

# Effect of roughness on Radiation Characteristic of Reflector Antenna Based on Multi-scale Modelling

**Abstract.** Firstly, the wavelet analysis was used as the filter tool to separate the different scale roughness from the reflector surface testing data, and a mathematical model of the multi-scale roughness was presented to investigate its impact on the radiation characteristic of the reflector antenna. Based on the roughness model, a formulation of density of the far-field was deduced and the definable modeling flowchart was given. Finally, a 3.7-m and a 3-m antenna were simulated to validate the model, formulation and the modeling method. The results showed that the roughness broadened the beam-width and raised the side lobe level of the antenna, while it had little impact on the antenna gain. The major roughness scale was found by the comparison of the different amplitude of the roughness, which will provide theoretical foundation for the control of the surface-working. Above all, the results of the proposed model were in good agreement with the test data and it could simulate the antenna more accurately and obtained better electrical properties with define machining cost than the existing modeling and analysis method.

**Streszczenie.** Z pomiarów badań testowych powierzchni reflektora, wstępnie odseparowano wyniki dla chropowatości różnych rozmiarów, wykorzystując, jako filtr, analizę falkową. Aby zbadać wpływ chropowatości na charakterystykę promieniowania anteny, wykorzystano wielowymiarowy, matematyczny model chropowatości. Na podstawie tego modelu utworzono wyrażenie na gęstość pola dalekiego zasięgu oraz skonstruowano blokowy schemat przepływu obliczeń. W końcu, udowodniono poprawność zastosowanych wzorów i metody modelowania przez symulację anten 3,7m i 3m. Wyniki modelowania pokazują, że chropowatości wywołują rozszerzenie listka głównego i wzrost poziomu listków bocznych charakterystyki promieniowania anteny, natomiast mają niewielki wpływ na wzmocnienie. Wyznaczono wpływ rozmiarów chropowatości na charakterystykę, co jest teoretyczną podstawą kontroli wykonania powierzchni reflektora. Ponadto, w stosunku do dotychczas stosowanych metod modelowania, przy pomocy zaproponowanego modelu można symulować antenę z większą dokładnością i otrzymać dobre własności elektryczne ze zdefiniowanymi kosztami obróbki. **Wpływ chropowatości na charakterystyki promieniowania anteny reflektorowej oparty o modelowanie wielowymiarowe**

**Keywords:** Multi-scale Modeling, Random error, Reflector Antenna, Radiation Characteristic.

**Słowa kluczowe:** Modelowanie wielowymiarowe, Błąd przypadkowy, Antena reflektorowa, Charakterystyka promieniowania

## Introduction

Antennas are of great important in microwave communication and relay system engineering. The error factors (random error and system error), as the significant factors affecting the antenna radiation characteristic, are the neck-bottle against its rapid development [1-4]. Since Ruze published the first work on this subject and pursued effect of small random phase and amplitude errors on the average side-lobe level. The effect of the random error on the radiation characteristic have been studied by a number of authors and made a very significant achievement. But the researches on effect of the random error on the radiation characteristic were mainly focused on the error in the geometric form and the waviness, while the roughness profile in micro-nano scale was pay little attention on. It is generally assumed that the random error uniformly distributed throughout the reflector surface of the antenna. So some random functions were chosen to simulate the part random error and the entire surface error was gained by integrating all the local error. But the random error radiation characteristic of the antenna is significantly different. In recent years, wavelet analysis is more and more applied in filter arithmetic, which has the character of time-frequency localization and the character of "microscope" and "magnifier"[5-6]. Using the wavelet analysis as the filter tool, the roughness affecting spheres was determined based on the antenna working frequency, and the affecting sphere was regard as an assessment reference surface. The different scale roughness was separated from the reference surface by the wavelet filtering. Finally, the multi-scale roughness mathematics model, which includes three scales error information, was built in this paper.

