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Blind separation of delayed sources based on second-order Taylor approximation

***Abstract**. Conventional linear instantaneous mixing model becomes unsuitable if propagation time delays are taken into account. A blind separation algorithm based on second-order Taylor approximation for delayed sources (SOTADS) is presented, under the constraint that time delays are small in comparison with the coherence time of each source. Simulation results validate that the proposed algorithm performs superior than related approaches even when the constraint is violated.*

***Streszczenie.** Zaprezentowano algorytm ślepej separacji bazujący na aproksymacji Taylora drugiego rzędu dla źródeł z opóźnieniem SOTADS. Założono że czas opóźnienia jest mały w porównaniu z czasem koherencji obu źródeł. (**Ślepa separacja sygnałów bazująca na aproksymacji Taylora drugiego rzędu**)*

Keywords: blind source separation, time-delayed source, Taylor series expansion, joint diagonalization.
Słowa kluczowe: ślepa separacja sygnałów, aproksymacja Taylora

Introduction

Blind source separation (BSS) denotes extracting latent sources only from several observed mixtures, without available knowledge about the source signals and the mixing process. According to the assumptions made about the mixing model, BSS traditionally can be categorized as: instantaneous and convolutive. The instantaneous mixing case has been achieved with good results in a large number of ways [1-2]. As a matter of fact, the hypothesis that all the sources reach the sensors simultaneously can hardly be satisfied in real-world applications. For example, in acoustics the propogation of the audio signals through the medium is not instantaneous (like signals through air or water), or in electrical engineering signals from multiple antennas are received asynchronously, or in biomedicine myoelectric signals recorded in multiple locations over the skin surface are delayed versions due to the transmission of the intracellular potentials along the muscle fibers. Taking the signal propagation delay into account, mixtures are approximated by linear combinations of time-delayed source signals, which is called anechoic mixing. And it can be deemed as a particular case of the convolutive mixing model [3], where each mixture is a differently filtered combination of source signals. Obviously, approaches for the convolutive mixing are over-parameterized for the anechoic mixing.

So far only several literatures [4-11] have addressed the blind separation of delayed sources in anechoic environment. Some algorithms [6-8] are derived by applying stochastic time-frequency analysis. Other much more popular ways by using truncated Taylor series of each delayed source has been successfully explored [8-11]. When the propagation delays are sufficiently small in comparison with the coherence time of each source, it shows that such mixtures can be modelized by linear instantaneous combinations of different temporal derivatives of sources [12]. These derivatives act as new dependent sources, and separation can be achieved by using second-order statistical analysis. However, most of existing research concentrate on first-order Taylor approximation, which have a positive effect on the performance for relatively small delays, such as the recent Rtau-Delay algorithm [8]. To handle relatively larger delays, it will be helpful to expanding the Taylor series to higher orders.

Therefore in this paper, we have presented a blind separation algorithm based on second-order Taylor approximation for delayed sources (SOTADS). We propose in a first step to whiten observations as the classical SOBI algorithm [13] does, and in a second step to recover original source signals by joint diagonalization [13-14] of the covariance matrix of the whitened observations. What should be mentioned is that we also utilize the symmetry of the covariance matrix to reduce the approximation error and improve the estimation accuracy. It can be said that SOTADS is the extension of SOBI. Simulation results validates the outstanding performance of our proposed algorithm over both Rtau-Delay and SOBI, especially when confronted with relatively larger propagation delays.

The rest of this paper is organized as follows. In section II, the signal model is briefly introduced. Section III presents the proposed SOTADS algorithm. Simulation results and discussion are provided in section VI. At the end of the paper, a concise conclusion is given.

