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Research on Coverage Performance of GNSS

Abstract. The theory of Ground Coverage Performance (GCP) and Space Coverage Performance (SCP) of GNSS constellations is analysed. These performances are influenced by restriction of both the signal's beam angle and the mask angle which is used by the receivers of users. Then the judgment methods and simulation results are acquired. In practical application, relevant constellations should be selected according to different tasks which are based on the simulation results to ensure the maximum number of visible satellites at the same time.

Streszczenie Przeanalizowano teoretycznie obszary zasięgu naziemne (GCP) i w przestrzeni około-ziemskiej (SCP) dla konstelacji nadajników satelitarnych systemów nawigacji (GNSS). Sprawność zasięgu jest ograniczona zarówno przez kąt promieniowania nadajnika, jak i przez kąt zasięgu odbiorników użytkownika. Przedstawiono metody oceny i wyniki symulacji. W praktycznych zastosowaniach, w zależności od różnych zadań i wyników symulacji, konstelacja satelitów musi być dobrana tak, aby była widoczna maksymalna ich ilość w tym samym czasie. Badania dotyczą systemów: GPS, GLONASS, GALILEO i chińskiego BD2. **Badania sprawności zasięgu globalnych systemów nawigacji satelitarnej GNSS**

Keywords: GNSS, Ground Coverage Performance, Space Coverage Performance, Simulation. **Słowa kluczowe:** Globalny system nawigacji satelitarnej, Sprawność zasięgu odbioru naziemnego, Symulacja

Introduction

The four Global Navigation Satellite Systems (GNSSs) which are currently on-orbit operation include the American GPS, the Russian GLONASS, the European GALILEO system and the Chinese BD2 system (the Second Generation of BeiDou Navigation System). The coverage performance of GNSS is one of the most important indicators which ensure the availability of the GNSS [1, 2]. Therefore, researches on Ground Coverage Performance (GCP) and Space Coverage Performance (SCP) can provide essential references for the application of GNSS. Experts have worked a lot over coverage performance of each GNSS respectively, they also have analyzed and simulated the coverage scope of a single satellite through geometrical methods [2, 3, 4, 5]. However, these methods are relatively complex, yet comparisons of the advantages and disadvantages of that among the four GNSSs have rarely been made. In this paper, the theory of GCP and SCP in different orbital altitudes of GNSS is analyzed. Corresponding judgment methods are put forward, which take restriction of both the signal's beam angle and the receiver's mask angle into consideration. Then 32 GPS satellites, 24 GLONASS satellites, 27 GALILEO satellites [6] and 12 BD2 satellites are selected to be simulated and the results are collected. Finally, GCP and SCP of the four constellations are compared respectively.



Fig.1. Ground coverage scope of a single satellite signal

Analysis and simulation of ground coverage performance

Theory of ground coverage performance

A GNSS satellite-borne sensor transmits the signal to ground with a certain beam angle and thus forms a conical coverage scope. The beam angle of the satellite signal is about $32^{\circ} \sim 46^{\circ}$ [5]. Meanwhile, the ground user receives the signal with no less than the mask angle which is used by the receiver of the user. Therefore, the ground coverage scope is not only affected by location of the ground user, that is to say whether the ground user is in the visible scope of the satellite, but also by the position of satellite, whether it is in the observation scope of the user [7]. The ground coverage scope of a single satellite signal is shown in Figure 1.

The shadow part in the figure presents the ground coverage scope of a single satellite signal. Supposing that the earth is a sphere with radius *R*. The orbital altitude of the satellite *S* is H_S . The one of the satellite's tangents to the earth is *SD* which is shown in the figure. If $\angle OSD = \alpha$,

(1)
$$\alpha = \arcsin\left(\frac{R}{R+H_s}\right)$$

The corresponding values α of four GNSSs satellites in different orbital altitudes are presented in Table 1.

	annudes						
	GNSSs	Orbital Types	Orbital Altitudes [km]	α[°]			
	GPS	MEO	20200	13.87			
	GLONASS	MEO	19100	14.49			
	GALILEO	MEO	23600	12.27			
GALILEO BD2	GEO	25960	0 60				
	BD2	IGSO	33000	0.00			
		MEO	21500	13.21			

Table 1. The values $\boldsymbol{\alpha}$ of GNSSs satellites in different orbital altitudes

In the table, MEO is short for Medium Earth Orbit, GEO is short for Geostationary Earth Orbit, and IGSO is short for Inclined GeoSynchronous Orbits.

