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Distribution Optimization Based on Feasible Direction Method for Horizontal Well Perforation

***Abstract**. The horizontal wellbore pressure and reservoir heterogeneity are major factors in the horizontal well uneven inflow. Given the situation, it is to establish balanced production of horizontal wells as the objective function, and holes dense as the optimization which is a typical constrained nonlinear optimization. By feasible direction method, it is to obtain distribution optimization results, in order to control the production pressure within horizontal section and to realize uniform advancing of flow profile in the horizontal segment.*

Streszczenie.** W artykule przedstawiono metodę optymalizacji rozmieszczenia i gęstości odwiertów oraz zbalansowania ilości studzien poziomych, co zapobiegnie nierównomiernym rozpływom cieczy. Dodatkowo pozwoli to na kontrolę ciśnienia przy wytwarzaniu poziomych sekcji oraz jednorodne przedłużanie profilu przepływu w poziomym segmencie. **(Optymalizacja rozmieszczenia perforacji studzien poziomych – metoda realnego kierunku)

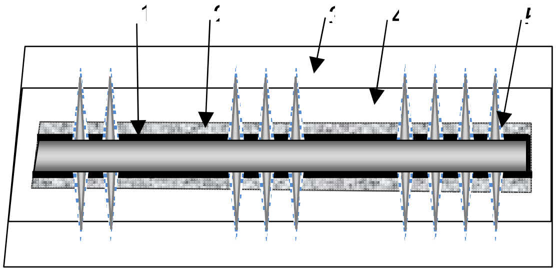
Keywords: Horizontal well; Perforating; Feasible direction method
Słowa kluczowe: studzienka pozioma, perforowanie, metoda realnego kierunku

Introduction

Oil wells are drilled as round holes from the ground surface to the deep underground, and the multi-layer steel pipes are laid down into holes, perforating the pipes to let oil flow through the channel, therefore the fluids flow into well, lifted to the surface through these channels. Previous wells are vertical wells, and the wellbores cross through the reservoir vertically. Recent years, horizontal well drilling has been applied more widely because it can develop more reservoirs, yield higher production. Meanwhile, Perforation completion is a main stream of horizontal well drilling field, as it can separate different pressure reservoirs and support loose production layers more effectively. During fluid flow process in the horizontal wells, due to changes in fluids and pressure, it can cause uneven flow within perforated intervals, as a result in emerging water too early at borehole bottom of horizontal wells. Under this circumstance, it will affect the production, therefore it needs to optimize horizontal well perforating parameters and control the horizontal section inflow profile.

1 Characteristics of Horizontal Wells

Fluid flow through Horizontal well is variable mass flow; there is pressure drop along the production segment wellbore flow direction, so pressure distribution unevenly within production segment. As for the wellbore downstream, the larger pressure difference at the end of production, the higher radial flow is; for upstream wellbore, the smaller pressure, the lower radial flow, which causes uneven distribution[1]. The reservoir flow and fluids in the wellbore of production segment is a coupling relationship, while oil and water often coexist in the formation, and the water flow is greater than that of oil, it's resulted in water come out in different time in different segment in horizontal wells, Fig. 1.



1、Casing；2、Cement sheath；3、Reservoir；4、Contaminated areas；5、Perforation compacted zone
Fig.1. Horizontal Well Perforating diagram

Therefore, it is important to perforate holes and control inflow profile for the horizontal wells. A large number of studies has been done worldwide [2-4], the common practice is to couple the percolation model [5] and the wellbore fluid flow model so as to establish objective function 'production', perforating distribution as variable parameter to compute design variables optimization [6,7].

