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Research on Interval Concept Lattice and its Construction Algorithm

***Abstract**. In classic concept lattice and rough concept lattice, the concept extents have all the attributes or only one attribute sometimes. So the support and confidence degree of the extracted association rules would be reduced greatly. To solve this problem, authors have put forward a new concept lattice structure: interval concept lattice $L_\beta^\alpha(M^\alpha, M^\beta, Y)$ based on the parameter interval $[\alpha,\beta]$ ($0 \le \alpha \le \beta \le 1$). The concept extent is an object sets which meet the properties in the intent in the interval $[\alpha,\beta]$ $0 \le \alpha \le \beta \le 1$. It has been proved that interval concept lattice degenerate into classic concept lattice when $\alpha = \beta = 1$, and when $\beta = 1, \alpha > 0$, interval concept lattice degenerate into rough concept lattice. Then some unique properties of interval concept lattice have been proved. The construction algorithm of interval concept lattice was designed. Finally, the necessity and practicability were verified through a case study.*

Streszczenie:** W klasycznej i przybliżonej kracie pojęć ich obszar obejmuje każdy lub czasami tylko jeden atrybut. A więc podstawa i stopień poufności wydobywanych relacji mogą zostać poważnie zredukowane. Aby rozwiązać ten problem autorzy proponują nową strukturę kraty pojęć: przedziałową kratę pojęć $L^\alpha{}_\beta$ (M^α, M^β, Y) zdefiniowaną w przedziale [α,β] (0 ≤ α ≤ β ≤ 1). Udowodniono, że przedziałowa krata pojęć przekształca się w klasyczną jeśli (α = β = 1) i w przybliżoną gdy (α > 0, β = 1). Zbadano unikalne własności kraty przedziałowej i zaprojektowano algorytm jej budowy. W końcu zweryfikowano , w przypadku studialnym, potrzebę jej wprowadzenia i możliwość wykonania. **Badania przedziałowej kraty pojęć i algorytm jej budowy

Keywords: Concept Lattice, Rough Concept Lattice, Interval Concept Lattice, Construction Algorithm of Lattice.
Słowa kluczowe: Krata pojęć, Przybliżona krata pojęć, Przedziałowa krata pojęć, Algorytm budowy kraty

Introduction

Concept lattice[1] is a powerful tool for data analysis which proposed by Professor Wille R in 1982. Each node in concept lattice is in a form of formal concept which contain the intent (concept description) and the extent (objects covered by intent). The construction process of concept lattice from the form context is a concept clustering process in fact. The concept lattice vividly embodies the generalizations and specialized relationships between the concepts through the Hasse diagram. Concept lattice, as a kind of data analysis tool, has the completeness and accuracy, and has been widely applied in information retrieval, digital library, knowledge discovery, and so on[2-4]. At present, the domestic and overseas scholars have carried various researches on concept lattice that mainly include construction algorithm and improvement[5-7], rule mining based on concept lattice[8-9], and the fusion of other theories such as fuzzy theory, predicate logic, and rough set theory.

Some expand concept lattices have obtained, such as, fuzzy concept lattice, weighted concept lattice, constraint concept lattice, quantitative concept lattice, expansion concept lattice, rough concept lattice etc[10-12].

In particular, in classic concept lattice, fuzzy concept lattice and weighted concept lattice, the extent contains the objects which meet all the attributes in the intent. To find the concepts which have partial attributes, we must scan the concept lattice and combine the concepts. The time cost is so larger especially for large concept lattice. While, in the rough concept lattice[12], although the concepts which have partial attributes can be searched, but there may be a lot of objects which only have an attribute of the intent, thus the support and confidence degree of constructing association rules will be greatly reduced. In practical applications, we often care the object set which have a certain number or percent of attributes in intent. Then, some pertinent association rules will be mined through the correlation analysis. For example, in the supermarket shopping system, the promotional manager often pay more attention to the customers who purchase k (k>1) kinds of goods or more and the potential demand of these customers, and then carry out the product propaganda to get the greatest benefit through the minimum promotion. However, in the existing concept lattice structure, this kind of query can not be operated directly, and some merger connections or filtrations must be performed. The time and space cost price are too higher. For this problem, this paper puts forward a new concept lattice structure-interval concept lattice, and proves that the interval concept lattice is the expand of classic concept lattice and rough concept lattice and the classic concept lattice and rough concept lattice are the special cases of interval concept lattice. Then the construction algorithm of interval concept lattice is given and its effective was proved through comparison.

