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The Local Importance of Node in Complex Networks

Abstract. *This paper presents a new weighted graph to represent the strength of relationships between nodes, this graph can be a direct reflection of the interaction frequency of interconnected nodes, through the calculation of the weights, it can measure the local importance of the node results show that the local importance of a node is proportional to the degree of the node and the interaction frequency of the node. Under certain conditions, the local importance is inversely proportional to the degree of its adjacent nodes.*

Streszczenie. *W artykule przedstawiono nowy diagram ważony do prezentacji siły relacji między korzeniami sieci. Graf może być bezpośrednim odbiciem wzajemnych oddziaływać częstotliwościowych współpracujących korzeni. Poprzez obliczenia wag może oceniać znaczenie korzenia lokalnego. Wyniki wskazują, że znaczenie korzenia lokalnego jest proporcjonalne do stopnia korzenia i częstotliwości współpracy. W szczególnych warunkach lokalne znaczenie jest odwrotnie proporcjonalne do stopnie korzenia przylegającego. (**Lokalne znaczenie korzenia w sieci złożonej**).*

Keywords: weighted graph, local importance, interaction frequency.
Słowa kluczowe: graf ważony, znaczenie lokalne, częstotliwość współpracy.

Introduction

In nature there are a variety of network systems, over the past decade, people are trying to describe the things around us through the network system: the ecosystem, the neural network, human society and so on. The concept of Complex networks [1] attracted so many researchers. The highest frequency which they discussed is the model of small world network [2] and scale-free network [3]. In small-world network model, the shortest path from one node to any other node is not too long, just like a small world. In this small world, you want to contact other member is not difficult. Scale-free network model found that there are some central nodes in the complex networks, the number of which is not very big, but the central nodes have high degree. The central nodes attract so many attentions. In human society, for example, there are some people which have more friends than others, in various fields and industries, Is such people important? This is the contents of this article. Do the nodes which have high degree play a more important role than the other nodes that degree is smaller? The link between the nodes is not the same category, and this article study how different categories of links between nodes to help measure the local importance of nodes in the networks.

The categories of relation between nodes

It is inevitable that the central node attract so much attention. A large number of researchers agree that the central node which have high degree, in many cases, play a more important role than small-degree nodes. The same time, many researchers fans in the betweenness centrality [4] of the nodes in networks. The formula to calculate the betweenness centrality is: Bet(v)= $\sum_{s\neq v\neq t}\frac{Q_{st}(v)}{Q_{st}}$, where Qst is the total number of shortest paths from node s to t and Qst(v) is the number of those paths which across node v. In this paper, we discuss the relationship between the local importance of node and the exchange frequency of the node and its neighboring nodes; we also discussed the categories of the relationships between nodes.

The researchers found that there are a lot of "group" in the complex networks [5], the nodes within the group are connected by dense connections, The connections between groups are relatively sparser. The connection between groups can be measured by many algorisms. The strength of the relationship between the nodes is different in the real network [6]. Can we measure the strength of the relationship between two nodes? The answer is positive. The relationships can be categorized by many factors, for example, the frequency of interaction between two nodes. The way to define the strength of relationship between the nodes is different in different real networks. Which need more discussion? In this paper, the strength measured by the frequency of interaction is used for the measure of local importance of nodes.

Relationship is connected to two nodes, strong or weak in this relationship should be defined by the two connected nodes respectively. In many cases, the relationship is in a state of inequality [7]. In this paper, the relationship between the nodes is divided into three types: weak relationship, a single strong relationship, strong relationship. In Fig.1, a weak relationship between two nodes is denoted by a straight line (Fig. 1(a)), a single strong relationship is denoted by a straight line with a arrow (Fig. 1(b) and ©), a strong relationship is denoted by a double arrow straight (Fig.1(d)).

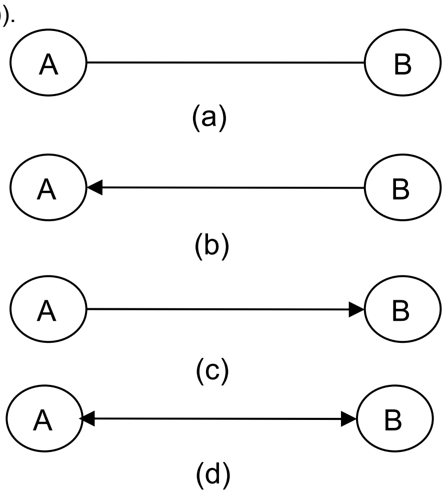

Fig. 1. The categories of relationship between nodes

In Fig.1(b), node A is important for node B; but in the opinion of node A, node B is not important. On the contrary, in Fig. 1(c), node A is not important for node B and node B is important for node A. In Fig. 1(d), the two nodes connected are both important for each other, this relationship is a strong relationship.