2 Production Capacity Model for Perforated Horizontal Well

2.1 Fluid Flow Model for Non-perforated Sections

For non-perforated sections, wellbore flow rate is constant, the fluid pressure loss is mainly caused by the wall friction dP_f; when horizontal wells have a certain dip, it is also to consider the pressure drop dP_g caused by gravity, that is, the total pressure drop dP_w equal to friction pressure drop dP_f and gravity pressure drop dP_g. That is:

$$(1)\quad dP_w = dP_f + dP_g$$

$$(2)\quad \mathrm{d}P_\mathrm{f} = \frac{1}{2} f \frac{\rho l}{D} v^2$$

where friction factor *f* by calculating the Reynolds number to determine the flow pattern:

If it is linear flow (laminar flow), then

$$f = \frac{64}{\mathrm{Re}}$$

If turbulence (rough wall), then

$$f = \left\{ -1.8\lg\left[\frac{6.9}{\mathrm{Re}} + \left(\frac{\varepsilon}{3.7D} \right)^{1.11} \right] \right\}^{-2}$$

Friction of the transition section can be obtained from the linear interpolation coefficient between the linear flow and turbulence.

The deviation angle θ, the pressure drop caused by the gravity of the inclination for horizontal wells may be expressed as:

$$(3)\quad dP_g = \rho g l \cos\theta$$

2.2 Fluid Flow Model for Perforated Sections

The pressure drop of the fluid in the perforated section comprising: a pressure drop caused by the frictional resistance generated by fluid flow in the wellbore *dP_f*, Reservoir fluids flow into the well and confluence with the mainstream fluid, which causes the mixed pressure drop dP_m; Meanwhile, radial fluid inflow makes well section become variable mass flow, so the acceleration occurs and produces accelerated pressure drop dP_a; besides, when there is inclination in the horizontal well, the pressure drop by gravity dP_g should be taken into consideration.

Divided the perforated section into the length of ΔL and the number of n perforation unit according to the number of holes, each the unit contains a hole. The pressure loss $dP_{w,i}$ of the *i* unit is obtained from the sum of above several pressure loss.

$$dP_{w,i} = dP_{f,i} + dP_{m,i} + dP_{a,i} + dP_{g,i} \tag{4}$$

The total pressure loss of the horizontal wells perforated sections dP_wcan be stated as follows:

$$dP_w = \sum_{i=1}^{n} dP_{w,i} \tag{5}$$

Where *n* is the number of perforated holes.

In perforated section, each unit of wall friction pressure drop algorithm is the same as the non-perforated section:

$$dP_{f,i} = \frac{1}{2} f_i \frac{\rho \Delta L}{D} v_i^2 \tag{6}$$

When the wall surface inflow and mainstream outflow ratio (the perforations radial flow and wellbore axial flow ratio) is less than the critical value, the radial fluid flow smooth the pipe flow, reduce the pressure drop. At this point, the mixing pressure drop caused by the perforations friction and radial inflow can be written as follows:

$$dP_{m,i} = dP_{p,i} - 0.031 \times \mathrm{Re}_i \left(\frac{q}{Q} \right)_i \tag{7}$$

Where *q* is radial flow of a single perforation, m^3/s; *Q* is axial flow of horizontal wellbore, m^3/s. dP_p is perforations friction pressure drop, Pa.

When the flow rate ratio is greater than the critical value, the radial inflow of the fluid will block pipe flow, increasing pressure drop. In this situation, the mixing pressure drop is written in the form:

$$dP_{m,i} = 760 \times \mathrm{Re}_i \left(\frac{q}{Q} \right)_i \tag{8}$$

Perforation friction pressure drop $dP_{p,I}$ can be obtained by perforation friction coefficient $f_{perf,i}$ shown below:

$$dP_{p,i} = \frac{1}{2} f_{perf,i} \frac{\rho \Delta L}{D} v_i^2 \tag{9}$$

The perforation friction coefficient f_{perf} to calculate wall friction coefficient, and obtain constant in the universal velocity distribution law, then calculate roughness function according to the Gardel method, last obtain perforation friction coefficient by Newton's iterative method, as the following implicit form:

$$\sqrt{\frac{8}{f_{fper}}} = 2.5 \ln \left(\frac{\mathrm{Re}}{2} \sqrt{\frac{f_{pref}}{8}} \right) + B - \frac{\Delta u}{u^*} - 3.75 \tag{10}$$

