R_{n1}hR_{R1}}{2} + \frac{\sqrt{3}X_{n1}hX_{R1}}{2} = 0
\]

Analysis of performance of filter linear circuit with RLC parameters

Simulation scheme of measurement of voltage and current symmetric components and influence of changes of linear filter impedances due to frequency changes has been obtained from the equation (2):

\[
\left[\hat{Z}(s)\right] = \left[\frac{V(s)}{I(s)}\right]
\]

Filter impedance in Laplacian domain is:

\[
Z(s) = \frac{V(s)}{I(s)} = \frac{LCs^2 + Rs + 1}{Cs}
\]

If from the scheme on Fig. 3 resistance, inductivity or capacitance is excluded, values must be alternative 0, 0, infinity (inf). Diagram for $Z$ and argument $Z$ (for ex.) on Fig. 4 relates to measuring values of voltage and current of elementary test for determination of $Z$ for frequency $f = 300\text{Hz}$. For obtaining of frequent characteristic of impedance it is necessary to establish ($A \ B \ C \ D$ matrix) a space model of system condition. At the input is one voltage source and at the output measuring current block system. Function $Z(s)$ can be transformed form the matrix of space condition and Bode function as follows:

\[
[A,B,C,D] = \text{power2sys}('\text{psbseriesbranch}')
\]

\[
\text{freq} = \text{logspace}(1,4,500);
\]

\[
\text{w} = 2 \ast \text{freq};
\]

\[
\text{Ymag}, \text{Yphase}
\]

\[
\text{lab('Impedance Zmag')}
\]

\[
\text{subplot}(2,1,1)
\]

\[
\text{loglog(freq,Zphase)}
\]

\[
\text{grid}
\]

\[
\text{title('1,3,5 th harmonic filter')}
\]

\[
\text{xlabel('Frequency, Hz')}
\]

\[
\text{ylabel('Impedance Zmag')}
\]

\[
\text{subplot}(2,1,2)
\]

\[
\text{semilogx(freq,Zphase)}
\]

\[
\text{xlabel('Frequency, Hz')}
\]

\[
\text{ylabel('phase Z')}
\]

\[
\text{grid}
\]

Fig.3. Measuring scheme with voltage source whose parameters are: ideal sinusoidal AC voltage, amplitude 100 V; phase position (0); frequency (Hz): 10-104; initial time: 0 source impedance $Z_{source}$ parallel link $RIL$, $R(\text{IL}) = 10$; $L(\text{IL}) = 10e-03$, RLC – parameters filter: resistance $R(\text{IL}) = 1$; inductivity $L(\text{IL}) = 1e-03$, capacitance $C(\text{IL}) = 1e-06$
Next, the block impedance $Z(f)$ can be measured, and
drawn diagram of impedance as the frequency function.
Requirement of impedance measurement is to disconnect
current source from the measuring scheme, Fig. 3. In
addition are presented two diagrams: first with the values of
impedance module: $|\hat{Z}(f)|$ and second with phase values,
that is, $\arg \hat{Z}(f)$ impedance.

**Example:** Source parameters: ideal $AC$ voltage source,
amplitude 100 V, phase position (deg): 0; elementary
frequencies (Hz):50; initial time 0; frequency adjustment $f =
(10 – 10000) \text{ Hz}$; source impedance – parallel link $RLL$,
$R(f)=10$; $L(f)=10e^{-03}$.

Standard measuring impedance: $Z(f)$ and multiplication
factor 1. Sample parameters of linear filter $RLC$ [resistance
$R(f)=1$; inductivity $(H)=100e^{-03}$; capacitance $(F)=10e^{-06}$]
and variants: a) full $RLC$, b) serial $RL$, c) serial $RC$.

a) Measurement of basic harmonic of voltage and
current with application of linear filter with $RLC$ impedance
$[R(f)=1]$; $L(f)=100e^{-03}$; $C(f)=10e^{-06}$, scheme and diagrams: voltage ordinate $\pm200V$, current ordinate $\pm10A$, $f=50Hz$.

Measuring scheme on Fig. 3, by which the following
diagrams are obtained, has been used.

**Conclusion**
As it can b seen from equations (24) and (25) for
adjustment of filter accuracy there must be two elements,
and appropriate filter adjustment is possible only in the case
if obtained discrepancies form given values are not present
in more than two or four independent filter parameters.

From obtained diagrams in tests, on Fig. 4-7, it can be
seen that basic current harmonic in measurement follows
b) Measurement of basic harmonic of voltage and
current by linear filter with serial $RL$ impedance $[R(f)=1;
L(f)=100e^{-03}]$, scheme and diagrams: voltage ordinate
$\pm200V$, current ordinate $\pm10A$, $f=50Hz$.
Reduced scheme on Fig. 3 with excluded capacitance
$C$ [3, 7] is used.

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basic voltage harmonic. Diagrams on Fig. 4.a and 4.b confirm that values of module and argument $Z$ depend on developed harmonics and that this is necessary to be taken into consideration in projection of filter through constructive correction of elements parameters values given in Table 1.

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