The large caliber and high precision characteristic of the reflector antenna result in the high costs and long cycle in the process of antenna manufacturing. So producing numerous prototypes is a romantic method, and the verification of relatively theory mainly by means of the simulation experiment analysis. But any the existing

commerce software can analysis the nonideal antenna model, which confines the development of the theory. Aim to this problem, a modeling flowchart of the reflector antenna was represented in this paper. The alterable random error and systemic error were considered in the process of the antenna modeling and analysis by adding the interrelated software module compiled into the existing commerce software. The values of the errors were determined based on the machining technology and the working environment of the reflector antenna. It expanded the applicability scope of the commerce software and improved its analysis precision.

For research on the effect of the error, most existing relative literatures revised the radiation characteristic formulation after the antenna has been manufactured and tested, but not in the process of the antenna design even before the design. These are really experimental methods, which can gain engineering experience, but still cannot find the essential of the influence and build the quantitative influence formulation. What is worse, the reference data for the revision was obtained by assuming some probability models. These factors inevitably resulted in inaccurate analytical results [7-8]. The work presented found that, effect of the multi-scale roughness on the surface of the reflector antenna is to increase the phase error. A formulation of density of the far-field based on the multi-scale roughness model was deduced. In where, the error was determined according to the machining technology and the working environment of the reflector antenna, which considered the effect of all the error and provided the theoretical foundation for analysis and design of the antenna.

## Multi-scale Roughness

Microscopically the real reflector surface of the antenna is considered as the linear superposition of roughness, waviness, and form with the addition of flaws. The divided regulation of the three scales depended on the ratio of the roughness to the working length of the antenna. With the

different scale, the influence is different. Therefore, it is imperative to establish a roughness model contains all scales information [9,10]. Based on the microwave surface theory, 98.2% of the current concentrates in the scope of the  $4\pi$  from the surface. Therefore, the current flowing in the depth of the  $4\pi$  from the surface rather than the exact shape of the surface profile is considered as a more reasonable view. The roughness on the scale too large or too small has little influence on the electrical properties. Using the wavelet analysis as the filter tool, the roughness affecting spheres was determined based on the antenna working frequency, and the affecting sphere was regard as an assessment reference surface. The different scale roughness was separated from the reference surface by the wavelet filtering. Finally, a multi-scale roughness model was built as follow [13,14]:

$$(1) \quad z(x, y) = L \left( \frac{G}{L} \right)^{(D-2)} \left( \frac{\ln b}{M} \right)^{1/2} \sum_{m=1}^M \sum_{n=0}^{n_{\max}} b^{(D-3)n} \left\{ \cos \phi_{m,n} \right. \\ \left. - \cos \left[ \frac{2\pi b^n (x^2 + y^2)^{1/2}}{L} \cos \left( \tan^{-1} \left( \frac{y}{x} \right) - \frac{\pi m}{M} \right) + \phi_{m,n} \right] \right\}$$

where,  $x$  – abscissa,  $y$  – ordinate of the rough surface,  $L$  – sampling length of the rough surface,  $G$  – frequency independent scaling parameter,  $D$  ( $2 < D < 3$ ) – fractal dimension,  $b$  – fundamental spatial frequency,  $N$  and  $M$  – numbers of tones,  $\phi_{m,n}$  – phase term that has a uniform distribution. over the interval  $[-\pi, \pi]$ . The purpose of normalize is giving the  $F(x, y)$  function of the rough surface with an arbitrary  $R_a$ , which is the root-mean-square (rms) roughness of this surface. When  $D=2.5$  and  $b=1.6$ , the value of  $N$  drops from 2048 to 1024, which is equivalent that the resolution drops by one half, the surface contour still similar to its former state. It shows that the dimension of the roughness function was a parameter no concern with the resolution of the measuring instrument and the function can be used to represent the essential of the roughness surface. Fig.1 showed a plane view of a 3D multi-scale roughness surface constructed from Equation (1) using  $G = 3.78 \times 10^{-3} \text{ nm}$  and  $D = 2.25$ , the  $R_a = 13.6 \text{ nm}$ .

