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An Algorithm for Construction of Triangulation with Nesting Islands

Abstract*. Islands are composed by closed arcs without intersection with each other. The relationship between islands can be either nesting or mutually independent. Through the analysis on nesting relationship among the islands, the arcs compositing islands are divided into four basal types, moreover, every type is provided an algorithm for calculation. The inner of islands is automatically identified according to the types of arc segments and then a triangulation algorithm with constrained condition was carried out. According to previous method, complex triangulations are structured. Proved by experiments, the efficiency of the algorithm is mainly determined by the efficiency of formatting delaunay of triangulation.*

Streszczenie*. W artykule przedstawiono analizę związków zagnieżdżonych, występujących między wyspami, poprzez którą dokonano podziału łuków, tworzących wyspy na cztery podstawowe kategorie, a ponad to dla każdej z nich określono sposób obliczenia. Wnętrze wysp jest określane automatycznie, na postawie typów łuków, co w wyniki pozwoliło określić algorytm triangulacji. Badania eksperymentalne potwierdzają skuteczność algorytmu, wynikającą z zastosowanej triangulacji Delone (****Algorytm tworzenia triangulacji z wyspami zagnieżdżonymi****).*

Keywords: *Nesting Islands, Delaunay Triangulation, Polygon, Constrained Data*
Słowa kluczowe: wyspy zagnieżdżone, triangulacja Delone, poligon, ograniczone dane.

1 Introduction

Delaunay triangulation algorithm has been very mature and applied widely, in whose application discrete data is not independent, but exist certain constraint relationship mutually, such as model boundary of reconstructed object, the ridge and valley line in the surface model, etc. Constructing triangulation net with such a constrained condition known as Constrained Delaunay Triangulation [1,2,3,4](Constrained Delaunay Triangulation abbreviated as CDT), researchers have done a lot of studies on the CDT algorithm, in which two-step method[5] is more commonly used. In most of algorithms it takes into account that all the non-constraint original data and the constraint form the network, and then are forcibly embedded in the constrained edge beyond Delaunay. The above algorithms are considered to establish CDT under the independent constraint line. There are little literatures about the islands group composed of bounded line segments, not to mention CDT made under the condition of islands nested. Although the litrature[6,7] giving algorithm on the islands original data, it is not applicable to multi-layer nested islands, where the multi-layer refers to the each island might still containing. multiple islands, and the contents of this paper is about CDT multi-nested constructed with multi-layer nested islands who are border lines with direction "surround" closed areas and attach to different properties, which are in practical applications common; Especially in geological field. therefore, researches on CDT of the nested multi-constrained data field islands are of great significance.

2 Classification and data structure

An island is a region closed with one or more end to end sections brims which does not intersect with each other.

The two island contracts: (1) The composition of island arc is the within and outer boundary alternative for the border, the point on the outer boundary is clockwise ordered, within the boundary is arranged by counter-clockwise. (2) The arc itself cannot be intersected and at the same time can not intersect with other island brims.

2.1 Classification of Brims

Before CDT construction, first establish a hierarchical relationship between the brims. In Figure 1, only the adjacent brims can form islands, such as brims 1, 2 and 4 may compose islands, similar to 2, 5 and 6. There are three arc relationships as follows: (1) Inclusion relation: arc 1 includes 2 and 4 but does not include 5 and 6; Arc 2 includes 5 and 6; Arc 3 includes 7. (2)Being Include relation: 2 and 4 were included by arc 1; 5 and 6 were included by arc 2 and 7 was included by arc 3. (3)Independent relation: neither includes any brims nor it is being included, such as brims 8.

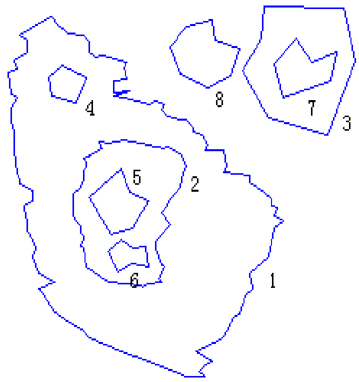

Fig.1. Type of brim

From this we can draw the shapes of four types of brims: A class, which brims only with inclusion relation but not being included by any brims, in Figure 1, such as the brims 1,3; B class, which brims were not only being included by other brims but also include other brims, such as brim 2; C class, only being included by other brims but does not include any brims, such as brims 4,5,6,7; D class, which brims are neither included by other brims nor include other brims, such as arc 8.

