Workflow Region Recognition Algorithm and Its Time Complexity

Abstract. Based on polychromatic sets theory, a new workflow model – special net structure model (SNS model) is proposed, and a concept of workflow region is defined. And then, the recognition algorithm of workflow region is presented and its time complexity is analyzed quantitatively. The compound workflow SNS model can be reduced level by level from inner to outer by invoking the recognition algorithm of workflow region. This recognition algorithm is executed only once for a lot of workflow dynamic changes. Therefore, the computing complexity is feasible and acceptable.


Keywords: workflow region, recognition algorithm, time complexity, polychromatic sets theory
Słowa kluczowe: obszar pracy strumieniowej, algorytm rozpoznawania, złożoność czasowa, teoria ustawień polichromatycznych.

Introduction
Workflow is defined as the automation of a business process, in whole or part. Workflow management technology offers a promising approach for realizing process-aware information systems. Recently, workflow dynamic changes management has emerged as the most important issue in the research area of workflow. Compared with the workflow modeling, analysis and verification, however, workflow instance migration supporting dynamic changes remains an unsolved problem completely [1,2]. In fact, the workflow region recognition is the precondition of selecting workflow instance migration policies.

Ellis et al. defined change region as the part of the net containing all the activities directly affected by the change, and proposed three policies for dynamic change of workflow: Flush, Abort, and SCO (Synthetic Cut-Over), which kept consistency of workflow [3]. However, the authors did not provide a method for identifying the change regions, i.e. change regions did need to be identified manually. Based on workflow nets (WF-nets), Van der Aalst proposed a term dynamic change region and its generation algorithm [4]. There was no need to computer the dynamic change region for individual cases and any case not marking the dynamic change region could be transferred without jeopardizing the correctness criteria. However, this was a sufficient but not necessary condition and the implementation of the algorithm was too complex.

Gao and Li proposed a business process integrated modeling method based on UML and polychromatic sets [5]. Furthermore, Gao et al. established the workflow dynamic change models and described formally the frequent dynamic changes, such as addition, deletion and construction transformation of nodes/branches [6]. On the basis of these achievements, this paper focuses on the workflow region recognition algorithm and its time complexity analysis.

Polychromatic sets theory
The traditional set theory is unable to represent the different characteristics of set itself and its elements. Similarly, the traditional graph theory does not describe the diverse properties of nodes and edges. These shortcomings restrict their application ranges in complex systems modeling. The idea of extending and improving the traditional set theory and traditional graph theory has been proposed and some theoretical and practical achievements have been gained. One of them is polychromatic sets theory (PST) proposed by V.V. Pavlov [7]. PST is a relatively new mathematics theory and information processing tool and has two important concepts: polychromatic set (PS) and polychromatic graph (PG). More details on polychromatic set theory can be found in the reference [8].

Workflow SNS model
In the engineering practice, a special net structure (SNS) is often used. By using prior node and next node, the nodes of SNS can be classified into start node, end node and activity node. From the compositions of SNS, every node itself need not be pigmented any color. The edges of SNS not only describe whether the interrelation exists in any two nodes, but also can be pigmented a certain color to represent a connected kind of interrelation existing in any two connected nodes. In fact, the SNS is a concolorous graph with colorless nodes and concolorous edges, and also is a special polychromatic graph, which can be described by polychromatic sets theory formally.

According to the definition of polychromatic graph, the unified pigmentation of SNS, PS of nodes and PS of edges can be described respectively as follows:

- $F(G) = F(C)$
- $PS_A = A$
- $PS_C = (C, F(C), \{C \times F(C)\})$

where: $A$ – node set, $C$ – edge set, $F(C)$ – edge unified pigmentation, $\{C \times F(C)\}$ – relationship of edge set and unified pigmentation.

