Behavior Consistency Analysis Based on the Behavior Profile about Transition Multi-Set of Petri Net

Abstract. Duplicate activities often appear in the business process modeling, analyzing the consistency of corresponding model containing duplicate activities is a problem, the existing behavior consistency methods can not analyze effectively the process model with the multi-set of transition. In the paper, by analyzing of three kinds of weak order relations of multi set of transition, a kind of consistency measure methods based on behavior profile of multi-sets of transitions of Petri net is proposed. Finally, an example is given out, which shows the method is effective.

Streszczenie. W artykule przedstawiono metodę określenia regularności związków słabych w systemie wielowątkowym o powielających się danych. Analiza, trzech wybranych rodzajów relacji oparta została na profilu wielowątkowej sieci Petriego. Opisano także przykład, potwierdzający skuteczność działania (Analiza regularność zachowań, na podstawie profilu zachowań w sieci wielowątkowych Petriego).

Keywords: Behavior profile, Petri net, Multi-sets of transition, Consistency.

Introduction

In business processes, duplicate activities often appear, which sometimes due to the needs of models, sometimes faults of the business analysts and system modelers. In this case, it’s too much trouble to delete these repetitive activities or refine them, sometimes we only need to analyze the relationship between them, and compute its consistency with the target model[1]. At this point, a major challenge in this area is be able to compute the degree of consistency between the duplication of activities of the models.

For consistency measure, some literatures has proposed lots of analysis methods, but the majority was on the basis of the expendable transitions, [2] describes the observed consistency, and it considered not only the transitions but also the state. Using trace equivalence or bisimulation to analyze the behavior consistency between the models, there are some problems [3]. Trace equivalence or bisimulation can only produce a Boolean value that is consistent with (a value of 1), inconsistent (value 0) to show consistency, but can not explain the inconsistent extent [4]. [5] proposed the behavior constraint of a weaker equivalence than trace equivalence - behavior profiles, so it can visually see the degree of similarity between the models, such that we can determine the match of the models. But they can not handle the duplication activities in models. While for multi-sets, [6] proposes the method of measuring the behavior precision and behavior recall, but which has no specified answer to the degree of consistency. On this basis, [7] introduced a quantitative method based on observed behavior, which only considers the order relation, but also can not effectively distinguish the difference between repeated transitions.

Based on the above background, we present a consistency analysis method based on behavior profiles of multi-set of transitions of petri net.

Motivating Example

Here we look at an example of a work flow system, as shown in the Fig.1. In the Fig.1, if we want to describe the degree of consistency between the four models, what can we do? We find that there are two same transitions in Fig.1 (b), Fig.1 (c), Fig.1 (d), so there is a problem for the same transitions, while the previous literatures have not described in such cases. Therefore, a major challenge is be able to calculate the match between the duplication of activities of the model.

For the remaining part of the article, under no explanation, we assume that the workflow system network is defined as \( N = (P, T, F) \). Workflow system means that the establishment of a process model, therefore, we also use activities and transitions.

**Fig. 1. The Petri net model of workflow system**

The Calculation of the Consistency Based on the Transition Multi-Sets

Consistency assessment may be based on behavior equivalences or behavior relations, while profile based on behavior equivalences can be expected to be computationally hard [8], so we need to study the behavior relations. All relations for tuples of transitions in one model are compared to the relations for tuples of corresponding transitions in the other model. Hence, consistency assessment translates into comparing relations for all pairs of transitions that are part of correspondences. [9] reviews several relational semantics. The major difference between them is their focus on either direct causal dependencies or indirect dependencies, such as the behavior profile[4,5,8]. Against the background of alignments between related process models, indirect dependencies seem to be more suited to assess consistency. Our approach, on the basis of behavior profiles for the multi-sets of transitions, considers three weak order relations between multi-sets of transitions, identifying consistent aligned transitions, to analyze the consistency between them.
Definition 1. Let \((N_i[i])\) be a net system with \(N = (P, L, F)\). The weak order relation \(\succ\subseteq L \times L\) contains all pairs \((x, y)\), such that there exists a firing sequence \(\sigma = t_1 \cdots t_n\) with \((N_i[i]()[\sigma], j \in \{1, \ldots, n-1\}\), and \(j < k \leq n\), for which holds \(t_j = x\) and \(t_k = y\).

Definition 2. Let \((N_i[i])\) be a WF-system. The strict order relation \(\to\subseteq L \times L\) contains all pairs \((x, y)\) with \(x \succ y\) and \(y \not\succ x\).

In Fig.1(a), only when A occurs, B and C can occur, therefore, \(A \rightarrow B\), \(A \rightarrow C\). Similarly, for Fig.1(b), the left A and B is in a strict order relation.