Basic Concept
Concept Lattice

Definition1 [1] (U, A, R) is a formal context, where $U = \{x_1, x_2, ..., x_n\}$ is the object sets and each $x_i (i \le n)$ calls an object; $A = \{a_1, a_2, ..., a_m\}$ is the attribute set, and each $a_j (j \le m)$ calls an attribute; R is the binary relationship between U and A . $R \subseteq U \times A$. If $(x,a) \in R$, then we said x has the attribute a , and write as xRa .

Definition 2 [1] For the formal context (U,A,R) , operators f,g are defined as follows:

$\forall x \in U$, $f(x) = \{y \mid \forall y \in A, xRy\}$ i.e. f is the mapping between x and its attributes;

$\forall y \in A$, $g(y) = \{x \mid \forall x \in U, xRy\}$ i.e. g is the mapping between y and its objects.

Definition 3 For formal context (U,A,R) , if $f(X) = Y, g(Y) = X$ for $X \subseteq U, Y \subseteq A$, we said the sequence $< X,Y >$ is a formal concept, or concept for short. X is the extent and Y is the intent.

Definition 4 $L(U,A,R)$ refers to all the concepts of the formal context (U,A,R) , and:

(1) $$(X_1,Y_1) \le (X_2,Y_2) \Leftrightarrow X_1 \subseteq X_2 (\Leftrightarrow Y_1 \supseteq Y_2)$$

Then " $\le$ " call the partial order relationship on $L(U,A,R)$.

Definition 5 If all the concepts in $L(U,A,R)$ meet the relationship " $\le$ ", then we call $L(U,A,R)$ as the concept lattice of (U,A,R) .

Definition 6 If $(X_1,Y_1) \le (X_2,Y_2)$, and there is no concept (X_3,Y_3) meet:

(2) $$(X_1,Y_1) \le (X_3,Y_3) \le (X_2,Y_2)$$

Then (X_1,Y_1) is the child concept of (X_2,Y_2) , and (X_2,Y_2) is the father concept of (X_1,Y_1) .

Rough Concept Lattice

In rough concept lattice[12], the upper approximation extent and lower approximation extent refer to the maximal concept set and the minimal concept set respectively which have all the attributes in $Y \subseteq U$. The merge operations are avoided when we query the concept of partial attributes.

Definition 7[12] For the formal context (U,A,R) , $L(U,A,R)$ the classic concept lattice, and $< X,Y >$ is the classical concept on L . The upper approximation extent is defined as:

$$M = \{x \mid x \in U, f(x) \cap Y \ne \phi\}$$

Y is the intent of the concept, and $f(x)$ is the operator in Definition 2. M refers to the object set which might be covered with the intent. .

Definition 8[12] For the formal context (U,A,R) , $L(U,A,R)$ the classic concept lattice, and $< X,Y >$ is the classical concept on L . The lower approximation extent is defined as:

$$N = \{x \mid x \in X, f(x) \cap Y = Y\}$$

X is the extent of classic concept, and Y is the intent of the concept. $f(x)$ is the operator in Definition 2. N refers to the object set which have all the attribute in the intent. .