The above four class brims can all be used as island's internal and external borders, if internal boundary is included in external boundary, shown in Figure 1, brim 1 is the external boundary, 2 and 4 are the internal boundary. So brims 1,2,4 are composition of island and the area enclosed is the island's "internal" and 2 as the internal boundary line, then 5 and 6 will be the outer boundary, this time, 2,5 and 6 as island's "external", and the enclosed area are called "empty", the area within 5 and 6 as the island's "internal". On the contrary, if the internal boundary contains the external boundary, in Figure 1,brim 1 as internal boundary, 2 and 4 as external boundary, then region surrounded by 1,2,4 brims as the island's "external" and formation of "empty ", 2 for the outside line, then 5 and 6 will be used as the internal boundary, this time, the area formed by the 2,5 and 6 for the island's" internal ", while the 5 and 6 of the region for the island's "external" formed" empty", for only one brim island as the external borders, such as brim 8.

We draw the following conclusion through the above analysis: the arc can be "internal" or "external" of the island made by the adjacent island which is largely depend on the inclusion relationship of inside or outside of the boundary line. If internal boundary is included by external boundary then the formed area for the island's "internal", in the contrary, "external" formed "empty"; adjacent B class(brims 2 and 4) or adjacent C class (brims 5 and 6) with the same attributes are either outside boundary or within boundary; D class brims are considered to be outside boundary; region outside of A Class or D Class brims are considered "empty" (area out of 1,3 and 8 in Figure 1) .

2.2 The data structure of island

```
class Brim::Object// Brim class composition of island
{
    UINT m_BrimNo;     // No.of brims
    stPoint m_vetexArray[MaxCount]; // Point composed
    of brims, stPoint point structure composed of brims

    UINT m_Type;// The types of brims

    UINT m_Count;// Number of brims Numbered

    m_brimNo brims.
    UINT m_Relation[i];// The number of the i brim
    numbered m_brimNo
    int m_Direction;// Point arrangement direction,
    clockwise of the outer boundary for 1,
    counterclockwise of the inner boundary for -1
……}
CtypedPtrArray <COBArray, Brim * > m_BrimArray;// Brims
are stored in an array.
int m_Ploygon[polyNo][brim];// Brim No of brim in polyNo
island.
```

Brims can be arranged in the direction of using the formula below to calculate that if $M > 0$ brims are arranged clockwise and vice versa for the counter-clockwise arrangement.

(1) $$M = \sum_{i=1}^{n} (x_{i+1} - x_{i-1}) y_i$$

where: M– current Area, x,y –Coordinate.

3. Automatic identification of brim type

Following these steps to automatically identify the relationship among the arcs:

(1) All brims, regardless of the order, load up at one time, the number brims automatically numbered in the order by read and determine the direction of arrangement of brims in accordance with the symbols of calculation M
(2) Identify relationship between brims n (starts from n = 1) and the other arc i (i = n +1, n +2 ... ...).
(3) Determine whether the first point of i(any point) in polygons formed by n, if not, n = n +1 return to (2), otherwise the next step.
(4) Determine whether the first point is in the polygon composed of brims m except n, if not then the next step; otherwise further determine whether the first point of brim m is in the polygon whose components are brims n, if not also continue to the next step, otherwise n = n +1 and return to (2).
(5) arc n contains i, make m_Relation [m_Count] = i, the number of arc n plus 1 to make m_Count ++.
(6) Make n = n +1 and return to (2) until n <arc segments number.

The arc and its type can be known through m_Relation and m_Count of the arc. In figure 1, arc 1 contains two arcs whose numbers were 2 and 4, meaning m_BrimArray[1]. m_Relation[1]=2,m_BrimArray[1].m_Relation[2]=4, m_BrimArray[1]. m_Count = 2. In the mean time, all types of brims can be identified by the two arrays, such as when judging the arc of No. 3, if m_BrimArray[3]. m_Count does not equal to 0, indicating arc 3 is A or B class, so when the other brims' m_Relation array values are equal to 3, indicating arc 3 is B class, i.e. m_BrimArray[3]. m_Type = 1, otherwise, arc 3 is A Class, meaning m_BrimArray[3]. m_Type = 0.When m_CountMark[3] equals 0, indicating arc 3 belong to C or D class, in this case, if m_Relation array values are equal to 3, this shows that arc is C class, i.e. m_BrimArray[3]. m_Type = 2, otherwise it is D class, meaning m_BrimArray[3]. m_Type = 3.

As shown in the plane space in Figure 1, what kinds of islands that are composed of various brims is largely depending on whether the A class arcs and D class arcs ((1,3,8 in Figure 1) are inside or outside boundaries. We only concern structuring CDT in the "internal".The composition of the island only needs to determine what arcs enclose the islands' "internal".

4. Triangulation net within the island

First, along the direction of the arc, taking binding edge (arc formed by two adjacent points) as a triangle edge, on the right of the edge searching for a suitable point by the principle of the largest angle[9], constructing a triangle with constrained edge, such triangle is called the base triangle, meeting the nature of CDT, and then expand new triangles inward based on the former base triangle. The first step, the base triangle is constructed by binding edge, shown in Figure 2 a), area of CDT enclosed by 1, 2, 4; the second step is the expansion of the base triangle to structure DT(Delaunay Triangulation), shown in Figure 2 b).