Signal Model

Let the *M* observations $\boldsymbol{x}(t)=[x_1(t),...,x_j(t),...,x_M(t)]^T$ be linear combinations of *N* delayed source signals $\boldsymbol{s}(t)=[s_1(t),...,s_i(t),...,s_N(t)]^T$, with unknown time delays t_{ji} in addition to unknown attenuation coefficients a_{ji} :

(1) $$x_j(t)=\sum_{i=1}^{N}a_{ji}s_i(t-t_{ji})+p_j(t)$$

where $\boldsymbol{p}(t)=\left[p_1(t),...,p_j(t),...,p_M(t)\right]^T$ is the Gaussian noise vector. Note that $\boldsymbol{s}(t)$ are assumed to be zero-mean, unit-variance, mutually uncorrelated, and uncorrelated with $\boldsymbol{p}(t)$.

As it is shown in [12] that if all delays are considered sufficiently small for the inequality below to be verified

(2) $$t_{ji}<<T_d=\frac{1}{\sqrt{2}\pi f_{\max}}$$

where $f_{\max}$ is the maximum frequency of the sources, then observations can be approximated by first-order Taylor series expansion:

(3) $$x_j(t)\approx\sum_{i=1}^{N}a_{ji}s_i(t)-\sum_{i=1}^{N}a_{ji}t_{ji}\dot{s}_i(t)+p_j(t)$$

where $\dot{s}_i(t)$ denotes the first derivatives of $s_i(t)$. Moreover, the tolerable delays can be enlarged to

(4) $$t_{ji}<<\sqrt{3}T_d$$

if the second-order Taylor series expansion is introduced:

(5) $$x_j(t)\approx\sum_{i=1}^{N}a_{ji}s_i(t)-\sum_{i=1}^{N}a_{ji}t_{ji}\dot{s}_i(t)+\sum_{i=1}^{N}a_{ji}\frac{t_{ji}^2}{2}\ddot{s}_i(t)+p_j(t)$$

where $\ddot{s}_i(t)$ denotes the first and second derivatives of $s_i(t)$. Eq.(5) can be expressed in matrix form as:

(6) $$\boldsymbol{x}(t) \approx \tilde{\mathbf{A}}\tilde{\boldsymbol{s}}(t) + \boldsymbol{p}(t)$$

with the extended source vector $\tilde{\boldsymbol{s}}(t) = \left[s_1(t),...,s_N(t),\dot{s}_1(t),...,\dot{s}_N(t),\ddot{s}_1(t),...,\ddot{s}_N(t)\right]^T$ and the extended mixing matrix

$$\tilde{\mathbf{A}} = \begin{bmatrix} a_{11} & \cdots & a_{1N} & -a_{11}t_{11} & \cdots & -a_{1N}t_{1N} & a_{11}t_{11}^2 & \cdots & a_{1N}t_{1N}^2 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ a_{M1} & \cdots & a_{MN} & -a_{M1}t_{M1} & \cdots & -a_{MN}t_{MN} & a_{M1}t_{M1}^2 & \cdots & a_{MN}t_{MN}^2 \end{bmatrix}.$$

Obviously, Eq.(5) is the linear instantaneous approximation of Eq.(1). To ensure that $\tilde{\mathbf{A}}$ has full rank, it is necessary to assume the number of observations is at least three times that of the sources, i.e., $M \geq 3N$. This limitation can be overcomed by using antenna array to replace traditional antennae in practical applications.

The proposed SOTADS algorithm

Paoulis [15] has pointed out the following properties: (i) $\dot{s}_i(t)(1 \leq i \leq N)$ are mutually uncorrelated, so are $\ddot{s}_i(t)$; (ii) $s_i(t)$ and $\dot{s}_i(t)$ are only uncorrelated for $\tau = 0$, and so are $\dot{s}_i(t)$ and $\ddot{s}_i(t)$; (iii) $s_i(t)$ and $\ddot{s}_i(t)$ are even not uncorrelated for $\tau = 0$. Hence in our signal model in Eq.(6), besides the original sources, these first-order and second-order derivatives act as new dependent sources. Referring to the classical SOBI algorithm which is designed for temporally correlated sources, the proposed SOTADS algorithm to separate delayed sources is also composed of two steps: whitening and joint diagonalization.