Definition 9[12] Suppose a formal context (U,A,R) , the triads sequence (M,N,Y) is any node of the lattice structure generated from the partial order relationship R . Y is the intent. M is the upper approximation extent. N is the upper approximation extent. If the concept lattice meets:

(3) $$M = \{x \mid x \in U, \exists y \in A, xRy\}$$

(4) $$N = \{x \mid x \in U, \forall y \in A, xRy\}$$

(5) $$Y = \{y \mid y \in A, \forall x \in M, \exists y \in A, xRy\}$$

Then we call L as rough concept lattice generated by (U,A,R) (RL for short). Each node (M,N,Y) is a rough concept.

Definition 10[12] Suppose $H_1 = (M_1,N_1,Y_1)$ and $H_2 = (M_2,N_2,Y_2)$ are two different nodes in rough concept lattice, and $H_1 \le H_2 \Leftrightarrow Y_2 \subseteq Y_1$. If there doesn't exist $H_3(M_3,N_3,Y_3)$ which meets $H_1 \le H_3 \le H_2$, then we call H_2 is the father node (direct precursor) of H_1 , and H_1 is the child node (direct successor) of H_2 .

Theorem 1 (M,N,Y) is a node of rough concept lattice. The rough concept lattice degeneration into classic concept lattice when $M = N$, namely, rough concept lattice is the improvement of classic concept lattice.

The Definition and Properties of Interval Concept Lattice

Definitions

Definition 11 For the formal context (U,A,R) and its rough concept lattice $RL(U,A,R)$, (M,N,Y) is the rough concept. Set an interval $[\alpha,\beta]$ ($0 \le \alpha \le \beta \le 1$), then α upper bound extent M^α is:

(6) $$M^\alpha = \{x \mid x \in M, |f(x) \cap Y| / |Y| \ge \alpha, 0 \le \alpha \le 1\}$$

Y is the intent of concept and $f(x)$ is the operator in the definition 2. $|Y|$ refers to the number of elements in Y , i.e. the Card. M^α refers to the objects which may be covered by $\alpha \times |Y|$ attributes or more in Y .

Definition 12 For the formal context (U,A,R) and its rough concept lattice $RL(U,A,R)$, (M,N,Y) is the rough concept. Set an interval $[\alpha,\beta]$ ($0 \le \alpha \le \beta \le 1$), then β lower bound extent M^β is:

(7) $$M^\beta = \{x \mid x \in M, |f(x) \cap Y| / |Y| \ge \beta, 0 \le \alpha \le \beta \le 1\}$$

X is the extent of rough concept and Y is the intent of concept. $f(x)$ is the operator in the definition 2. M^β refers to the objects which may be covered by $\beta \times |Y|$ attributes or more in Y .

Definition 13 Suppose (U,A,R) is a formal context, and (M^α,M^β,Y) is an interval concept. Y is the intent. M^α is the α upper bound extent and M^β is the β lower bound extent.

Definition 14 $L_\alpha^\beta(U,A,R)$ refers to all the $[\alpha,\beta]$ interval concepts on the context (U,A,R) , then:

(8) $$(M^\alpha{}_1,M^\beta{}_1,Y_1) \le (M^\alpha{}_2,M^\beta{}_2,Y_2) \Leftrightarrow Y_1 \supseteq Y_2$$

Then " $\le$ " call the partial order relationship.

Definition 15 If all the concepts in $L_\alpha^\beta(U,A,R)$ meet " $\le$ ", then $L_\alpha^\beta(U,A,R)$ call interval concept lattice on the formal context (U,A,R) .

Theorem 2 The node (M^α,M^β,Y) of interval concept lattice meets the following properties:

(9) $$M^{\alpha}=\{x\in U \mid \exists Y_1\subseteq Y,|Y_1|/|Y| \geq \alpha(0\leq\alpha\leq 1),\forall y\in Y_1,xRy\}$$

(10) $$M^{\beta}=\{x\in U \mid \exists Y_2\subseteq Y,|Y_2|/|Y| \geq \beta(0\leq\alpha\leq\beta\leq 1),\forall y\in Y_2,xRy\}$$

(11) $$Y=\{y\in A \mid \forall x\in M^{\alpha},\exists Y_3\subseteq A,,|Y_3|/|Y| \geq \alpha(0\leq\alpha\leq 1),\forall y\in Y_3,xRy\}$$

Definition 16 $G_1=(M^{\alpha}{}_1,M^{\beta}{}_1,Y_1)$ and $G_2=(M^{\alpha}{}_2, M_2^{\beta},Y_2)$ are two nodes of interval concept lattice, and $G_1\leq G_2 \Leftrightarrow Y_1\subseteq Y_2$.