It can be seen that the value of α is less than half of the satellite signal's beam angle whose value is about 16 ° ~23 °. The beam angle is large enough to be ignored when researching on GCP of GNSS satellite. But the receiver's mask angle should be taken into consideration. Because of the interference of both heat loss of the ground and the occlusion of ground things, a valid observation scope should be considered in practical application [8]. It is to say that the mask angle σ which is used by the receiver of the user should be taken into account.

In Earth-Centered Earth-Fixed (ECEF) coordinate system, supposing that the coordinate of a satellite *S* is (x_i, y_i, z_i) , the coordinate of a ground user *U* is (x_u, y_u, z_u) . As is

shown in Figure 1, only the users in the shadow part can view the satellite. So the judgment condition of the visibility between ground users and the satellite is as follows: (2) $\angle SUO \ge 90^\circ + \sigma$

Where:

$$\angle SUO = \arccos\left(\frac{|SU|^2 + |UO|^2 - |SO|^2}{2|SU||UO|}\right)$$
$$|SU| = \sqrt{(x_s - x_u)^2 + (y_s - y_u)^2 + (z_s - z_u)^2}$$
$$|UO| = \sqrt{x_u^2 + y_u^2 + z_u^2} \quad |SO| = \sqrt{x_s^2 + y_s^2 + z_s^2}$$

Simulation and results analysis

If there are at least four simultaneous and continuous satellites of one GNSS in view of users in one area, the coverage probability of the GNSS is 100% for this area [1]. The larger the coverage probability, the better GCP, SCP and navigation ability. In the simulation, 32 GPS satellites (according to Yuma almanac), 24 GLONASS satellites (uniformly locate in 3 orbital planes 120° apart in right ascension), 27 GALILEO satellites (3 orbital planes, equally spaced and with 56° nominal inclination and 9 satellites per plane) [6] and 12 BD2 satellites are chosen. The satellites in the first three systems are all MEO satellites. The satellites in BD2 system include 5 GEO satellites, 3 IGSO satellites and 4 MEO satellites [4]. The number of simultaneous satellites of each GNSS in view of the ground users in global area is simulated, and it is counted for 24 hours statistically. The simulation conditions are set as follows: sampling step of latitude is $\Delta B=2^{\circ}$, sampling step of longitude is $\Delta L=2^{\circ}$, sampling step of time is $\Delta T=5min$, the mask angle is $\sigma=5^{\circ}$. After statistical analysis, the probability of the simultaneous satellites of each GNSS in view of the ground users in global area is shown in Figure 2.



Fig.2. The probability of the simultaneous satellites of each GNSS in view

From the figure above, we can see that the coverage probability of 32 GPS satellites, 24 GLONASS satellites and 27 GALILEO satellites to the global ground users is 100%. The percent in areas where the users can view no less than 4 BD2 satellites at the same time is about 50%. The ground coverage scope of 12 BD2 satellites is shown in Figure 3.



Fig.3. Ground coverage scope of 12 BD2 satellites

The shadow part in the figure presents the ground coverage scope of 12 BD2 satellites. It can be seen that 12 BD2 satellites have been able to provide the navigation ability in Asia Pacific region. The planned BD2 system should also include additional 23 MEO satellites. The whole system is expected to be finished in 2020, the global ground area will be covered completely at that time.

Analysis of space coverage performance

Theory of space coverage performance

The orbital altitude of the user should be taken into consideration when researching on SCP of GNSS. The space coverage scope of a single satellite signal is shown in Figure 4.



Fig.4. Space coverage scope of a single satellite signal

The shadow part in the figure presents the space coverage scope of a single satellite signal. In ECEF coordinate system, supposing that the coordinate of a satellite *S* is (x_s, y_s, z_s) , the orbital altitude of the satellite is H_s , the corresponding orbital altitude with the beam width of the satellite signal is *h*, and half of the signal's beam angle is γ . So:

(3) $h = (R + H_s) \sin \gamma - R$

As to MEO satellites, the value of h is about $1000km \sim 5000km$; as to GEO satellites and IGSO satellites, the value of h is about $5000km \sim 10000km$. It is to say that the restriction of the signal's beam angle can be ignored when the orbital altitude of the space user is less than h. Otherwise, both the beam angle and the mask angle should be considered.

Different from the ground users, the space users can view the GNSS satellites with a negative elevation with the increase of users' orbital altitudes. However, the influence of the mask angle σ which is similar to that of the ground users should be considered. The valid observation scope of a space user is shown in Figure 5.