Definition 3. Let \((N_i[i])\) be a WF-system. The exclusiveness relation \(\ll\subseteq L \times L\) contains all pairs \((x, y)\) with \(x \ll y\) and \(y \not\ll x\).

Definition 4. Let \((N_i[i])\) be a WF-system. The interleaving order relation \(\parallel\subseteq L \times L\) contains all pairs \((x, y)\) with \(x \parallel y\) and \(y \parallel x\).

Definition 5. For a WF-system \((N_i[i])\) the set of behavior profiles \(MBP_i = 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parallel problem (1) By the definition 2-4, analyze the relationship between the transitions, and determine the transition satisfying the behavior profile.

(2) By definition 6, in \(L_1\), loop from \(t_{i1}\) to \(t_{im}\), look for \(t_j\) in \(L_2\), if there is \(t_j \in L_2\), such that \(t_{i1} \sim t_j\), output \(\tilde{L}_1 = L_1\), otherwise remove \(t_{i1}\) from \(L_1\), output \(\tilde{L}_1 = L_1 \setminus \{t_{i1}\}\), to locate the set of aligned transitions.

(3) By definition 7, analyze \((t_{i1}, t_{j2}) \in \tilde{L}_1 \times \tilde{L}_2\), with \(t_{i1} \sim t_{j2}\) \((1 \leq m \leq n)\), if it meets \((t_{i1}, t_{j2}) \in L_1 \times L_2\), otherwise, remove \((t_{i1}, t_{j2})\) from \(\tilde{L}_1 \times \tilde{L}_2\), output \(\tilde{L}_1 \setminus \tilde{L}_2\), \((t_{i1}, t_{j2})\), to locate the set of consistent transitions.

Definition 8. Let \((N_i[i])\) and \((N_j[i])\) be net systems with \(N_i = (P_i, L_i, F_i)\) and \(N_j = (P_j, L_j, F_j)\), and \(L_1, L_2\) a correspondence relation for \(T_1, T_2\) (with \(T_1 \subseteq L_1\), \(T_2 \subseteq L_2\)), respectively. \(C_{\tilde{L}_1} \subseteq (\tilde{L}_1 \times \tilde{L}_2)\) and \(C_{\tilde{L}_2} \subseteq (\tilde{L}_1 \times \tilde{L}_2)\) a set of consistent transition pairs based on the multi-set of transitions. The degree of behavior profile consistency is defined as:

\[ MBP_\ell = \frac{\omega_1 |(\tilde{L}_1 \times \tilde{L}_2)|}{|\tilde{L}_1| + |\tilde{L}_2|} \]

Here:

\[ \omega_1 = \sum_{(t_i, t_j) \in C_{\tilde{L}_1}} (\tilde{z}(t_i) + \tilde{z}(t_j)) \]

\[ \omega_2 = \sum_{(t_i, t_j) \in C_{\tilde{L}_2}} (\tilde{z}(t_i) + \tilde{z}(t_j)) \]

\[ \tilde{z}(t_i) = \frac{1}{|\{t_j \in T_2 : (t_i, t_j) \in C_{\tilde{L}_1}\}|} \]

\(\sim\) (\(\sim\) is an alignment with \(\sim \subseteq L_1 \times L_2\).) According to the definition 8, we can compute the the degree of behavior consistency of two models with the multi-set of the transition. In the calculation process, we need compute the \(\tilde{z}(t_i)\) and the weight \(\omega\), then obtain the degree of the behavior consistency.

Algorithm 2: calculation for \(\tilde{z}(t_i)\).

Input: Two workflow systems \(S_i = (N_i[i])\) and \(S_j = (N_j[i])\), the sets of aligned transitions \(\tilde{L}_1 = \{t_{i1}, t_{i2}, \ldots, t_{im}\}\) and \(\tilde{L}_2 = \{t_{i1}, t_{j2}, \ldots, t_{jn}\}\) (obtained from Algorithm 2), the sets of consistent transitions \(C_{\tilde{L}_1} = \{t_{i1}, t_{i2}, \ldots, t_{im}\}\) and \(C_{\tilde{L}_2} = \{t_{i1}, t_{j2}, \ldots, t_{jn}\}\).

Output: \(\tilde{z}(t_i)\).
(1) Combine \( t_1, t_2, \ldots, t_n \) for each other, output results that \( C_l^t = \{ (t_1, t_2), \ldots, (t_k, t_{k+1}) \} \), and then perform steps (2).

(2) By definition 6, analyze \((t_s, t_s)\) if it meets the requirements of definition 7; output \( \bar{\tau}(t_s) = 1 \), then return to step (3); otherwise output \( \bar{\tau}(t_s) = 1 - \frac{1}{n} \) (where \( n \) is the total number which meets \((t_s, t_s) \in \mathcal{E})\), then return to step (3).