The local importance of node

The local importance of node is the level of attention among all adjacent node of the node. It is local because it depends on the adjacent node directly in stead of all other nodes.The strength of relationship is a vague concept, and how to measure it with a more accurate way? For example, in social network, which relationship is important? Frequency of interaction between nodes can be use to represent the strength of this relationship [8]. Based on this method, this paper proposes a way to measure the relationship between the nodes. For example, there is a msn account M. Within a month, the number of chat messages of the account M is total_msn. Those chat messages are produced with other d msn account (denoted as M1, M2 ,M3 ...... Md).The number of messages that account M with each other account is denoted as N1, N2, ... , Nd. In this example: N1 + N2 + ... + Nd = total_msn, Using a node wiring diagram with weights to describe this example, shown in Fig. 2.

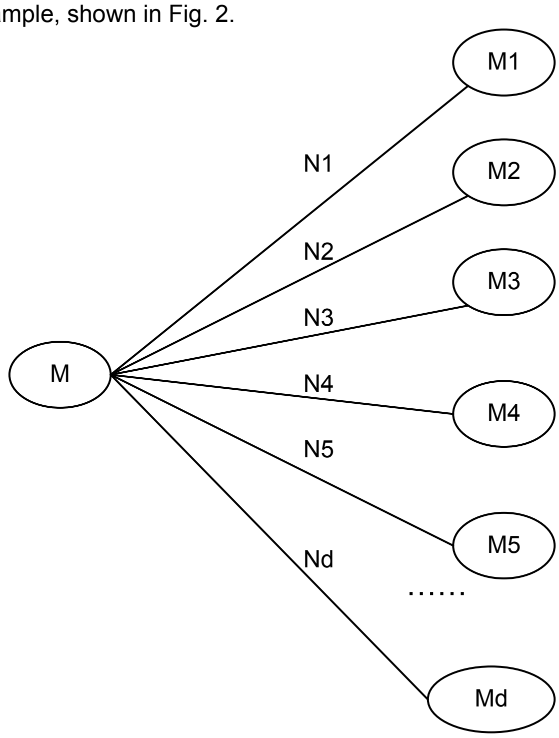

Fig. 2. the diagram of nodes-connection with weights

Assume that the number of all messages of adjacent node M1, M2, M3 … Md is Num1, Num2, Num3 ... Numd respectively. For a neighbor node Mi, the attention of node M can be expressed as follows: Ni / Numi, the sum of all these values can be used to represent the local importance of node M for all adjacent nodes: Weight(M)= $\sum_{i=1}^{d}\frac{Ni}{Numi}$. Where d is degree of the node M. In fact, the local importance of a node is not depends on the node itself, but on the views of other neighboring nodes. Which is specially common in social network[9]. For node M, the importance of adjacent nodes Mi is $\frac{Ni}{\sum_{i=1}^{d}Ni}$.

In this way, the frequency of interaction between nodes instead of fuzzy concept of strength is used to denote the strength of relationship between nodes. In scale-free network model, central node is usually considered to have a number of adjacent nodes (i.e., the high-degree node). Because the degree of nodes showing a power law distribution in the scale-free model, some nodes have a great degree, while most nodes have small degree. And numerous studies have shown that the high-degree nodes play a crucial role in many cases, such as connectivity. Are that high-degree nodes important consequentially? In this article, the local importance of node is denoted as weight (Node).

In order to facilitate the description, a simple calculation of the first extreme case, suppose that in Fig. 2, node M is connected to all other nodes in the network, each adjacent node is a leaf node, then for each adjacent node, the node M is 100% important, the local importance of the node of M to obtain the maximum Weight

(M) = $\sum_{i=1}^{d}\frac{Ni}{Numi}$ = d ,where $\frac{Ni}{Numi}$ = 1.

Then consider the second extreme case, that the degree of neighboring nodes is equal to the degree of node M. For convenience, assume that each adjacent node treat all adjacent nodes of it fairly, which distract its attention averagely. For node Mi, the attention of node M is $\frac{1}{d}$. For all adjacent nodes, the local importance of node M's represented as:

Weight（M）= $\sum_{i=1}^{d}\frac{1}{d}$ =1.

The more general case, the degree of the nodes M is d, assuming that each degree of the neighboring node is dmi (1 <dmi <d), each adjacent node treat all adjacent node fairly, then Ni= $\frac{Numi}{dmi}$ .for each adjacent node, the attention of node M is: $\frac{1}{dmi}$. The local importance of node M is Weight (M)= $\sum_{i=1}^{d}\frac{1}{dmi}$ .Assume that the average degree of all adjacent nodes is k, then the local importance of node M is

Weight（M）= $\sum_{i=1}^{d}\frac{1}{k}$ = $\frac{d}{k}$, shown in Fig. 3. Another scenario, suppose the degree of adjacent nodes of node M is distributed from 1 to d uniformly, namely: dm1 =1, dm2 = 2 ... dmi = i ...... dmd = d. The local importance of node M is

weight（M）= $\sum_{i=1}^{d}\frac{1}{dmi}$ = $\sum_{i=1}^{d}\frac{1}{i}$.