If there doesn't exist $G_3(GM_3,GN_3,Y_3)$ which meets $G_1\leq G_3\leq G_2$, then we call G_2 is the father node (direct precursor) of G_1, and G_1 is the child node (direct successor) of G_2.

Definition 17 If a node in $L_{\alpha}^{\beta}(U,A,R)$ is the precursor of all the nodes (except itself), then it call the root node of the interval concept lattice; If a node in $L_{\alpha}^{\beta}(U,A,R)$ is the successor of all the nodes (except itself), then it call the end node of the interval concept lattice.

Properties

Interval concept lattice is the development of rough concept lattice and its applicable scope is more extensive.

Theorem 3 Interval concept lattice is the development of rough concept lattice and rough concept lattice is the special case of interval concept lattice.

Proof:

Suppose a formal context (U,A,R), the interval concept lattice is $L_{\alpha}^{\beta}(U,A,R)$. $G=(M^{\alpha},M^{\beta},Y)$ is an $[\alpha,\beta]$ interval concept. And the rough concept lattice is $RL(U,A,R)$. $H=(M,N,Y)$ is an rough concept.

α : upper bound extent:

$M^{\alpha}=\{x \mid x\in U,|f(x)\cap Y|/|Y|\geq\alpha,0\leq\alpha\leq 1\}$

When $\alpha=0$, and $|f(x)\cap Y|/|Y|>\alpha$, its α upper bound extent: $M^{\alpha}=\{x \mid x\in U,|f(x)\cap Y|/|Y|>0\}$, i.e.

$$M^{\alpha}=\{x \mid x\in U, f(x)\cap Y\neq\phi\}$$

At this time, $M^{\alpha}=M$, that is, α upper bound extent is equal to the upper approximation extent in the rough concept lattice.

β : lower bound extent:

$M^{\beta}=\{x \mid x\in X,|f(x)\cap Y|/|Y|\geq\beta,0\leq\alpha\leq\beta\leq 1\}$

When $\beta=1$, and $|f(x)\cap Y|/|Y|=\beta$, its β lower bound extent: $M^{\beta}=\{x \mid x\in X,|f(x)\cap Y|/|Y|=1\}$, i.e.

$$M^{\beta}=\{x \mid x\in X, f(x)\cap Y=Y\}$$

At this time, $M^{\beta}=N$ that is, β lower bound extent is equal to the lower approximation extent in the rough concept lattice.

DEFINITION 18 When $\beta=1,|f(x)\cap Y|/|Y|=1$; and $\alpha=0$, $|f(x)\cap Y|/|Y|=0$, the interval concept lattice depredate into the classic concept lattice.

Theorem 4 Suppose $(M^{\alpha}{}_1,M^{\beta}{}_1,Y_1)$ is a node of the interval concept lattice,

$M^{\alpha}=\{u_1,u_2,...,u_r\}$, $M^{\beta}=\{v_1,v_2,...,v_s\}$,

$Y=\{y_1,y_2,...,y_t\}=\bigcup_{i=1}^{p}Y_i$,

$P(Y)$ is the power set of Y, $Y_i\in P(Y)$, then

(1) $M^{\alpha}=g(Y_1)\cup g(Y_2)\cup...\cup g(Y_p)$, $|Y_i|/|Y|\geq \alpha$.

