

# Procedure for Uncertainty Management in the Next Generation of GPS Based on Monte Carlo Simulation

**Abstract.** In order to remove the impact of the implementation uncertainty in the actual measurement process, Monte Carlo simulation was adopted to estimate the method uncertainty, assess the measurement program and improve the measurement program by Process of Uncertainty Management given in the next generation of Geometrical Product Specification. A simulation prototype system of uncertainty estimation based on Monte Carlo method is presented by using the professional software Matlab. An engineering example is introduced to test the proposed method.

**Streszczenie.** W celu usunięcia wpływu niepewności wykonania w bieżącym procesie pomiaru, aby oszacować niepewność metody i programu pomiarów, zaadoptowano symulację Monte Carlo. Program pomiarowy ulepszono przez zastosowanie procesu zarządzania niepewnością dołączonego do kolejnej generacji Geometrycznej Specyfikacji Produktu (GPS). Wykorzystując profesjonalny program Matlab zaprezentowano prototyp symulacji systemu oszacowania niepewności oparty o metodę Monte Carlo. Do testu proponowanej metody wprowadzono przykład inżynierski. **Procedura zarządzania niepewnością w GPS następnej generacji oparta o symulację Monte Carlo**

**Keywords:** Monte Carlo; measurement uncertainty; Geometrical Product Specification

**Słowa kluczowe:** Monte Carlo, Niepewność pomiarów, Geometryczna Specyfikacja produktu

## Introduction

The next generation of generation of Geometrical Product Specification (GPS) standard system expands the concept and meaning of measurement uncertainties, and normalizes the Procedure for Uncertainty of measurement Management (PUMA) by using the next generation of thoughts based on the GUM and the systematic quantifying-unifying in the next generation of generation of GPS. Furthermore, the concept of uncertainty has been expanded to the whole GPS system beyond just measurement uncertainty, and the functions, specifications and measurement verification of workpieces are quantified and integrated by using the statistic and quantificational characteristic of expanded uncertainty [1,2].

In the next generation of generation of GPS standard system, measurement uncertainty is divided into method uncertainty and implementation uncertainty. In order to remove the impact of the implementation uncertainty in the actual measurement process, Monte Carlo simulation was adopted to estimate the method uncertainty, assess the measurement program and improve the failure measurement program by Process of Uncertainty Management given in the next generation of GPS.

Monte Carlo method is a mathematical method to obtain an approximate value through the simulation of random variable. The basic idea of Monte Carlo simulation is: establish the probability distribution model which is similar to the described problems firstly to make some features of the distribution model link with the answers of the questions, and then solve the problems through calculating the samples obtained through random sampling and analyzing the final results [3].

We took the measurement of a cone angle with double ball method as an experiment, established the mathematical relationship model between the measurand and the direct measures, then conducted the sampling and synthesis by using Monte Carlo simulation, developed a prototype system of Monte Carlo simulation for uncertainty evaluation by using Matlab as a platform. Finally, we evaluated and improved measurement program based on Procedure for Uncertainty Management, which is given by next generation of GPS (Geometrical Product Specification). A simulation prototype system of uncertainty estimation based on Monte Carlo method is presented by using the professional software Matlab. An engineering example is introduced to test the proposed method.

## Procedure for uncertainty management in the next generation of GPS

Assuming that a given target uncertainty is  $U_T$ . The first iteration of Uncertainty Evaluation usually make a rough assessment of the uncertainty  $U_{E1}$ . If  $U_{E1}$  does not meet the requirements ( $U_{E1} > U_T$ ), then the given measurement task does not meet the requirements. According to Procedure for Uncertainty Management (PUMA) (Fig.1), which is given in the next generation of GPS, we can seek the appropriate measurement process to meet the conditions of  $U_E$  (measurement uncertainty estimated)  $\leq U_T$  (target uncertainty).

