Xinqin Gao[1], Xueping Wang[2], Mingshun Yang[1], and Yong Liu[1]

Xi'an University of Technology (1), Xi'an Jiaotong University (2)

Workflow Region Recognition Algorithm and Its Time Complexity

***Abstract**. Based on polychromatic sets theory, a new workflow model – special net structure model (SNS model) is proposed, and a concept of workflow region is defined. And then, the recognition algorithm of workflow region is presented and its time complexity is analyzed quantitatively. The compound workflow SNS model can be reduced level by level from inner to outer by invoking the recognition algorithm of workflow region. This recognition algorithm is executed only once for a lot of workflow dynamic changes. Therefore, the computing complexity is feasible and acceptable.*

***Streszczenie.** W artykule przedstawiono propozycję nowego modelu pracy strumieniowej SNS (ang. Special Net Structure), opartego na teorii ustawień polichromatycznych oraz koncepcję obszaru pracy strumieniowej. Przeprowadzono analizę działania algorytmu rozpoznawania regionu pracy strumieniowej, jego skuteczności i stopnia komplikacji. Uzyskane wyniki wskazują na jego pełną wykonalność. (**Sposób działania i złożoność czasowa algorytmu rozpoznawania obszaru pracy strumieniowej**).*

Keywords: workflow region, recognition algorithm, time complexity, polychromatic sets theory
Słowa kluczowe: obszar pracy strumieniowej, algorytm rozpoznawania, złożoność czasowa, teoria ustawień polichromatycznych.

Introduction

Workflow is defined as the automation of a business process, in whole or part. Workflow management technology offers a promising approach for realizing process-aware information systems. Recently, workflow dynamic changes management has emerged as the most important issue in the research area of workflow. Compared with the workflow modeling, analysis and verification, however, workflow instance migration supporting dynamic changes remains an unsolved problem completely [1,2]. In fact, the workflow region recognition is the precondition of selecting workflow instance migration policies.

Ellis *et al*. defined change region as the part of the net containing all the activities directly affected by the change, and proposed three policies for dynamic change of workflow: Flush, Abort, and SCO (Synthetic Cut-Over), which kept consistency of workflow [3]. However, the authors did not provide a method for identifying the change regions, i.e. change regions did need to be identified manually. Based on workflow nets (WF-nets), Van der Aalst proposed a term dynamic change region and its generation algorithm [4]. There was no need to computer the dynamic change region for individual cases and any case not marking the dynamic change region could be transferred without jeopardizing the correctness criteria. However, this was a sufficient but not necessary condition and the implementation of the algorithm was too complex.

Gao and Li proposed a business process integrated modeling method based on UML and polychromatic sets [5]. Furthermore, Gao *et al*. established the workflow dynamic change models and described formally the frequent dynamic changes, such as addition, deletion and construction transformation of nodes/branches [6]. On the basis of these achievements, this paper focuses on the workflow region recogintion algorithm and its time complesity analysis.

Polychromatic sets theory

The traditional set theory is unable to represent the different characteristics of set itself and its elements. Similarly, the traditional graph theory does not describe the diverse properties of nodes and edges. These shortcomings restrict their application ranges in complex systems modeling. The idea of extending and improving the traditional set theory and traditional graph theory has been proposed and some theoretical and practical achievements have been gained. One of them is polychromatic sets theory (PST) proposed by V.V. Pavlov [7]. PST is a relatively new mathematics theory and information processing tool and has two important concepts: polychromatic set (PS) and polychromatic graph (PG). More details on polychromatic set theory can be found in the reference [8].

Workflow SNS model

In the engineering practice, a special net structure (SNS) is often used. By using prior node and next node, the nodes of SNS can be classified into start node, end node and activity node. From the compositions of SNS, every node itself need not be pigmented any color. The edges of SNS not only describe whether the interrelation exists in any two nodes, but also can be pigmented a certain color to represent a connected kind of interrelation existing in any two connected nodes. In fact, the SNS is a concolorous graph with colorless nodes and concolorous edges, and also is a special polychromatic graph, which can be described by polychromatic sets theory formally.

According to the definition of polychromatic graph, the unified pigmentation of SNS, PS of nodes and PS of edges can be described respectively as follows:

(1) $$F(G) = F(C)$$

(2) $$PS_A = A$$

(3) $$PS_C = (C, F(C), [C\times F(C)])$$

where: A – node set, C – edge set, $F(C)$ – edge unified pigmentation, $[C\times F(C)]$ – relationship of edge set and unified pigmentation.