For the α upper bound extent M^{α}, the logic relationship among attribute subsets in the intent Y is "Or";

(2) $Y\subseteq f(u_1)\cup f(u_2)\cup...\cup f(u_r)$;

(3) $M^{\beta}=g(Y_1)\cup g(Y_2)\cup...\cup g(Y_q)$, $|Y_i|/|Y|\geq\beta$.

For the β lower bound extent M^{β}, the logic relationship among attribute subsets in the intent Y is "Or";

(4) $Y\subseteq f(v_1)\cup f(v_2)\cup...\cup f(v_s)$

Proof: (1)

$M^{\alpha}=g(Y_1)\cup g(Y_2)\cup...\cup g(Y_p)$, $|Y_i|/|Y|\geq\alpha$,

First, we prove that the later is the subset of the former: $\exists Y_1\subseteq Y$ and $|Y_1|/|Y|\geq\alpha$ which meet $g(Y_1)\not\subset M^{\alpha}$.

Suppose $x\in g(Y_1)$ and $x\notin M^{\alpha}$, we can get $Y_1\subseteq f(x)$.

$\because |Y_1|/|Y|\geq\alpha$

$\therefore |f(x)\cap Y|/|Y|\geq\alpha$.

According to the Definition 11, we get $x\in M^{\alpha}$. This is contradictory to the suppose: $x\notin M^{\alpha}$.

So, For $\forall$ $Y_i\subseteq Y$ and $|Y_i|/|Y|\geq\alpha$, $g(Y_i)\subseteq M^{\alpha}$ and $\bigcup_{i=1}^{p}g(Y_i)\subseteq M^{\alpha}$;

Then, we should prove that the former is the subset of the latter.

According to the theorem3.1, for $\forall x_i\in M^{\alpha}$, $\exists Y_i\subseteq Y$, $|Y_i|/|Y|\geq\alpha,\forall y_i\in Y_i$ meet x_iRy_i, i.e. $x_i\in g(Y_i)$, so $M^{\alpha}=\bigcup_i x_i\subseteq\bigcup_i g(Y_i)$.

(2) $Y\subseteq f(u_1)\cup f(u_2)\cup...\cup f(u_r)$

For $\forall Y_i\subseteq Y$, and $|Y_i|/|Y|\geq\alpha$, we can get that $g(Y_i)\subseteq M^{\alpha}$.

Suppose $x\in g(Y_i)$, $Y_i\subseteq f(x)$. According to the randomness of Y_i and x, $Y\subseteq f(u_1)\cup f(u_2)\cup...\cup f(u_r)$.

(3) $M^{\beta}=g(Y_1)\cup g(Y_2)\cup...\cup g(Y_q)$, $|Y_i|/|Y|\geq\beta$, Slimily to (1).

(4) $Y\subseteq f(v_1)\cup f(v_2)\cup...\cup f(v_s)$. Slimily to (2).

Theorem 5 Suppose Y is series of attribute set, $Y_i\subseteq Y$ and

$|Y_i|/|Y|\geq\alpha$, $Y_j\subseteq Y$ and $|Y_j|/|Y|\geq\beta$.

X is the extent of classic concept lattice which covered by the attribute Y. M,N are respectively the upper approximation extent and lower approximation extent of the rough concept lattice which covered by the attribute Y. M^{α},M^{β} are respectively the α upper bound extent and β lower bound extent of the interval concept lattice which covered by the attribute Y. They meet the following relation: $X\subseteq N\subseteq M^{\beta}\subseteq M^{\alpha}\subseteq M$.

Proof:

(1) Suppose $x \in X$, according to the completeness of classic concept lattice, $f(x) = Y$, i.e. $f(x) \bigcap Y = Y$. According to the definition 8, $x \in N$, so $X \subseteq N$.

(2) Suppose $x \in N$, according to the definition 8, $f(x) \bigcap Y = Y$, i.e.

$|f(x) \bigcap Y| / Y = 1 \geq \beta$ （$1 \leq \beta < 0$）.