Whitening for SOTADS

The $3N \times 3N$ covariance matrix of the extended source is

(7) $$R_{\tilde{s}}(\tau) = \begin{bmatrix} R_s(\tau) & R_{s\dot{s}}(\tau) & R_{s\ddot{s}}(\tau) \\ R_{\dot{s}s}(\tau) & R_{\dot{s}}(\tau) & R_{\dot{s}\ddot{s}}(\tau) \\ R_{\ddot{s}s}(\tau) & R_{\ddot{s}\dot{s}}(\tau) & R_{\ddot{s}}(\tau) \end{bmatrix}$$

From above analysis it is easy to get

(8) $$\begin{cases} R_s(\tau) = diag\left[R_{s_1s_1}(\tau),...,R_{s_Ns_N}(\tau)\right] \\ R_{s\dot{s}}(\tau) = diag\left[R_{s_1\dot{s}_1}(\tau),...,R_{s_N\dot{s}_N}(\tau)\right] \\ R_{\dot{s}s}(\tau) = diag\left[R_{\dot{s}_1s_1}(\tau),...,R_{\dot{s}_Ns_N}(\tau)\right] \\ R_{s\ddot{s}}(\tau) = diag\left[R_{s_1\ddot{s}_1}(\tau),...,R_{s_N\ddot{s}_N}(\tau)\right] \\ R_{\ddot{s}s}(\tau) = diag\left[R_{\ddot{s}_1s_1}(\tau),...,R_{\ddot{s}_Ns_N}(\tau)\right] \\ R_{\dot{s}}(\tau) = diag\left[R_{\dot{s}_1\dot{s}_1}(\tau),...,R_{\dot{s}_N\dot{s}_N}(\tau)\right] \\ R_{\dot{s}\ddot{s}}(\tau) = diag\left[R_{\dot{s}_1\ddot{s}_1}(\tau),...,R_{\dot{s}_N\ddot{s}_N}(\tau)\right] \\ R_{\ddot{s}\dot{s}}(\tau) = diag\left[R_{\ddot{s}_1\dot{s}_1}(\tau),...,R_{\ddot{s}_N\dot{s}_N}(\tau)\right] \\ R_{\ddot{s}}(\tau) = diag\left[R_{\ddot{s}_1\ddot{s}_1}(\tau),...,R_{\ddot{s}_N\ddot{s}_N}(\tau)\right] \end{cases}$$

and for $\tau = 0$,

(9) $$R_{\tilde{s}}(0) = \begin{bmatrix} R_s(0) & 0 & R_{s\ddot{s}}(0) \\ 0 & R_{\dot{s}}(0) & 0 \\ R_{\ddot{s}s}(0) & 0 & R_{\ddot{s}}(0) \end{bmatrix}$$

Here we approximately regard $R_{\tilde{s}}(0)$ to be a diagonal matrix, that is to say, the effect of $R_{s\ddot{s}}(0)$ and $R_{\ddot{s}s}(0)$ is neglected. Then the whitening method described in the SOBI algorithm [13] can be adopted, and it is briefly summarized as follows:

Step 1: Estimate the covariance $\hat{R}_x(0)$ from data samples. Denote $\lambda_1,...,\lambda_{3N}$ to be the $3N$ largest eigenvalues in descending order and $h_1,...,h_{3N}$ to be their corresponding eigenvectors.

Step 2: Estimate the noise variance $\hat{\sigma}_p^2$ by taking the average of the $M-3N$ smallest eigenvalues of $\hat{R}_x(0)$.

Step 3: Estimate the whitening matrix as

$$\mathbf{W} = \left[(\lambda_1 - \hat{\sigma}_p^2)^{-\frac{1}{2}}h_1,...,(\lambda_{3N} - \hat{\sigma}_p^2)^{-\frac{1}{2}}h_{3N}\right]^T$$

Step 4: Obtain the whitened observations $\boldsymbol{y}(t) = \mathbf{W}\boldsymbol{x}(t)$.