Therefore, SNS can be described formally in polychromatic graph as follows:

(4) $$PG_{SNS} = (A, C, F(C), [C\times F(C)])$$

A is node set of SNS, written as

(5) $$A = \{a_i \mid 1 \le i \le n\}$$

C is edge set of SNS, written as

(6) $$C = \{c_{i,j} \mid 1 \le i, j \le n\}$$

In the practical operation, the edge set of SNS is computed by using Boolean matrix $[A\times A]$ as follows:

(7) $$[A\times A] = \begin{array}{c} \begin{array}{ccccc} a_1 & \cdots & a_j & \cdots & a_n \end{array} \\ \begin{bmatrix} c_{11} & \cdots & c_{1j} & \cdots & c_{1n} \\ \vdots & \cdots & \vdots & \cdots & \vdots \\ c_{i1} & \cdots & c_{ij} & \cdots & c_{in} \\ \vdots & \cdots & \vdots & \cdots & \vdots \\ c_{n1} & \cdots & c_{nj} & \cdots & c_{nn} \end{bmatrix} \end{array} \begin{array}{c} a_1 \\ \vdots \\ a_i \\ \vdots \\ a_n \end{array}$$

All elements $c_{ij} = 1$ constitute a set, which is the C, edge set of SNS.

$F(C)$ is edge unified pigmentation of SNS, which represents a set of interrelation types existing in any two connected nodes and can be written as:

(8) $$F(C) = \{F_k \mid 1 \leq k \leq m\}$$

$[C\times F(C)]$ represents the relationship of edge set C and unified pigmentation $F(C)$ and can be written as:

(9) $$\left[C\times F(C)\right] = \begin{array}{c} \begin{matrix} F_1 & \cdots & F_k & \cdots & F_m \end{matrix} \\ \begin{bmatrix} r_{11} & \cdots & r_{1k} & \cdots & r_{1m} \\ \vdots & \cdots & \vdots & \cdots & \vdots \\ r_{i1} & \cdots & r_{ik} & \cdots & r_{im} \\ \vdots & \cdots & \vdots & \cdots & \vdots \\ r_{n1} & \cdots & r_{nk} & \cdots & r_{nm} \end{bmatrix} \end{array} \begin{matrix} c_{1,2} \\ \vdots \\ c_{i,j} \\ \vdots \\ c_{n-1,n} \end{matrix}$$

In the Boolean matrix $[C\times F(C)]$, if $c_{i,j}$, the edge of SNS, is pigmented a color F_k, a certain interrelation type, the Boolean element r_{ik} is 1.

According to equations (5)-(9), SNS can be described further in polychromatic graph as follows:

(10) $$PG_{SNS} = [(A\times A)\times F(C)]$$

Where: $(A\times A)$ – Cartesian product of the nodes set A and itself, $F(C)$ – edge unified pigmentation of SNS.

The workflow model is a formal expression of business process. In the workflow SNS model, basic model primitives have two kinds:

Nodes: start node, end node and activity node. All nodes constitute a node set A. Their graphical symbols are "●", "◉" and "⬭" respectively.

Edges: interrelations between any two nodes. All interrelations constitute an edge set C. SNS is a special polychromatic graph with colorless nodes and concolorous edges. The unified color F_k represents the interrelation types existing in any two connected nodes, such as link-arc, and-split, and-join, or-split and or-join. These interrelation types constitute the edge unified pigmentation $F(C)$, represented as

(11) $$F(C) = (F_1, F_2, F_3, F_4, F_5)$$

The graphical symbols of five interrelation types are "—>—", "—+>—", "—>+—", "—◇>—" and "—>◇—" respectively.

Applying the above basic model primitives to describe the workflow process, four basic model constructs can be gained: sequential construct, parallel construct, choice construct and iterative construct, which can constitute more complicated workflow model.

Workflow region

Compared with nodes and basic model constructs of workflow SNS model, workflow region is a higher-level, abstract and generalized concept and has relativity. It may be an activity node, a basic model construct, a nesting of several basic model constructs, even or a complex workflow model. During of instance migration, two types of workflow regions are identified, namely the current running region and the dynamic change region. The current running regions refer to the current running node a_i and the basic model constructs that include the current running node a_i. The dynamic change regions refer to the dynamic change node a_j and the basic model constructs that include the dynamic change node a_j.