Fig.5. The valid observation scope of a space user

The scope which is indicated by the arrow in the figure is the observation scope of the space user. The user can view the satellites with a negative elevation when the satellites in view are below the horizontal plane of the user. On the contrary, the user can view the satellites with a positive elevation when the satellites in view are above the horizontal plane of the user. In ECEF coordinate system, supposing that the coordinate of a space user U is (x_u, y_u, z_u) , the orbital altitude of the user is H_u . Combined with Figure 4 and Figure 5, it can be seen that the space user U can view the satellite S when U is in the shadow part of Figure 4 and S is in the scope where the arrow indicates in Figure 5. So the judgment condition of the visibility between space users and the satellite is as follows:

(4)
$$\begin{cases} \beta \leq \angle USO \leq \gamma \\ \angle SUO \geq \theta + \sigma \end{cases} \quad or \quad \begin{cases} \angle USO \leq \beta \\ \angle SUO > 90^{\circ} \\ \angle SUO \geq \theta + \sigma \end{cases}$$

where:

$$\angle SUO = \arccos\left(\frac{|SU|^{2} + |UO|^{2} - |SO|^{2}}{2|SU||UO|}\right)$$
$$\angle USO = \arccos\left(\frac{|SU|^{2} + |SO|^{2} - |UO|^{2}}{2|SU||SO|}\right)$$
$$|SU| = \sqrt{(x_{s} - x_{u})^{2} + (y_{s} - y_{u})^{2} + (z_{s} - z_{u})^{2}}$$
$$|UO| = \sqrt{x_{u}^{2} + y_{u}^{2} + z_{u}^{2}}$$

 θ is the included angle between one of the satellite's tangents to the earth and the line which is between the user and the earth centre.

(5)
$$\theta = \arcsin \frac{R}{R+H_u}$$

Simulation and results analysis

The space users in different orbital altitudes can view the different numbers of satellites. That is to say the SCP of satellites to the users in different orbital altitudes is different. Therefore, the space users in different orbital altitudes are chosen to simulate the visibility of the four GNSSs satellites respectively in this paper. On the condition of $\gamma=20^{\circ}$, $\sigma=5^{\circ}$, the other simulation conditions are set the same as that of GCP. The maximum number, the minimum number and the mean number of the GPS satellites in view of space users in different orbital altitudes are shown in Figure 6.



Fig.6. The number of GPS satellites in view of space users in different orbital altitudes



Fig.7. The number of GLONASS satellites in view of space users in different orbital altitudes

From the figure above, it can be seen that the number of visible satellites is getting larger with the increase of the user's orbital altitude. It reaches a peak when the altitude is about 2500km. The reason is that the user can view more satellites with a larger negative elevation with the increase of the user's orbital altitude. The number of visible satellites becomes smaller when the user's orbital altitude is higher than 2500km. With the restriction of the satellite signal's beam angle, the user can view less satellites with a positive elevation with the increase of the user's orbital altitude. Comparing the minimum numbers of the visible satellites in different orbital altitudes, it can be found that the user can view at least 4 GPS satellites when the orbital altitude of the user is lower than 3500km. In other words, the coverage probability of 32 GPS satellites is 100% for the users in that altitude. Comparing the maximum numbers of the visible satellites in different orbital altitudes, it can be figured out that the user can view less than 4 GPS satellites when the orbital altitude of the user is higher than *22000km* for the 32 GPS satellites signals cannot cover this area.

The maximum number, the minimum number and the mean number of the GLONASS satellites in view of space users in different orbital altitudes are shown statistically in Figure 7.

Similar to SCP of 32 GPS satellites, the number of visible satellites in GLONASS reaches the largest when the user's orbital altitude is up to about *2000km*. The number of visible satellites decreases when the user's orbital altitude is higher than *2000km*. The coverage probability of 24 GPS satellites is 100% when the user's orbital altitude is lower than *3000km*. The 24 GLONASS satellites signals cannot cover the area where the user's orbital altitude is higher than *18000km*.

The maximum number, the minimum number and the mean number of the GALILEO satellites in view of space users in different orbital altitudes are shown statistically in Figure 8.



Fig.8. The number of GALILEO satellites in view of space users in different orbital altitudes

The number of visible satellites in GALILEO reaches the peak when the user's orbital altitude is about *3500km*. The number of visible satellites falls when the user's orbital altitude is higher than *3500km*. The coverage probability of 27 GALILEO satellites is 100% when the user's orbital altitude is lower than *5500km*. The 27 GALILEO satellites signals cannot cover the area where the user's orbital altitude is higher than *30000km*.