Joint Diagonalization for SOTADS

Consider the properties shown in [16]:

(10) $$R_{s^{(q)}s^{(r)}}(\tau) = E\left\{s^{(q)}(t+\tau)s^{(r)}(t)\right\} = (-1)^r \cdot R_s^{(q+r)}(t)$$

where $s^{(k)}(t)$ denotes the k-order derivative of $s(t)$. The Eq.(7) can be converted into

(11) $$R_{\tilde{s}}(\tau) = \begin{bmatrix} R_s(\tau) & -R_s^{(1)}(\tau) & R_s^{(2)}(\tau) \\ R_s^{(1)}(\tau) & -R_s^{(2)}(\tau) & R_s^{(3)}(\tau) \\ R_s^{(2)}(\tau) & -R_s^{(3)}(\tau) & R_s^{(4)}(\tau) \end{bmatrix}$$

where $R_s^{(k)}(\tau)$ denotes the k-order derivative of $R_s(\tau)$.

Obviously, the covariance matrix $R_{\tilde{s}}(\tau)$ is far from diagonal for $\tau \neq 0$. The joint diagonalization of $R_y(\tau)$ for multiple nonzero delays cannot be applied directly. By exploiting the symmetry of $R_{\tilde{s}}(\tau)$, we find that

(12) $$\frac{R_{\tilde{s}}(\tau) + R_{\tilde{s}}^T(\tau)}{2} = \begin{bmatrix} R_s(\tau) & 0 & R_s^{(2)}(\tau) \\ 0 & -R_s^{(2)}(\tau) & 0 \\ R_s^{(2)}(\tau) & 0 & R_s^{(4)}(\tau) \end{bmatrix}$$

Therefore, it now seems much more reasonable for us to regard approximately the right part of Eq.(12) as a diagonal matrix, just like what we do in the previous whitenting process. And consequently,

(13) $$\frac{R_y(\tau) + R_y^T(\tau)}{2} = \mathbf{W}\tilde{\mathbf{A}}\begin{bmatrix} R_s(\tau) & 0 & R_s^{(2)}(\tau) \\ 0 & -R_s^{(2)}(\tau) & 0 \\ R_s^{(2)}(\tau) & 0 & R_s^{(4)}(\tau) \end{bmatrix}\tilde{\mathbf{A}}^T\mathbf{W}^T$$

It is observed that the size of the off-diagonal entries compared to that of the main diagonal entries depends on the bandwidth and frequency characteristic of each source [10].

A generalization of the Jacobi technique for the diagonalization of a single Hermitian matrix has been developed and implemented [13-14]. This extended technique consists of minimizing the so called "joint diagonality" criterion, to be exactly, the off-diagonal entries, as a product of successive Givens rotations. We will not explain its mechanism in detail in this paper.

After applying the extended Jacobi technique on $R_x(\tau) + R_x^T(\tau)$ at multiple nonzero delays, i.e., $\left\{R_x(\tau_k) + R_x^T(\tau_k) \middle| k = 1,...,K\right\}$, a unitary matrix $\mathbf{Q}$ is obtained as the joint diagonalizer. Thus the estimated sources can be calculated as $\hat{\tilde{s}}(t) = \mathbf{Q}^T\mathbf{W}\boldsymbol{x}(t)$, the estimated mixing matrix can be calculated as $\hat{\mathbf{A}} = \mathbf{W}^{\#}\mathbf{Q}$.