Fig.1 shows a partial workflow SNS model, which is given to illustrate the concept of workflow region. Where, $V_1=(a_2\otimes a_3\otimes a_4)$, $V_2=(a_1//V_1//a_5)$, $V_3=(V_2||a_6)$. If the node a_2 is the current running node, the node itself a_2, the parallel construct V_1, the sequential construct V_2, and iterative construct V_3 are all the different-level current running regions of current running node a_2, and they can be reasoned from inner to outer, i.e. $a_2\subset V_1\subset V_2\subset V_3$. If the node a_6 is the dynamic change node, the node itself a_6 and iterative construct V_3 are all the different-level dynamic change regions of dynamic change node a_6, and they can be reasoned from inner to outer, i.e. $a_6\subset V_3$. Here, the iterative construct V_3 is the common workflow region of current running node a_2 and dynamic change node a_6.

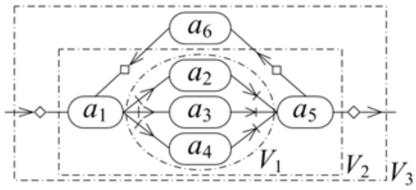

Fig.1. Workflow region

Recognition algorithm

In workflow regions, it is necessary to distinguish three types of relational symbols.

Operation relational symbols, such as "//", "⊗", "⊕" and "||". The operation of workflow regions constitutes a basic model construct.

Inclusion relational symbols, such as "=", "⊂" and "⊃". Inclusion relations can implement the reasoning of workflow regions level by level from inner to outer.

Position relational symbols, such as ">>", "<<" and "≠≠". In order to avoid confusion, position relational expression is placed into a bracket, such as [V_i>>V_j].

Based on Boolean operations of polychromatic sets, the reduction rules of basic model constructs are proposed [9].

According to the definition of workflow region and the reduction rules of basic model constructs, the recognition flowchart of workflow region is presented as shown in Fig. 2, and its main steps are described as follows.

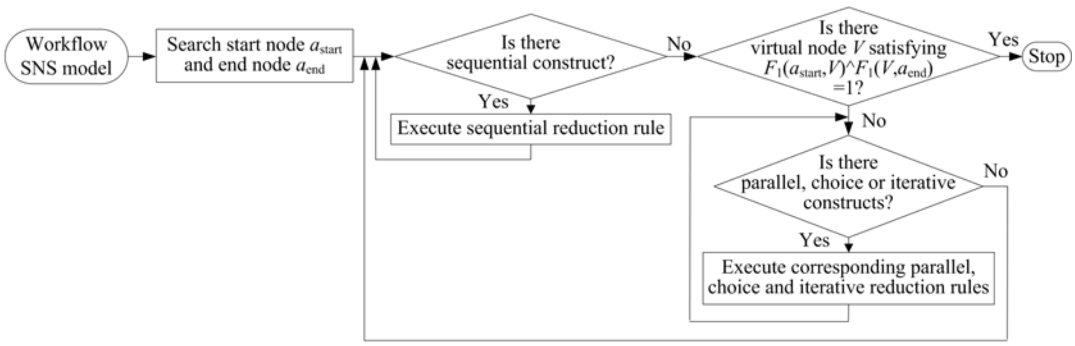

Fig.2. Recognition flowchart of workflow region

Step 1. Search the workflow SNS model to identify all sequential constructs and execute sequential reduction rules (excluding the start node a_{start} and the end node a_{end}).

Step 2. Search the reduced workflow SNS model to identify parallel constructs, choice constructs, and iterative constructs respectively, and execute corresponding reduction rules.

Step 3. Execute the above two steps repeatedly, until the result meets the condition that the start node directly links with the end node by a virtual node.

The recognition algorithm pseudo-codes of workflow region are listed as follows.

Time complexity analysis

In workflow SNS model with n nodes, the number of combinations of any two nodes (a_i, a_j) is n(n-1) at most. The execution number of each minor cycle in Line 04, 08, 09, 10 is all n(n-1) at most. Therefore, the complexity of major cycle from Line 02 to Line 12 is as follows:

```
BEGIN
//PG_SNS is workflow SNS model, a_start and a_end are start node and end node, and C_i and R_i (i=1,2,3,4) are reduction conditions and reduction rules of sequential, parallel, choice, iterative constructs.
01. Initialize PG_SNS; a_start; a_end
02. WHILE (TRUE)
03. {
04.     WHILE C1 DO R1
05.     IF F1(a_start,V)∧F1(V,a_end)=1
06.         BREAK
07.     ELSE
08.     WHILE C2 DO R2
09.     WHILE C3 DO R3
10.     WHILE C4 DO R4
11.     END IF
12. }
END
```

$$O\big(F(n)\big)=O\big(n(n-1)[n(n-1)+n(n-1)+n(n-1)]\big)=O\big(3n^2(n-1)^2\big) \quad (12)$$

In the worst case, an execution of major cycle can only reduce two nodes and the execution number of major cycle is n/2 at most. The approximate complexity of recognition algorithm of workflow region is as follows:

$$O\big(G(n)\big)=O\left(F(n)*\frac{n}{2}\right)=O\left(\frac{3n^3}{2}(n-1)^2\right)\approx O\big(n^5\big) \quad (13)$$

The above complexity analysis of recognition algorithm of workflow region is the worst case. In fact, the execution numbers of minor cycle and major cycle are impossible to achieve n(n-1) and n/2. Furthermore, they are opposite each other. The more the execution number of minor cycle is, the less the execution number of major cycle is.