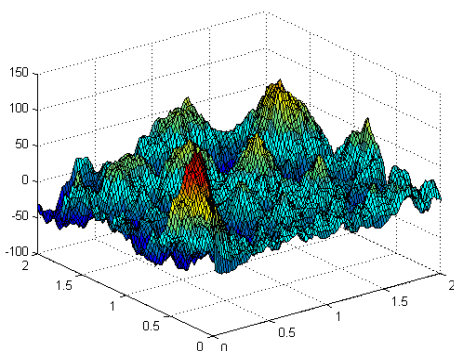


Fig. 1 The multi-scale roughness model

### Radiation Characteristic of the Reflector Antenna

The three central factors affecting the radiation characteristic of the reflector antenna are positional and direction error of the feed and the random error on the reflector surface. The existence of surface roughness changes the axial displacement. It makes the phase difference becomes a function of the location, and the aperture surface will be no more an equiphase surface, the radiation pater of the antenna will change too[12].

$$(2) \quad E(\theta, \phi) = \iint_{S'} f(\xi, \phi') e^{j\Delta\phi} e^{j\varphi(\rho', \phi')} \rho' d\rho' d\phi'$$

where,  $E(\theta, \phi)$  – density of the far-field in p point,  $f(\xi, \phi')$  – distribution function of the antenna aperture on the caliber surface  $S'$ ,  $e^{j\Delta\phi}$  – phase error caused by the roughness on the reflector surface,  $e^{j\varphi(\rho', \phi')}$  – phase error caused by the positional and direction error of the antenna feed. The equation for the phase error can be expressed as:

$$(3) \quad \Delta\phi = 4\pi z(x, y) \cos^2(\xi/2) / \lambda$$

where,  $z(x, y)$  – roughness on the reflector surface of the antenna,  $\lambda$  – working length of the antenna. And the roughness function in Eq. (1) is used as the axial displacement and substituted in Eq. (3). And the Eq. (3) is used to combine the roughness and the radiation characteristic of the antenna to obtain the gain reduction coefficient of the antenna with respect to distorted reflector.

Because any the existing commerce software can analysis the nonideal antenna model. A flow chart of the simulation was given as shown in the Fig.2 to verify the correctness and reliability of the modeling and analysis method above. The roughness model was built refer to the engineering data, which ensures the accuracy of the model. If the electrical properties result is meet the design requirement, the cycle is completed; otherwise, the system error and the random error will be remedied to gain the ideal result. It was clear from the flow chart that the systematic error and random error were added into the process of simulation. It remedied the deficiency of the existing method.