According to the definition 12, $x \in GN$, so $N \subseteq M^{\beta}$;

(3) Suppose $x \in GN$, according to the definition 12, $|f(x) \bigcap Y| / Y \geq \beta$，($0 \leq \alpha \leq \beta \leq 1$).

$|f(x) \bigcap Y| / Y \geq \alpha$.

According to the definition 11, $x \in GM$, so $M^{\beta} \subseteq M^{\alpha}$;

(4) Suppose $x \in M^{\alpha}$, according to the definition 11, $|f(x) \bigcap Y| / Y \geq \alpha$, ($0 < \alpha \leq 1$),so $f(x) \bigcap Y \neq \phi$, According to the definition 7, $x \in M$, so $M^{\alpha} \subseteq M$. (Note: $\alpha \neq 0$).

Construction Algorithms of Interval Concept Lattice
Basic Ideas

Algorithm steps:

(1) Calculate attribute sets' power sets

Extract attribute set A from the formal context, produce all subsets of A, constitute a power set $P(A)$.

(2) Determine the intent

Make the power set as the corresponding to the intent of the interval concept. Thus generating vertex set G of original interval concept lattice. For the convenience of algorithm and realization, suppose each node set be six-group $(M^{\alpha}, M^{\beta}, Y, Parent, Child, No)$, the intent Y is corresponding to the elements in the collection of power $P(A)$. The rest are initialized to empty.

(3) Determine upper bound extent of α

Suppose parameters α ($0 < \alpha \leq 1$). According to the intent of each node in node sets, take each of the attributes subset $Y_i \subseteq Y$ in intent and $|Y_i| / |Y| \geq \alpha$, traverse all records, find out all objects meeting all attributes in Y_i, incorporate into the upper bound extent M^{α} as corresponding to node α.

(4) Determine lower extension of β

Suppose parameters β ($0 \leq \alpha \leq \beta \leq 1$). According to the intent of each node in node sets, take each of the attributes subset $Y_i \subseteq Y$ in intent and $|Y_i| / |Y| \geq \beta$, traverse all records, find out all objects meeting all the attributes in Y_i, incorporate into the lower bound extent M^{β} as corresponding to node β, and merge with the above step.

(5) Form lattice structure

First, structure the root nodes and end nodes, and then other nodes are inserted into the lattice with the form of new nodes, so as to form the concept lattice structure.

Definition 19 Suppose L_{α}^{β} is the interval concept lattice $[\alpha, \beta]$, h is the new node, in the process of constructing, if:

(1) Only the root node and end node in L_{α}^{β}, or there is no other precursor nodes except roots node, named h the pope nodes.

(2) One or more nodes are directly subsequent of h that can be found in the direct precursor and its direct subsequent nodes in L_{α}^{β}, named h the clique nodes;

(3) No one node is directly subsequent of h that can be found in the direct precursor and its direct subsequent nodes in L_{α}^{β}, named h the department nodes.

The processes of structuring interval concept lattice, is the process of looking for direct precursor and direct subsequent of new node h, if h is pope node, its direct precursor is root node and direct subsequent is end node, then, directly modify child nodes of root nodes, the father nodes of end node and the nodes of the father and child set; while h is direct or following nodes, find all may be the father and child node nodes in the existing lattice, in this process h is not only a clique node, also can be in the node, or two kinds of identities appear alternately.

Algorithm Description

CAICL

(Construct Algorithm of Interval Concept Lattice)

Input: Formal context (U, A, R) and parameters interval [α,β]