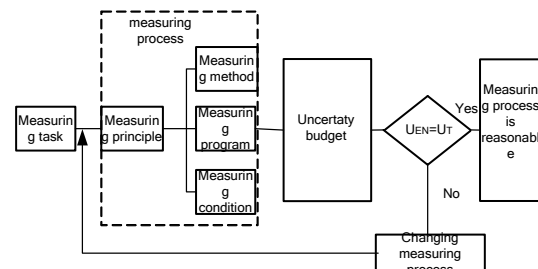


Fig.1 PUMA diagram with given target uncertainty

## Principle of Monte Carlo simulation for the estimation of measurement uncertainty

We analyzed the measurement uncertainty contributors, determined the probability distribution of each uncertainty contributor, and sampled the probability density function of each contributor, gained the sampling values by using Monte Carlo simulation. The propagation process of sampling distribution is shown in Fig.2 [4].

## Experimental research of uncertainty estimation by monte carlo method

We took an example in this section as the experimental research to study the application of Monte Carlo method. As can be shown in Fig.3 [5], the diameter of the bigger ball is  $d_1$ , the measurement uncertainty of  $d_1$  is  $u_{d1}$ ; the diameter of the small ball is  $d_2$ , the measurement uncertainty of  $d_2$  is  $u_{d2}$ . The distance from the top of the bigger ball to the bottom plane is  $l_1$ , the measurement uncertainty of  $l_1$  is  $u_{l1}$ ; the distance from the top of the small ball to the bottom plane is  $l_2$ , the measurement uncertainty of  $l_2$  is  $u_{l2}$ . Assuming that the instrument for measuring the

diameters and distances is a micrometer, according to the instructions, the indication error of the micrometer is 0.002mm.

The measurement method is shown as follows: put a ball into the slot firstly, then measure the vertical distance from the top of the ball to the bottom plane, carry this ball out, put into another ball, measure the vertical distance from the top of the ball to the bottom plane, as can be seen from Fig.2.

The basic principle of estimating uncertainty by Monte Carlo simulation method is: generate four groups of data that meet the normal

distribution firstly, that is:  $d_1, u_{d1}; d_2, u_{d2}; l_1, u_{l1}; l_2, u_{l2}$ . Then we conducted 106 times sampling operation to gain the mean value and uncertainty of the cone angle according to equation (5).

We applied Matlab as a platform, developed a prototype system of Monte Carlo uncertainty estimation. We used the prototype system to gain the result of uncertainty estimation by Monte Carlo simulation. As can be seen from Fig.3, the cone angle is 0.52rad, and the measurement uncertainty is  $1.5 \times 10^{-4}$  rad.

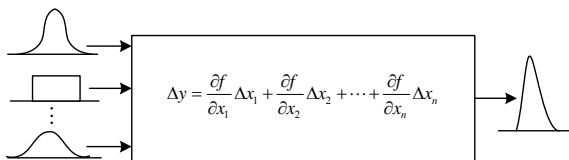


Fig.2 MCS diagram of the sampling distribution propagation

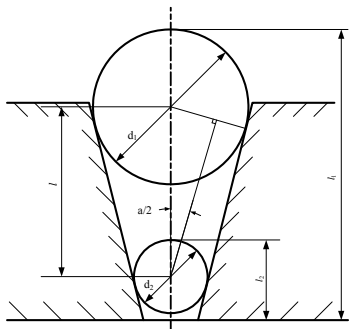


Fig.3 diagram of the cone angle measurement

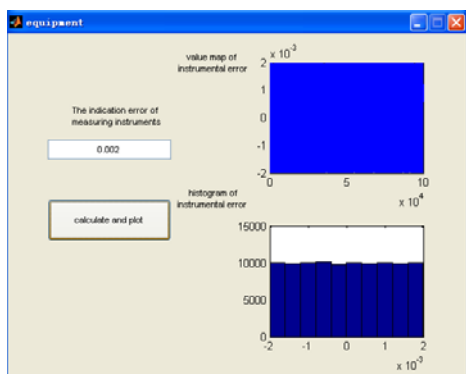


Fig.4 probability distribution of the measuring instrument error

Assuming a given target uncertainty is  $1.0 \times 10^{-4}$  rad. We can see,  $U_{E1} (1.51 \times 10^{-4} \text{rad}) > \text{target uncertainty } U_T (1.0 \times 10^{-4} \text{ rad})$ , therefore, this measurement process does not meet the requirement. According to Procedure for