Simulation Results and Discussion

Band-limited Gaussian noise signals with different but overlapping spectra were used as the original sources. The delays were generated randomly between 0 and a defined maximum value, that is, max. delay. Related parameters were: N=2, M=8, K=4, fmax=350Hz, Td=0.64ms. Three algorithms were compared, i.e., SOBI based on zero-order Taylor expansion, Rtan-Delay based on first-order Taylor expansion and the proposed SOTADS based on second-order Taylor expansion. Their performance was measured by the cross-correlation index (CCI) as

$$CCI = \frac{1}{N}\sum_{i=1}^{N}\max\left\{\left|R_{s_j\hat{s}_j}\right|\right\} \qquad (14)$$

where $\max\left\{\left|R_{s_j\hat{s}_j}\right|\right\}$ is the maximum value of the normalized and absolute cross-correlation between the original source sequence s_j and the estimated source sequence $\hat{s}_j$.

A simulation example of the original signals and their corresponding estimations obtained by the proposed SOTADS algorithm is presented in Fig.1.

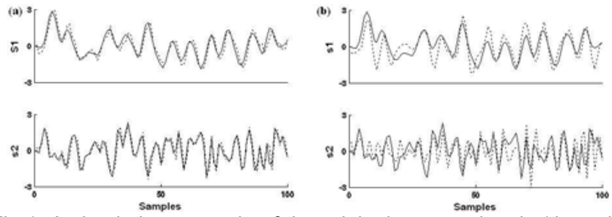

Fig.1. A simulation example of the original source signals (the solid line) and the corresponding estimations (the dashed line) obtained by the proposed SOTADS algorithm. (a) Mixture was generated with maximum delay of 1T_d, the CCI between the two sources and their estimations are 0.9984 and 0.9826 for top and bottom respectively. (b) Mixture was generated with maximum delay of 5T_d, the CCI between the two sources and their estimations are 0.8817 and 0.8081 for top and bottom respectively.

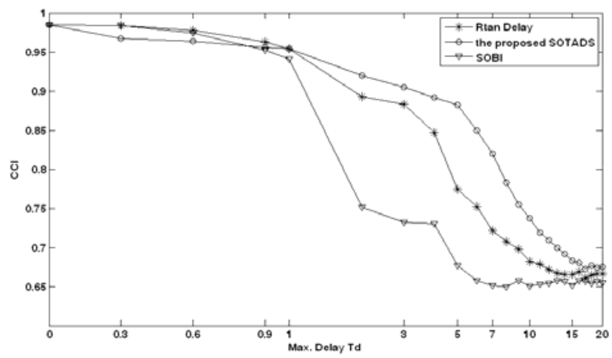

Fig.2. The average simulation results for SNR= ∞dB over 500 independent runs. The horizontal axis denotes the multiple of T_d, linear scale from 0 to 1 and log scale from 1 to 20 for the convenience of observation.

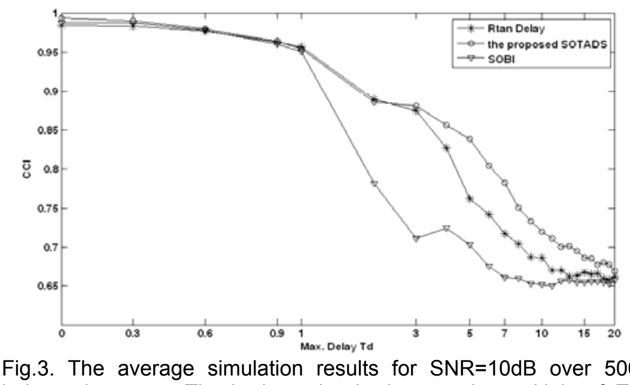

Fig.3. The average simulation results for SNR=10dB over 500 independent runs. The horizontal axis denotes the multiple of T_d, linear scale from 0 to 1 and log scale from 1 to 20 for the convenience of observation.