(3) By definition 6, analyze \((t_s, t_s)\) if it meets the requirements of definition 7; output \( \bar{\tau}(t_s) = \bar{\tau}(t_s) \), then return to step (4); otherwise output \( \bar{\tau}(t_s) = 1 - \frac{1}{n} \), then return to step (4).

(4) Similarly, repeat above steps, until the last transition pair \((t_s, t_s)\), if it meets the requirements of definition 7; output \( \bar{\tau}(t_s) = \bar{\tau}(t_s) \), the algorithm terminates; otherwise output \( \bar{\tau}(t_s) = 1 - \frac{1}{n} \), the algorithm terminates.

Algorithm 3: calculation for degree of the consistency.

Input: two workflow systems \( S_1 = (N, \mathcal{E}_1) \), \( S_2 = (N, \mathcal{E}_2) \), the set of aligned transitions \( \bar{\mathcal{E}_i} = \{ (t_{i1}, t_{i2}), \ldots, t_{ik} \} \), the set of consistent transitions \( C_l^t = \{ (t, t), \ldots, (t, t) \} \), two functions \( \bar{\tau}(t) \), \( \bar{\tau}(t) \).

Output: the degree of the consistency \( MBP \).

(1) By Algorithm 2, calculate the total number of \( \bar{\tau}(t) + \bar{\tau}(t) \), called \( m \), with \((t, t) \in \mathcal{E}_1 \times \mathcal{E}_1 \), then return to step (2).

(2) By Algorithm 2, calculate the total number of \( \bar{\tau}(t) + \bar{\tau}(t) \), called \( l \), with \((t, t) \in \mathcal{E}_1 \times \mathcal{E}_1 \), then return to step (3).

(3) \( \alpha_1 = \frac{1}{m} \), then according to the definition 8, compute the \( MBP = \frac{\alpha_1 \times m_1 + \alpha_2 \times m_2}{m_1 + m_2} \).

(4) Compute \( \bar{\mathcal{E}_1} \times \mathcal{E}_1 \) and \( \bar{\mathcal{E}_1} \times \mathcal{E}_1 \) respectively, obtain \( m_1 = \bar{\mathcal{E}_1} \times \mathcal{E}_1 \) and \( m_2 = \bar{\mathcal{E}_1} \times \mathcal{E}_1 \), then according to the definition 8, compute the \( MBP = \frac{\alpha_1 \times m_1 + \alpha_2 \times m_2}{m_1 + m_2} \).

Case Study

In Fig. 1 (a) and Fig. 1 (b), by Algorithm 1, we can see that the set of transitions meeting the behavior profile is \( L_{pr} = \{ A, B, C, D \} \), \( L_{pr} = \{ A, B, C, D \} \), respectively. The sets of aligned transitions of the two models \( \bar{\mathcal{E}_1} = \{ A, B, C, D \} \) and \( \bar{\mathcal{E}_1} = \{ A, B, C, D \} \) respectively, and \( \bar{\mathcal{E}_1} \times \mathcal{E}_1 = 16 \) and \( \bar{\mathcal{E}_1} \times \mathcal{E}_1 = 25 \), so the sets of consistent transitions are \( C_l^t = \{ A, B, C, D \} \) and \( C_l^t = \{ B, C, D \} \), respectively. Finally, according to Algorithm 2, we can compute the weight \( \alpha_1 \) and \( \alpha_2 \) of the two models, where \( \alpha_1 = \frac{15}{16}, \alpha_2 = \frac{23}{25} \), the degree of behavior profiles consistency is \( MBP = \frac{15}{16} \), \( \frac{23}{25} \), \( \frac{25}{16} + \frac{25}{25} \) = 0.927.

Similarly, in Fig. 1 (a) and Fig. 1 (c), the degree of behavior profile consistency is about 0.927. In Fig. 1 (a) and Fig. 1 (d), the degree of behavior profile consistency is about 0.885. In Fig. 1 (b) and Fig. 1 (c), the degree is about 0.9. The compliance degree between Fig. 1 (b) and Fig. 1 (d) is about 0.918; in Fig. 1 (c) and Fig. 1 (d), the degree is about 0.869.

Conclusions

In this paper, on the basis of previous study, we extend the behavior profile. Our contribution is the definition of multi-sets of transitions; for the set of transitions with the same name in the workflow system, and on this basis to extend the three weak order relations of the behavior constraints, and to propose measurement based on the behavior constraints of the multi-sets of transitions, and to use six algorithms to obtain transitions meeting the relations of behavior profile, aligned transitions, consistent transitions, function, weights and the degree of behavior profiles consistency, respectively. In the future, we plan to study the measuring causal behavior profile based on multi-sets of transitions.

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