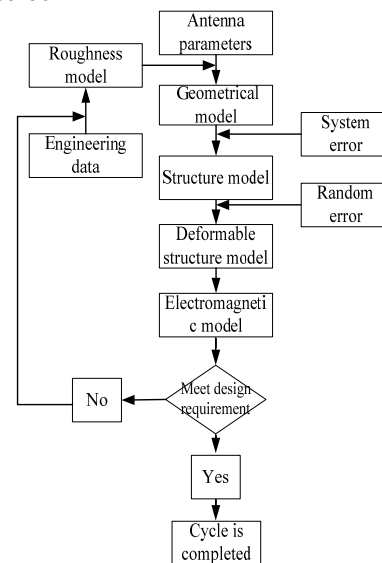


Fig. 2 The flow chart of the simulation

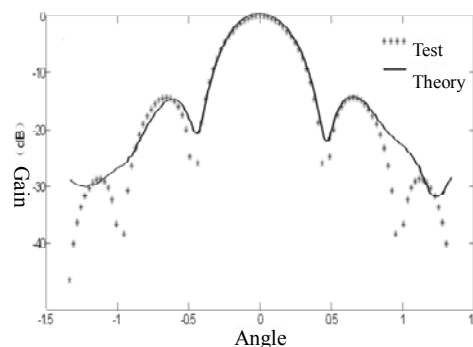


Fig.3 Comparison of results of the presented method

## Testing of the theory

In order to validate the presented modeling method and the advantages over the current method, the applications of both methods in the 3.7-m and 3-m cassegrain reflector antenna engineering of frequencies in the Ku band were demonstrated. Firstly, a comparison of test data and the presented modeling method for the 3.7-m antenna was made. The caliber of the main reflector surface of the antenna is 3.7m and that of the auxiliary reflector surface is 0.44m, the antenna used as the receiving one in the experiment and the working frequency is 12.5GHz. As shown in the Fig.3, the gain and the first side-lobe of the antenna along the angle were all highly consistent with the test data in the area of the main lobe, which manifests the truth of the method. The others side-lobe differed obviously in the phase and amplitude quantification because that the theory was based on the antenna working in the ideal circumstance, while such ideal experiment conditions were not readily reproduced in practical reflector antenna test. The original machining surface of the antenna reflector antenna, without being touched up in any way, are the linear superposition of roughness, waviness, and form with the addition of flaws. In order to analysis the different influence of the different scales roughness on the radiation characteristic of the reflector antenna, a comparison of the different scale of the roughness was made for the 3-m reflector antenna.

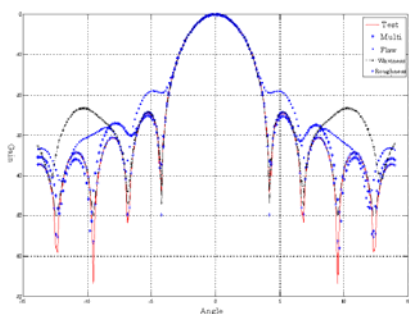


Fig. 4 Comparison of the different scale of the roughness

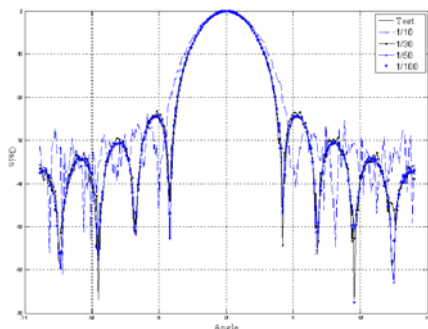


Fig. 5 Comparison of the different amplitude of the roughness

In the Fig.4, the different scale roughness was separated from the surface by the use of wavelet filter. The results of the single scale roughness have a lower first side-lobe than the test data. Because the single scale roughness lost some roughness information, which equivalent to a smoother surface. For the same systematic error and the random error of the reflector, the gain reduction coefficient obtained by the multi-scale roughness modeling model was worse than that by other single scales error model, due to the fact that the presented method took the three scales error into consideration simultaneously. In a word, the result of the multi-scale roughness model was most close to the test data, which indicated that the multi-scale roughness model could describe the rough surface more exactly than any single scale model. As shown in Fig.5, increasing of the

roughness amplitude broadened the beam-width and raised the side-lobe level of the antenna, while had little impact on the antenna gain. And when the amplitude of the roughness approximates the tenth part of the operation wavelength, the antenna was affected most. The finding suggested that determine the strongest factor based on the operation frequency and reducing it in the process of the manufacture was an effective way to make sure the effectiveness of antenna design, which will reduce the production cost and shorten the circle for production.

## Conclusions

Based on the analytical and experimental investigations presented, the following conclusions may be drawn. The method of simulating reflector surface by multi-scale roughness model obtained a satisfactory result in research on radiation characteristic of the reflector antenna. Firstly, the wavelet analysis method used as a filtering tool, a mathematical model of the multi-scale roughness was presented. And then, a formulation of density of the far-field was deduced and the definable modeling flowchart was given. The feasibility of the modeling and simulation method was checked by the comparison to the test data, and the accuracy of the theory was proved by the comparison among the single scale error and the multi-scale roughness to the test data. Finally, the influence degree of the different amplitude of the roughness were considered, which has not been fully discussed in the past.

In addition, two problems are still unsettled. Firstly, as one of the parameters, which can fully describe the morphology of the rough surface, the parameter of the multi-scale roughness model should have some inter-relationships with the traditional judge system to manifest the continuity of the science. On the other hand, it's our experience that in comparison, qualitative methods tend to be faster, less expensive, and more likely to provide useful answers to important questions that lead to superior design. And we deduced the formulation of density of the far-field based on the roughness model. But the qualitative analysis of the comparison result not quite finished. Further study is needed to give a qualitative comparison to meet the logic, preciseness, accuracy and application of the science.

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