How to ensure the only the points on one side of the arc participated in structuring the net is the key of the first algorithm, which can be resolved by vector cross product. Constrained edge is ordered in sequence, so we consider which composed by two adjacent points i and i+1 as vector V(i,i+1). Try to look for point k which can be either within the region or on the arc. So vector V(i,k) is made by vectors Vi and Vk, set: d=V(i,i+1)×V(i,k). If d>0, the point k is on the right side of constraint edge; if d<0, then the point k is on the left side of constraint edge; if d=0, the point k is right on the constraint edge.

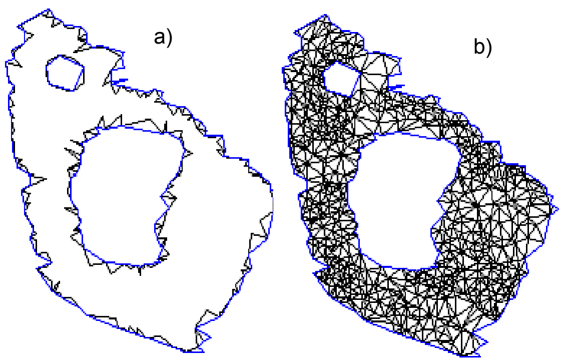

Fig.2. Constructing CDT with two steps

For the arc 1 in Figure 2 a), points searching from the right side of the constrained edge Pi,i+1(including points on the arc) when i equals to 1, according to the principle of the largest angle to construct the first base triangle with constrained edges. If the point found is the adjacent point i+2 of the constrained edges, which means two adjacent sides being edges of the triangle, then the next edge Pi+1,i+2 is identified as no longer being expanded for triangle. If constraint edge i is completed then continue to i+1 until all constraint edges are included in the construction of base triangle. Arc 2 and 4 in figure 2 a) compose triangle by the points on the right of constrained edge, because of direction of arc, CDT constructed by only expanding to the right side of the constrained edge is the island "inside".

Triangle growth[10] method is used to extend the edge that has never been expanded by base triangle, and then to generate new triangles, as shown in Figure 2 b) below.

5. Efficency analysis

Use different number of points to construct CD and CDT, count the relationship between time and point. AS can be seen from the figure 3, when the number of the point is closer, time efficiency is almost the same, so the time of the algorithm mainly depends on the structure of CD.

Apply the algorithm in Three-dimensional strata modeling; we can cut any irregular topography at will. The practice proved that the algorithm is feasible.

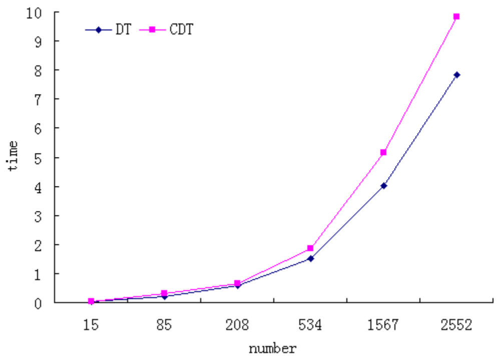

Fig.3. Time and points

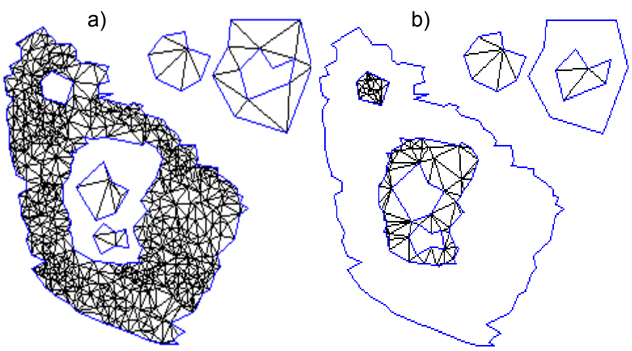

Fig.4. The CDT with holes

Figure 4 a) is showing the CDT which is structured by edges 1 and 3 in Figure 1 as the outer boundary. Figure 4b) CDT is constructed by an inner boundary.

6. Conclusions

Triangulation with constrained data is used in many practical applications such as geology, e.g. cutting area enclosed by rocks of different geological section, also establishing CDT with constraint data of nested islands is another common use. This paper analyzed the relationship between the compositions of island arcs, based on the automatic classification of the arcs, as long as the arc properties of A class are settled, the "inside" of islands can be automatically formed in which CDT can be structured. According to the algorithm, CDT within each island can be output easily. And the algorithm is simple and easy to implement.

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