The maximum number, the minimum number and the mean number of the BD2 satellites in view of space users in different orbital altitudes are shown statistically in Figure 9.



Fig.9. The number of BD2 satellites in view of space users in different orbital altitudes

In this paper, only 12 BD2 satellites are chosen to be simulated. From the figure above, it can be seen that when the user's orbital altitude is up to *36000km*, the users can still view 4 BD2 satellites. It illustrates that BD2 constellation has a certain coverage ability when the user's orbital altitude is up to that of GEO, this performance is better than that of the other three GNSSs. The reason is that there are 5 GEO satellites and 3 IGSO satellites which are included in the 12 BD2 satellites. The mean number of the satellites in view is more than 4 when the user's orbital altitude is lower than *10000km*. That's to say the coverage probability of 12 BD2 satellites is at least 50% when the user's orbital altitude is lower than *10000km*.

The detailed coverage probability of each GNSS constellation to the space users in different orbital altitudes are presented in Table 2 statistically.

Table 2.	The	coverage	probability	of each	GNSS	constellation	to
the space	e use	rs in differ	ent orbital a	altitudes			

H_u [km]	GPS	GLONASS	GALILEO	BD2					
1000	100%	100%	100%	93.44%					
2000	100%	100%	100%	97.52%					
3000	100%	99.98%	100%	99.19%					
4000	99.93%	91.36%	100%	98.99%					
5000	95.19%	73.91%	100%	99.24%					
6000	83.49%	41.10%	95.88%	99.31%					
7000	64.09%	21.83%	92.99%	99.45%					
8000	46.05%	12.36%	83.54%	99.50%					
9000	31.00%	6.59%	66.46%	77.71%					
10000	21.21%	4.53%	51.50%	52.55%					
15000	2.15%	0.03%	7.79%	11.43%					
20000	0.31%	0.00%	5.19%	4.85%					
36000	0	0	0	0.10%					
	•	•		•					

From this table, we can see that SCP of BD2 constellations is better than that of the other three GNSSs constellations obviously when the users' orbital altitudes are higher than *5000km*, and the coverage performance of GLONASS constellation is the worst in the same condition. The reason is that the orbital altitude of GLONASS satellite is the lowest in the four GNSSs, and satellites in the highest orbital altitude are included in BD2 constellation. The SCP of GPS constellation and GALILEO constellation have advantages when the space users' orbital altitudes are lower than *5000km*. It is because that the number of GPS satellites and GALILEO satellites which are simulated is larger.

Conclusions

In this paper, 32 GPS satellites, 24 GLONASS satellites, 27 GALILEO satellites and 12 BD2 satellites are chosen to analyze the theory of GCP and SCP in different orbital altitudes. Then the corresponding judgment methods are put forward and the simulation results are also acquired. The GCP of the first three GNSS constellations are perfect, because their ground coverage probabilities are all 100%. The GCP of the constellation which consists of 12 BD2 satellites is good for only Asia Pacific region, because the ground coverage probability to Asia Pacific region is 100%, while the probability is about 50% to the global area. The SCP of GNSS constellation is different from each other with different orbital altitudes of the users. The space coverage probabilities of the first three GNSS constellations are 100% when the user's orbital altitude is lower than a certain value. That's to say the SCP is good in this condition. But the value is different, which is related to the orbital altitude of satellites in each GNSS. The higher the orbital altitude, the larger the value. As to GPS, the value is about 3500km; as to GLONASS, the value is about 3000km; as to GALILEO, the value is about 5500km. With the increase of user's orbital altitude, the SCP becomes worse and worse. When the orbital altitude of the user is higher than another certain value, the GNSS satellites signals cannot cover this area. These values are also related to the orbital altitudes of satellites in each GNSS. As to GPS, the value is about 22000km; as to GLONASS, the value is about 18000km; as to GALILEO, the value is about 30000km. The SCP of BD2 constellations is better than that of the other three GNSS constellations when the users' orbital altitudes are higher than 5000km, because there are 5 GEO satellites and 3 IGSO satellites in BD2 system. Their orbital altitudes are the highest among all the navigation satellites, which is about 35860km. However, the simulated BD2 system is not complete, the results can only present a part of the coverage performance. In the practical application, relevant constellations should be selected according to different tasks which are based on the simulation results to ensure the maximum number of visible satellites at the same time.

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