Accelerated pressure drop is only associated with the density of the fluid, and the flow rate, it can be expressed as:

$$dP_{a,i} = \frac{\rho}{2} \left(v_{i+1}{}^2 - v_i{}^2 \right) \tag{11}$$

The perforated sections gravity pressure drop algorithm is the same as non-perforated intervals.

$$dP_g = \rho g l \cos \theta \tag{12}$$

2.3 Reservoir - Wellbore Coupled Control Equation

The perforated section is divided into n_1 perforating unit with length of Δx, and divide non-perforated intervals into n_2 units based on their numbers. Bringing the total number of units to *n*, from the toe end to the heel end, it is to number the unit from 1 to n, as shown in Figure 2.

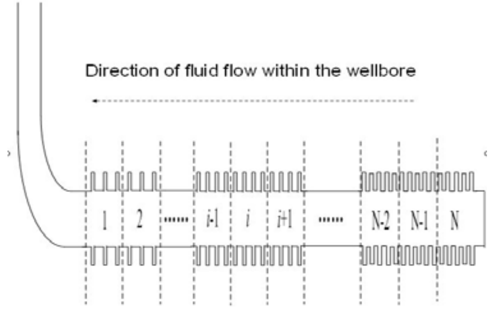


Fig.2. Horizontal Well Perforating unit for schematic diagram

To obtain a uniform inflow profile, the flow rate of the unit length in perforated section is:

$$q = q_L B_0 / L \tag{13}$$

Where q_L is horizontal well liquid production, m^3/s; B_0 is crude oil volume; *q* is fluid flow of any perforating unit, m^3/s

When horizontal well is operating, pressure difference from any perforating unit can be interpreted as fluid media seepage pressure drop Δp_r and the turbulence pressure drop through the perforations Δp_s. Then, two adjacent perforating unit wellbore pressure drop presented as:

$$\Delta p_{w,i} = p_{w,i} - p_{w,i-1} = (\Delta p_{r,i-1} - \Delta p_{r,i}) + (\Delta p_{s,i-1} - \Delta p_{s,i}) \tag{14}$$

Where, $p_{w,i}$、$p_{w,i-1}$ are respectively of fluid flow pressure of perforating unit *i*、*i*-1, Pa; $\Delta p_{r,i}$, $\Delta p_{r,i-1}$ are respectively the pressure drop of reservoir inflow in unit *i*、*i*-1, Pa; $\Delta p_{s,i}$, $\Delta p_{s,i-1}$are respectively the turbulent pressure drop in unit *i*、*i*-1, Pa.

Therefore, the wellbore pressure drop can be expressed as a function of the perforations flow rate of:

$$p_w = F(q) \tag{15}$$

2.4 Perforation Inflow Profiles Parameters Optimization

According to the above reservoir seepage, and perforation flow model analysis, it can be drawn reservoir inflow pressure drop from each unit, hole turbulence pressure drop and the wellbore flowing pressure drop. These parameters are adjacent to the previous perforating unit, so make the *i* hole parameters known to calculate the *i*-1 perforating unit hole density equation as follows:

$$\rho_{\mathrm{p},i-1} = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad i=1,\ 2,\ \ldots,\ \mathrm{N}-1 \tag{16}$$