Uncertainty Management (PUMA) (Fig.1), we can seek the appropriate measurement process to meet the conditions of  $U_E (\text{measurement uncertainty estimated}) \leq U_T (\text{target uncertainty})$ . According to equation of the cone angle standard deviation (standard uncertainty) calculation:

$$(1) \quad u_\alpha = \frac{2}{\cos \frac{\alpha}{2}} * \sqrt{\left(-\frac{d_1 - d_2}{2l^2}\right)u_{d1}^2 + \left(\frac{d_1 - d_2}{2l^2}\right)u_{d2}^2 + \left(-\frac{l_1 - l_2}{2l^2}\right)u_{l1}^2 + \left(-\frac{l_1 - l_2}{2l^2}\right)u_{l2}^2}$$

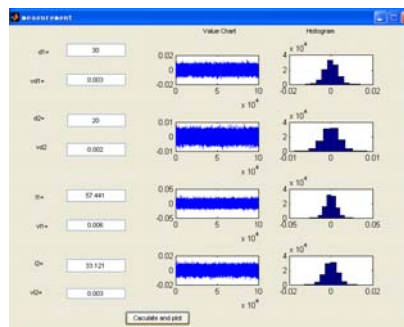


Fig.5 probability distribution of the measuring value

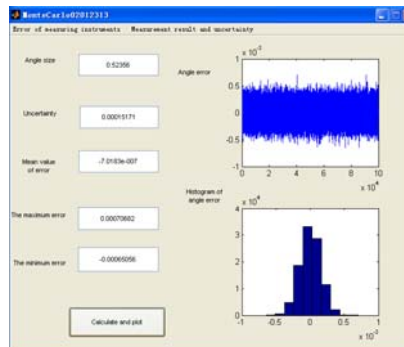


Fig.6 Monte Carlo simulation results of the first measurement process

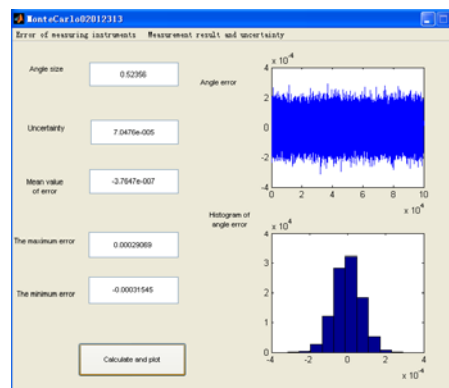


Fig.7 Monte Carlo simulation results of the improved measurement process

The diameters of the balls in the improved measurement program were  $d_1 = 45$ ,  $d_2 = 15$ , we can give the data as follows:

$$d_1 = 45.00, \\ u_{d1} = \sigma_{d1} = 0.0023, \\ d_2 = 15.00,$$

$$u_{d2} = \sigma_{d2} = 0.0008,$$

$$l_1 = 93.921,$$

$$u_{l1} = \sigma_{l1} = 0.0047,$$

$$l_2 = 20.961,$$

$$u_{l2} = \sigma_{l2} = 0.001.$$

The cone angle in the improved measurement program is 0.52rad, and the combined standard uncertainty is  $7.05 \times 10^{-5}$  rad. Because  $U_{E2} (7.05 \times 10^{-5} \text{ rad}) \leq U_T (1.0 \times 10^{-4} \text{ rad})$ , so the improved measurement process can meet the requirement.

### Conclusions

In this paper, Monte Carlo method for the estimation of uncertainty is proposed. We took the indirect measurement of the cone angle as an experiment, established the mathematical relationship model between the measurand and each uncertainty contributor firstly, and then conducted the sampling and synthesis of uncertainty contributors by Monte Carlo simulation. We applied Matlab as a platform to develop a prototype system of Monte Carlo simulation method for measurement uncertainty estimation. Finally, we evaluated and improved the measurement program based on uncertainty management process (PUMA), which is given in the next generation of GPS. The experimental results show that the Monte Carlo simulation method has a

good prospect in the uncertainty estimation and measurement program design.

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