Fig.2 shows the results for noise-free environment, i.e., SNR=∞dB. For small delays (≤1T_d), the difference among three algorithms is unconspicuous. To be exactly, SOTADS performs slightly worse than Rtau-Delay for 0 ≤ max. delay ≤ 1T_d. The reason for it may be due to the approximation error introduced by the diagonalization assumption in Eq.(9) and Eq.(12). As the delays increase, their performance inevitably deteriorates, especially for SOBI. It is easy to find that the proposed SOTADS significantly outperforms than the other ones. For example when the max. delay reaches 5T_d, the CCI of SOTADS remains 0.9, while Rtan-Delay decreases to 0.77 and SOBI slides to 0.67 dramatically. From another point of view, to maintain the CCI above 0.9, the tolerable max. delay for SOBI is only 1T_d, while 3T_d for Rtau-Delay and 5T_d for SOTADS respectively.

The results for SNR=10dB is presented in Fig.3. The same trend also supports the above analysis. Note that the performance of SOTADS is almost the same as that of Rtau-Delay for 0 ≤ max. delay ≤ 2T_d. Here the approximation error of the diagonalization assumption conteracts the negative impact imposed by the noise to some extent. Moreover, the robustness to noise of the proposed SOTADS becomes outstanding for larger delays (⩾3T_d).

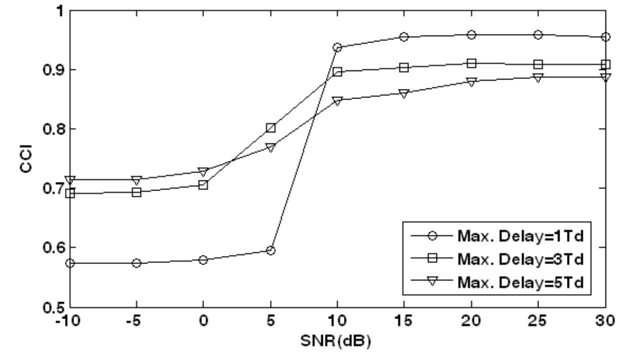

Fig.4. The effects of maximum delay and SNR on CCI performance for the proposed SOTADS algorithm over 500 independent runs.

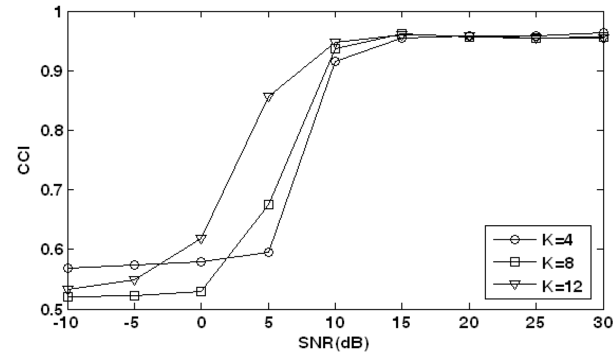

Fig.5. The effects of the non-zero delay number and SNR on CCI performance for the proposed SOTADS algorithm over 500 independent runs.

Fig.4 shows the effects of maximum delay for the proposed SOTADS algorithm under different SNRs. As it is known, the CCI performance improves as the SNR increases. Especially when SNR⩾10dB, three of them become satisfying and keep stable. And to our expect, the 1T_d max. delay case behaves the best, while the 5T_d max. delay case behaves the worest. As for lower SNR (⩽5dB), it is interesting that the larger delay can conteract the more negative effect of noise.

Fig.5 shows the effects of the non-zero delay number participated in the joint diagonalization for the proposed SOTADS algorithm under different SNRs. Similar with Fig.4, SNR=10dB is also a devide. And K=12 seems to be a better choice for 0dB⩽SNR⩽10dB. Their performance become so poor for SNR⩽0dB that it is meaningless to make a comparision.

Conclusion

To extract original source signals from mixtures with propagation delays, the SOTADS algorithm is proposed to handle relatively larger time delays, which makes sense in wide real-world applications. It simplifies this anechoic mixing problem into traditional linear instantaneous BSS by applying second-order Taylor series expansion. The SOTADS algorithm is based on principles from the SOBI algorithm through twice diagonalization approximations but with significant improved performance. Moreover, it enhances the estimation accuracy by utilizing the symmetry of the covariance matrix. How to eliminate the approximation error becomes the next research goal.

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