In this paper, we suppose that the old workflow model, the new workflow model and the compound workflow model are all sound modes, i.e. no structural conflict exists in them. After executing recognition algorithm of workflow region, it is inevitable that workflow SNS model is reduced level by level from inner to outer until the start node directly links with the end node by a virtual node. Hence we obtain various virtual nodes, i.e. workflow regions. Several workflow instances of the same workflow SNS model may be active at the same time. However, the recognition algorithm of workflow region is executed only once for all changes. That is to say, there is no need to compute the workflow regions for individual instances.

Conclusions

The workflow region recognition is the precondition of selecting workflow instance migration policies. Based on polychromatic sets theory, a new workflow model – special net structure model (SNS model) and a concept of workflow region are proposed. Further, the recognition algorithm of workflow region is presented and its time complexity is analyzed quantitatively.

The workflow region recognition algorithm is based on polychromatic sets theory and has the well-defined mathematical foundation and the significant progress and advantage in problem formalization and universality. The recognition algorithm of workflow region is executed only once for all changes, i.e. there is no need to compute the workflow regions for individual instances. Therefore, the computing complexity is feasible and acceptable.

Acknowledgments

The authors gratefully acknowledge the support of the National Natural Science Foundation, China (No. 60903124), the Specialized Research Fund for the Doctorial Program of Higher Education, China (No. 20096118120003), the China Postdoctoral Science Foundation, China (No. 20100480080), the Research and Development Program of Shaanxi Province for Young Science & Technology Star, China (No. 2012KJXX-34), and the Scientific Research Plan of Shannxi Provincial Education Department, China (No. 12JK0659).

REFERENCES

[1] Lin H., Fan Y., Newman S., Manufacturing process analysis with support of workflow modelling and simulation, International Journal of Production Research, 47 (2009), No. 7, 1773-1790

[2] Pan H.L., Wu J.Q., Wu C.H., A workflow instance migration algorithm for improving time complexity, Communications in Computer and Information Science, 153 (2011), 314-320

[3] Ellis C., Keddara K., Rozenberg G., Dynamic change within workflow systems, Proceedings of the ACM Conference on Organizational Computing Systems, (1995), 10-18

[4] Van der Aalst W.M.P., Exterminating the dynamic change bug: a concrete approach to support workflow change, Information Systems Forntiers, 3 (2001), No. 3, 297-317

[5] Gao X., Li Z., Business process modeling and analysis using UML and polychromatic sets, Production Planning & Control, 17 (2006), No. 8, 780-791

[6] Gao X., Li Z., Du X., Dynamic change model and instance migration approach for workflow management system, Computer Integrated Manufacturing Systems, 14 (2008), No. 8, 1521-1528

[7] Pavlov V.V., Polychromatic Sets and Graphs for CALS, Moscow, Russia: STANKIN Press, 2002

[8] Zhang Y., Li Z., Xu L., Wang J., A new method for automatic synthesis of tolerances for complex assemblies based on polychromatic sets, Enterprise Information Systems, 5 (2011), No. 3, 337-358

[9] Gao X., Li Z., Li L., A process model for concurrent design in manufacturing enterprise information systems, Enterprise Information Systems, 2 (2008), No. 1, 33–46

***Authors**: dr. Xinqin Gao, School of Mechanical and Precision Instrument Engineering, Xi'an University of Technology, South Jinhua Road 5, 710048 Xi'an, China, E-mail: xinqingao@gmail.com; dr. Xueping Wang, School of Economics and Finance, Xi'an Jiaotong University, West Yanta Road 74, 710061 Xi'an, China, E-mail: wangxueping006@xjtu.edu.cn; dr. Mingshun Yang, School of Mechanical and Precision Instrument Engineering, Xi'an University of Technology, South Jinhua Road 5, 710048 Xi'an, China, E-mail: yangmingshun@xaut.edu.cn; dr. Yong Liu, School of Mechanical and Precision Instrument Engineering, Xi'an University of Technology, South Jinhua Road 5, 710048 Xi'an, China, E-mail: liuyong@xaut.edu.cn.*