Therefore, SNS can be described formally in polychromatic graph as follows:

- $PG_{SNS} = (A, C, F(C), \{C \times F(C)\})$

$A$ is node set of SNS, written as

- $A = \{a_i \mid 1 \leq i \leq n\}$

$C$ is edge set of SNS, written as

- $C = \{c_{ij} \mid 1 \leq i, j \leq n\}$

In the practical operation, the edge set of SNS is computed by using Boolean matrix $[A \times A]$ as follows:

$$[A \times A] = \begin{bmatrix}
    a_1 & \cdots & a_1 & \cdots & a_i & \cdots & a_n \\
    c_{11} & c_{12} & \cdots & c_{1i} & \cdots & c_{1n} & a_1 \\
    \vdots & \vdots & \cdots & \vdots & \cdots & \vdots & \vdots \\
    c_{n1} & c_{n2} & \cdots & c_{ni} & \cdots & c_{nn} & a_n \\
\end{bmatrix}$$
All elements $c_{ij} = 1$ constitute a set, which is the edge set of SNS. $F(C)$ is edge unified pigmentation of SNS, which represents a set of interrelation types existing in any two connected nodes and can be written as:

$$F(C) = \{F_k | 1 \leq k \leq m\}$$

$(9)$

$[C \times F(C)]$ represents the relationship of edge set $C$ and unified pigmentation $F(C)$ and can be written as:

$$[C \times F(C)] = \begin{bmatrix} F_1 & \cdots & F_k & \cdots & F_m \\ r_{11} & \cdots & r_{1k} & \cdots & r_{1m} \\ \vdots & \cdots & \vdots & \cdots & \vdots \\ r_{k1} & \cdots & r_{kk} & \cdots & r_{km} \\ \vdots & \cdots & \vdots & \cdots & \vdots \\ r_{mk} & \cdots & r_{mk} & \cdots & r_{mm} \end{bmatrix} c_{ij}$$

$(9)$

In the Boolean matrix $[C \times F(C)]$, if $c_{ij}$ is 1, the edge of SNS, is pigmented a color $F_i$, a certain interrelation type, the Boolean element $r_{ij}$ is 1.

According to equations (5)-(9), SNS can be described further in polychromatic graph as follows:

$$PG_{SNS} = [((A \times A) \times F(C))$$

$(10)$

Where: $(A \times A)$ – Cartesian product of the nodes set $A$ and itself, $F(C)$ – edge unified pigmentation of SNS.

The workflow model is a formal expression of business process. In the workflow SNS model, basic model primitives have two kinds:

Nodes: start node, end node and activity node. All nodes constitute a node set $A$. Their graphical symbols are $\bullet$, $\circ$ and $\bigcirc$ respectively.

Edges: interrelations between any two nodes. All interrelations constitute an edge set $C$. SNS is a special polychromatic graph with colorless nodes and concolorous edges. The unified color $F_i$ represents the interrelation types existing in any two connected nodes, such as link-arc, and-split, and-join, or-split and or-join. These interrelation types constitute the edge unified pigmentation $F(C)$, represented as

$$F(C) = \{F_1, F_2, F_3, F_4, F_5\}$$

$(11)$

The graphical symbols of five interrelation types are “”, “”, “”, “” and “” respectively.

Applying the above basic model primitives to describe the workflow process, four basic model constructs can be gained: sequential construct, parallel construct, choice construct and iterative construct, which can constitute more complicated workflow model.

**Workflow region**

Compared with nodes and basic model constructs of workflow SNS model, workflow region is a higher-level, abstract and generalized concept and has relativity. It may be an activity node, a basic model construct, a nesting of several basic model constructs, even or a complex workflow model. During of instance migration, two types of workflow regions are identified, namely the current running region and the dynamic change region. The current running regions refer to the current running node $a_i$ and the basic model constructs that include the current running node $a_i$. The dynamic change regions refer to the dynamic change node $a_i$ and the basic model constructs that include the dynamic change node $a_i$.