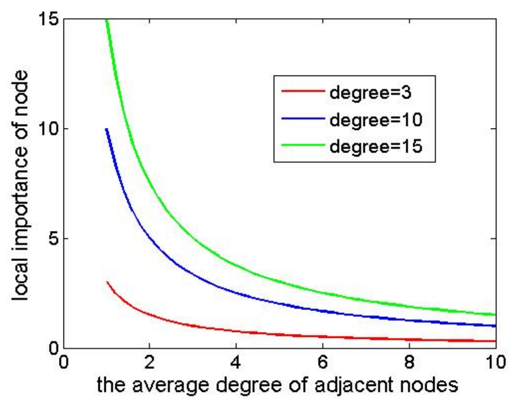

Fig. 3. relationships between local importance of node and k

The conclusions of the above calculation is: assume the average allocation of attention in each node, the local importance of node M depending on its degree and the degree of its adjacent node, the greater the degree of the node M is, the greater the local importance is; the greater the degree of its adjacent nodes is, the local importance of node M smaller is. In this case, if the degree of node M is greater than or equal to the degrees of its neighboring nodes, the local importance of node M is smaller than or equal to 1, i.e. Weight（M）≧1.

In the real network environment, nodes are usually not so average to treat all adjacent nodes, but different, the treatment of adjacent nodes is not fair in real. In this case, the local importance of node M is Weight (M) = $\sum_{i=1}^{d}\frac{Ni}{Numi}$ (where Ni ≦ Numi), which depends on the frequency of interaction between node M and the adjacent nodes Mi and on its share in the total number of interaction of node Mi. If the value of d is fixed, the value of Weight (M) is ranging from 0 to d. When all adjacent nodes consider that node M is not important, the value of Weight (M) is 0, when all adjacent nodes are leaf nodes or all adjacent nodes consider that node M is 100% important, Then weight (M) = d. In addition to these two special cases, Weight (M) may be any value ranging from 0 to d. Assume that the average attention of node M among all adjacent nodes is ave-i, then the local importance of node M is Weight (M) = $\sum_{i=1}^{d}\frac{Ni}{Numi}$ =d*ave-i.

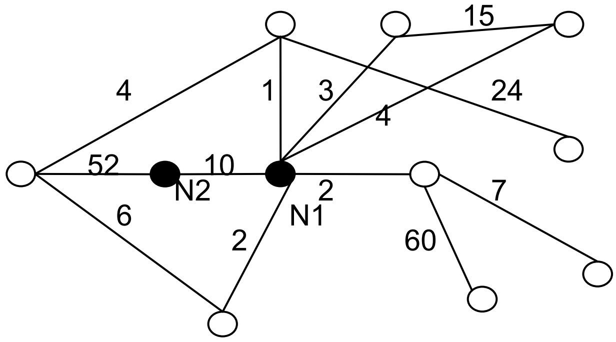

Fig. 4. a weighted network

In Fig. 4, in such a weighted network, Weight (N1) = 0.852, Weight (N2) = 1.293. It can be seen, although the degree of node N1 is larger than that of node N2, and the betweenness of node N1 is larger than the betweenness of N2 , the local importance of node N2 is larger than that of node N1.Weight (N2)> Weight (N1).Position Figures and tables at the tops and bottoms of columns. Avoid placing them in the middle of columns. Large Fig.s and tables may span across both columns. Fig. captions should be centered below the Fig.s; table captions should be centered above. Avoid placing Fig.s and tables before their first mention in the text. Use the abbreviation “Fig. 1,” even at the beginning of a sentence.

Conclusions

Based on the different strength of the connection between nodes, the local importance of the node is discussed in this paper. The local importance of a node is proportional with the degree of the node and the interaction frequency of the node. Under certain conditions, the local importance is inversely proportional to the degree of its adjacent nodes.

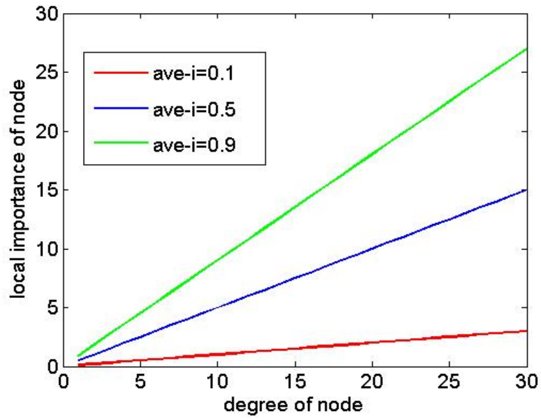

Fig.5. The Local Importance of node

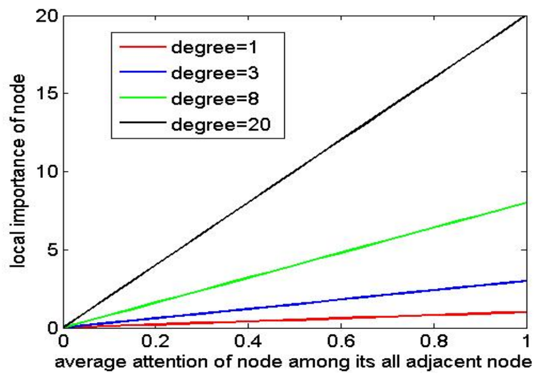

Fig.6. average attention of node

Acknowledgments

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