Where

$$a = \Delta p_{\mathrm{w},i} + \Delta p_{\mathrm{r},i} + \Delta p_{\mathrm{s},i} - \Delta p_{\mathrm{r},i-1};$$

$$b = -\left(\frac{\mu q}{2\pi k_{\mathrm{c},i-1} L_{\mathrm{p}}} \ln \frac{r_{\mathrm{c}}}{r_{\mathrm{p}}} + \frac{\mu q}{2\pi k_{\mathrm{d},i-1} L_{\mathrm{p}}} \ln \frac{r_{\mathrm{o}}}{r_{\mathrm{c}}} \right);$$

$$c = -\left[\frac{\lambda \rho q^2}{\left(2\pi L_{\mathrm{p}}\right)^2} \left(\frac{1}{r_{\mathrm{p}}} - \frac{1}{r_{\mathrm{c}}} \right) + \frac{\lambda \rho q^2}{\left(2\pi L_{\mathrm{p}}\right)^2} \left(\frac{1}{r_{\mathrm{c}}} - \frac{1}{r_{\mathrm{o}}} \right) \right]$$

where k_i is the i reservoir average effective permeability corresponding to its perforating unit, m^2; $k_i=(k_{hi}k_{vi})^{1/2}$, $k_{j,i}$ is the reservoir level permeability corresponding to the i perforation unit, m^2 ; $k_{v,i}$ is the reservoir vertical permeability corresponding to the i perforation unit, m^2; h_i is the thickness of reservoir corresponding to its perforating unit i, m; S_i is reservoir epidermal coefficient corresponding to its perforating unit i; $k_{p,i}$ is compaction zone permeability corresponding perforations unit i, m^2; $\rho_{p,i}$ is perforating hole density of unit i, m; $\Delta p_{w,i}$ is the pressure difference between the hole of unit i and unit i-1.

Thus, when given liquid production of horizontal wells and toe end unit density (perforating unit number N), it can obtain wellbore pressure drop $\Delta p_{w,N}$, the reservoir seepage pressure drop $\Delta p_{r,N}$, $\Delta p_{r,N-1}$ and perforation pressure drop $\Delta p_{s,N}$, it can deduce $N-1$ perforating unit perforation density, followed by analogy, the density distribution throughout the horizontal wellbore hole of the perforating unit can be calculated. The equation as follows:

$$(17)\quad \left|\begin{aligned} q^{n+1} &= G^{-1}p^{n} \\ p^{n+1} &= F(q^{n+1}) \end{aligned}\right.$$

3 Distribution Optimization Based on Feasible Direction Method for Horizontal Well Perforation

3.1 Feasible Direction Method for Perforating Distribution Model in Horizontal Well

Horizontal Well Perforating distribution optimization problem can be described a nonlinear programming. Taking into account the friction pressure drop, flow pressure drop, and gravity pressure drop, the Horizontal Well Perforating distribution model is written as:

$$(18)\quad \left|\begin{array}{l} \min\ f(X) = -\sum_{i=1}^{N}[G^{-1}(X)P]_i \\ s.t.\begin{cases} p_i - p_{wi} = 0 \quad i = 1,2,\ldots,N \\ \sum_{i=l(k+1)}^{l(k+1)} q_i = \sum_{i=1}^{N} q_i(X_{k+1} - X_k)/L \\ k = 0,1,2,\ldots,J-1 \\ X_l \ge 0 \\ X_{j+1} - X_j \ge 0,\ j = 1,2,\ldots,j-1 \\ L - X_j \ge 0 \end{cases} \end{array}\right.$$

The steps：

Step 1：any $x_1 \in \Omega\,(\Omega \subset R^n$ is a nonempty closed convex), $k=1$.

Step 2：solving the following linear programming, get d_k and z_k ,

$$\left|\begin{array}{l} \min \qquad z \\ s.t.\begin{cases} d^T\nabla f(x_k) - z \le 0 \\ d^T\nabla g_j(x_k) - z \ge -g_j(x_k) \qquad j\in I \\ -1 \le d_j \le 1 \qquad 1 \le j \le n \end{cases} \end{array}\right. \quad \text{if } z_k = 0,$$

process suspended；or $z_k < 0$, go to next step.

Step 3：Step length $\alpha_k = \arg\min\{f(x_k+\alpha d_k)\,|\,0\le\alpha\le\alpha_{\max}\}$, where ，$\alpha_{\max} = \sup\{\alpha\,|\,g_j(x_k+\alpha d_k)\ge 0,\ j\in I\}$.