Fig.1 shows a partial workflow SNS model, which is given to illustrate the concept of workflow region. Where, $V_4=(a_2 \circ a_3 \circ a_4)$, $V_5=(a_4 \circ V_1(a_3))$, $V_1(V_2(a_3))$. If the node $a_2$ is the current running node, the node itself $a_2$, the parallel construct $V_1$, the sequential construct $V_2$, and iterative construct $V_3$ are all the different-level current running regions of current running node $a_2$, and they can be reasoned from inner to outer, i.e. $a_2 \subset V_1 \subset V_2 \subset V_5$. If the node $a_6$ is the dynamic change node, the node itself $a_6$ and iterative construct $V_3$ are all the different-level dynamic change regions of dynamic change node $a_6$, and they can be reasoned from inner to outer, i.e. $a_6 \subset V_3$. Here, the iterative construct $V_3$ is the common workflow region of current running node $a_2$ and dynamic change node $a_6$.

**Recognition algorithm**

In workflow regions, it is necessary to distinguish three types of relational symbols.

Operation relational symbols, such as “$\|$”, “$\otimes$”, “$\oplus$” and “$\|$”. The operation of workflow regions constitutes a basic model construct.

Inclusion relational symbols, such as “$\subset$” and “$\supset$”. Inclusion relations can implement the reasoning of workflow regions level by level from inner to outer.

Position relational symbols, such as “$\gg$”, “$\ll$” and “$\neq$”. In order to avoid confusion, position relational expression is placed into a bracket, such as $[V_{rg} V_r]$. Based on Boolean operations of polychromatic sets, the reduction rules of basic model constructs are proposed [9].

According to the definition of workflow region and the reduction rules of basic model constructs, the recognition flowchart of workflow region is presented as shown in Fig. 2, and its main steps are described as follows.

**Fig.2. Recognition flowchart of workflow region**
Step 1. Search the workflow SNS model to identify all sequential constructs and execute sequential reduction rules (excluding the start node \( a_{\text{start}} \) and the end node \( a_{\text{end}} \)).

Step 2. Search the reduced workflow SNS model to identify parallel constructs, choice constructs, and iterative constructs respectively, and execute corresponding reduction rules.

Step 3. Execute the above two steps repeatedly, until the result meets the condition that the start node directly links with the end node by a virtual node.

The recognition algorithm pseudo-codes of workflow region are listed as follows.

**Time complexity analysis**

In workflow SNS model with \( n \) nodes, the number of combinations of any two nodes \((a, a)\) is \( n(n-1) \) at most. The execution number of each minor cycle in Line 04, 08, 09, 10 sequential constructs and execute sequential reduction is all \( n(n-1) \) at most. Therefore, the complexity of major execution number of each minor cycle in Line 04, 08, 09, 10 is as follows:

\[
O(F(n)) = O\left(\frac{3n^3}{2} - (n-1)^2\right) = O\left(n^3\right)
\]

In the worst case, an execution of major cycle can only reduce two nodes and the execution number of major cycle is \( n/2 \) at most. The approximate complexity of recognition algorithm of workflow region is as follows:

\[
O(G(n)) = O\left(F(n)^n \frac{n^2}{2}\right) = O\left(\frac{3n^3}{2} - (n-1)^2\right) = O\left(n^3\right)
\]

The above complexity analysis of recognition algorithm of workflow region is the worst case. In fact, the execution numbers of minor cycle and major cycle are impossible to achieve \( n(n-1) \) and \( n/2 \). Furthermore, they are opposite each other. The more the execution number of minor cycle is, the less the execution number of major cycle is.

In this paper, we suppose that the old workflow model, the new workflow model and the compound workflow model are all sound modes, i.e. no structural conflict exists in them. After executing recognition algorithm of workflow region, it is inevitable that workflow SNS model is reduced level by level from inner to outer until the start node directly links with the end node by a virtual node. Hence we obtain various virtual nodes, i.e. workflow regions. Several workflow instances of the same workflow SNS model may be active at the same time. However, the recognition algorithm of workflow region is executed only once for all changes. That is to say, there is no need to compute the workflow regions for individual instances.

**Conclusions**

The workflow region recognition is the precondition of selecting workflow instance migration policies. Based on polychromatic sets theory, a new workflow model – special net structure model (SNS model) and a concept of workflow region are proposed. Further, the recognition algorithm of workflow region is presented and its time complexity is analyzed quantitatively.

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