Output: Interval concept lattice L_{α}^{β}

(1) Calculating the power set $P(A)$

Power sets of attribute set can be calculated to use recursive method. One of the most important methods recursive methods is retrospective method. The dependency of each element in set has only two kinds of state: the elements belong to power set or not. The process of recursive method is first traverse sequence a "state tree", the process of calculation power set *P(A)* can be as the choice in set A, and use a binary tree to show elements' state change process in power set: the root node is as to the initial state of elements in power set (empty set); leaf nodes shows its end state, and the ith layer of the branch node, shows its current state of the above i-1 elements in set A (left branch is gotten, right branch is abandoned) [13]. The basic idea of computing power sets in attribute set is to find out all the subsets by Subset function (char * List, int m, char * Buffer, int flag). The HTML code is as follows:

```
void SubSet(char *List, int m, char *Buffer, int flag)
{
    if(m <= ListLength-1)
    {
        if(m==0)
        {
            Buffer[0]=List[0];
        }
        Buffer[flag]=List[m];
        if(flag==0)
        {
            Buffer[flag]=List[m];
        }
        for(int i=(flag==0) ? 0 : Index(List,Buffer[flag-1])+1; i<=ListLength-1; i++)
            //When flag==0， Buffer has no element， then i=[0...ListLength-1]
        //When flag>0，find out the last element in position i in set List of Buffer, make the element
        //[i....ListLength-1] in the last of element Buffer
        {
            Buffer[flag]=List[i];
            Output(Buffer,flag);
            SubSet(List, m+1, Buffer,flag+1);
        }
    }
    return;
}
```

(2) Determine intent, generate original concept node sets G

To ensure the completeness, make subsets in power set as the intent of the interval concept. Each attributes subset Y_i is corresponding to interval concept $(M_i^{\alpha}, M_i^{\beta}, Y_i)$, initial

$M_i^{\alpha} = M_i^{\beta} = \phi$. $G = \{(M_i^{\alpha}, M_i^{\beta}, Y_i) \mid i = 1,2,\cdots\}$.

(3) Determine upper bound extent set M_i^{α} and lower bound set extent set M_i^{β} of α

For each attributes subset Y_i, calculate the number of attributes $n = |Y_i|$. Scanning each object x in formal context, and make its attribute set $f(x)$ and Y_i is in meet operation:

If $| f(x) \cap Y_i | / | Y_i | \geq \alpha$, then $M_i^{\alpha} = M_i^{\alpha} \cup x$ 。

If $| f(x) \cap Y_i | / | Y_i | \geq \beta$, then $M_i^{\beta} = M_i^{\beta} \cup x$ 。

If the scan of M_i^{α} is empty, delete the corresponding node from G .

The HTML code is as follows:

```
ComputeUp_LwEx(P(A),G)
{           M_i^α = M_i^β = φ;
For each Y in P(A)
    For each x∈U
    {           If (|f(x)∩Y|/|Y|>=α)
                        M_i^α = M_i^α ∪ x
                        If (|f(x)∩Y|/|Y|>=β)
                        M_i^β = M_i^β ∪ x
                }
                If ( M_i^α = φ ) G = G − (M_i^α, M_i^β, Y_i)
}
```

(4) Form lattice structure

For the existing lattice nodes G , according to the precursor-subsequent relationship that is defined above, determine nodes' levels and relationship between parent and child.

Step1: determine attribute intent is the end node of ϕ, attribute intent that all of the sets is root nodes, if not; it will take the most number of attributes nodes as the root node, and delete the two nodes from G.

Step2: set $n = 1$, select the nodes that have one connotation attribute in G as parent nodes of endings node. If not, n = n + 1, it will take the number of attribute node be two, which is in turn on. Delete them from the G.

Step3: $n = n + 1$, select the nodes $G_i = (M_i^{\alpha}, M_i^{\beta}, Y_i)$ that the number of intent attribute is n from G, compare Y_i with Y_j , if $Y_i \supset Y_j$, then G_i is the father node of G_j , G_i is the child node of root nodes. $G = G - G_i$

Step 4: if $G \neq \phi$, turn step 3, and otherwise, end.

Examples

A formal context is listed in the table 1.

Table 1. A formal context

U \ A	a	b	c	d	e
1	0	0	1	1	1
2	0	1	0	1	0
3	1	0	0	0	1
4	1	1	1	0	0

The corresponding formal concept is listed in the table2.