Step 4: $x_{k+1} = x_k + \alpha_k d_k$， $k = k+1$.

Go to Step 2. [8,9]

3.2 Basic Parameters

Taking a horizontal well for example, it is to analyze optimizing parameters and its optimization effect. The parameters details as shown in Table 1.。

Table 1 Horizontal Wells Parameters

Reservoir Type	Bottom Water reservoir	Hole depth mm	400
Reservoir thickness m	12	Perforation density hole/m	16
Well drainage area km^2	0.04	Perforation interval numbers	1
Crude oil density g/cm^3	0.9	Section 1 top slant depth m	3400
Crude oil viscosity mPa·s	25	Section 1 bottom slant depth m	3600
Oil volume coefficient	1.1	Section 1 top vertical depth m	3180
Wellbore thickness mm	9.17	Section 1 bottom vertical depth m	3180
Wellbore outer diameter mm	139.7	Section 1 top horizontal permeability 10^{-3}um^2	1500
Daily liquid production m^3/d	50	Section 1 bottom horizontal permeability 10^{-3}um^2	1500
Hole diameter mm	10	Section 1top vertical permeability 10^{-3}um^2	600
Horizontal well depth m	8	Section 1 bottom vertical permeability 10^{-3}um^2	600

3.3 Optimization Model Output

Seeing from Figure 3, The density distribution comparison chart, the optimized hole density from the perforated sections toe end gradually reduced to the bottom end. This is due to the impact of the wellbore pressure drop, production pressure is larger at end of the horizontal section than that of at toe end. To obtain an even inflow profile, it is to reduce the perforation density at the end of section to obtain the even production pressure, so as to achieve the flow cross-sectional uniform advance.

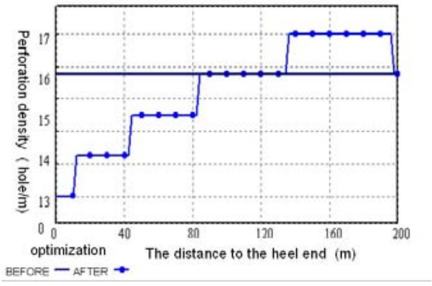


Fig 3 Perforation density distribution comparison before and after optimization

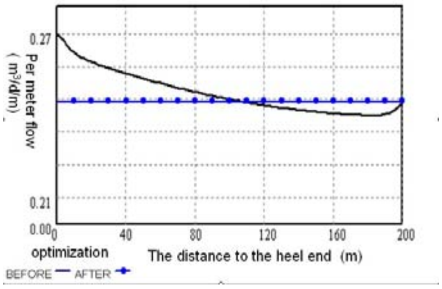


Fig 4 Comparison of Inflow profile before and after optimization

From Figure 4, it can be seen that, before optimization, the perforated intervals toe flow is low due to the effect of wellbore pressure drop, with the high flow at the end of the section. In addition, a significant increase occurs on the flow from both ends of the perforated intervals due to large drainage area at the end of the horizontal wells. After optimization, the more uniform flow profile appears.

4 Conclusions and recommendations

(1) Horizontal well perforating distribution optimization problem is typical nonlinear optimization problems with constraints, feasible direction method can be adopted to solve the practical problems ideally;

(2) In accordance with the principle of continuity of reservoir and wellbore pressure and quality of the hole wall, the horizontal wells of reservoirs - wellbore flow coupled model based on horizontal wells percolation model and mass flow pressure drop model, shows that the horizontal wellbore pressure drop and oil heterogeneity are of the major factors that lead to uneven horizontal well inflow profile。

(3) Setting horizontal well balanced production as the objective function, under the constraints of reservoir - wellbore flow coupling model, taking holes density as the optimization variables, it is to establish the optimization model for horizontal well perforation. By optimizing the distribution of horizontal well perforating, it can control the horizontal wells production pressure drop, in order to realize uniform advance of inflow profile in the horizontal.

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