Table 2. The formal concepts generated by the Table 1

Concept	Extent	Intent	Concept	Extent	Intent
c0	ϕ	abcde	C5	12	d
C1	1	cde	C6	13	e
C2	2	bd	C7	14	c
C3	3	ae	C8	24	b
C4	4	abc	C9	34	a
C10	1234	ϕ			

Let $\alpha = 0.5$, $\beta = 0.6$, then we can get the interval concepts based on the interval [α, β]. See table4.

Table 3. The interval concepts of the formal context in the table.1

Interval concept	α upper bound extent	β lower bound extent	Intent
c0	ϕ	ϕ	ϕ
c1	{12}	{12}	d
c2	{13}	{13}	e
c3	{14}	{14}	c
c4	{24}	{24}	b
c5	{34}	{34}	a
c6	{234}	{4}	ab
c7	{134}	{4}	ac
c8	{1234}	{ф}	ad
c9	{134}	{3}	ae
c10	{124}	{4}	bc
c11	{124}	{2}	bd
c12	{1234}	ϕ	be
Interval concept	**α upper bound extent**	**β lower bound extent**	**Intent**
c13	{124}	{1}	cd
c14	{134}	{1}	ce
c15	{123}	{1}	de
c16	{4}	{4}	abc
c17	{24}	{24}	abd
c18	{34}	{34}	abe
c19	{124}	{124}	bcd
c20	{14}	{14}	bce
c21	{1}	{1}	cde
c22	{124}	{4}	abcd
c23	{1234}	ϕ	abde
c24	{124}	{1}	bcde
c25	{1234}	{14}	abcde

The interval concept lattice is in the Fig.1.

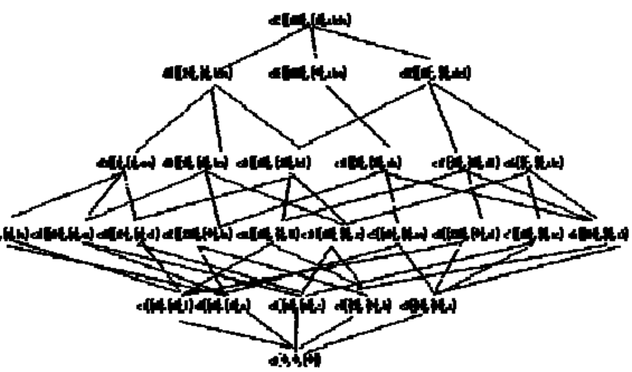

Fig.1. Interval concept lattice

In this concept lattice, some interval meet a certain precision are listed and the upper and lower bound of every concept all meet a given precision. For example, c24's lower bound extent "1" has at least $\beta \times 4 = 0.6 \times 4 = 2.4 \approx 3$ attributes in $\{b, c, d, e\}$. And the upper bound extent "1, 2, 4" at least meet $\alpha \times 4 = 0.5 \times 4 = 2$ attributes in $\{b, c, d, e\}$. In the actual shopping system, we can consider that the upper bound extent is the set of customers who buy at least two kinds of goods, and lower bound extent is the set of customers who buy at least three kinds of goods. Then we can analysis these customers to dig out the potential rules between the customers and the goods, and provide the basis for decision-making.

Conclusion

In the practical application, we not need consider if the objects have all the attributes or an attribute, but also in more cases, a certain precision is required, so this paper put forward a new kind of concept lattice through the comprehensive analysis of concept lattice and rough concept lattice. Its α upper bound and β lower bound extents both meet the attributes which have the given precision. Then the interval concept lattice was defined and its structure characteristics were discussed. Besides, the concept precision and coverage degree were defined to measure the concept lattice. It is proved that the interval concept lattice is the development of classic concept lattice and rough concept lattice. Further, the construction method of interval concept lattice was presented and the availability was explained through an example. The future research work focus on the node features of interval concept lattice, the association rules extraction method, and its application in practical problems.

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