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Numerical simulation and analytical model of electrical arc impedance in the transient processes

***Abstract**. In the paper are derived analytical expressions for calculation of values of transient resistances of electric arc in the case of interphase short circuits and also is performed numerical MATLAB simulation of electric arc in transient process. Modelling and simulation of electric network and arc was conducted in MATLAB Simulink surrounding, within the mathematical program MATLAB for simulation of transient processes. Empirical approach based on experiment was used in forming of analytical model of arc. Analysis was performed for two-phase and three-phase short circuits in the network, and real parameters of arc impedance are presented in Tables. Model has practical application because all parameters used in analysis can be obtained by measurement.*

***Streszczenie.** W artykule opisano sposób wyznaczenia wyrażenia określającego wartości chwilowe rezystancji łuków elektrycznych w przypadku zwarć międzyfazowych w sieci elektroenergetycznej. Przedstawiono wyniki badań symulacyjnych w programie Matlab. Stworzono empiryczny model łuku elektrycznego, którego wszystkie parametry mogą zostać uzyskane w pomiarach. (**Symulacje i analiza modelu impedancji łuku elektrycznego w stanach nieustalonych**).*

Keywords: electrical arc, short circuits, transient processes, numerical and analytical procedure.
Słowa kluczowe: łuk elektryczny, zwarcie, stan nieustalony, procedury numeryczne i analityczne.

Introduction

In the analyses of transient processes usually is presumed the resistance of electric arc, which has non-linear active character, and that the value of its inductivity equals to zero. Therefore, the assumption is reasonable about the simultaneous passage of currents and voltages through zero value and about active character of arc resistance.

In "Black box" model, the arc is described by a simple mathematical equation and obtained the relation refers to arc conductance, which can be obtained by measurement of parameters such as voltage and current of the arc. Black box model is not suitable for observation of open circuit, but is very reliable for simulation of occurrences in electric arcs circuits in the studies of electric networks. Although it is based on the model of physical conditions, it is just a mathematical model. Black box model is consisted of one or two differential equations. Model parameters can change according to different functions from different data. According to tables used, the accurate parameters of black box model are obtained.

Cassie model refers to study of electric arc conductance behaviour at high currents in time intervals which correspond to plasma temperatures that are higher than 8000 K.

The Mayr model describes the arc conductance in the cases when currents close to value zero. Mayr argues that arc channel has cylindrical shape with the constant diameter. Other black box models capable of simulating thermodynamic occurrences at breaking of the arc channel are the Avdonin model, the Hochrainer model, the Kopplin model, the Schwarz model, and the Urbanek model [1, 2].

All models show that at currents higher than I = 100A, gradient of electric arc voltage has value grad(E)=15V/cm and the value is in invert ratio to values of electric current. Arc model has non-linear electric conductance.

The classical black box models are the Cassie model and the Mayr model. Model Cassie & Mayr with generalized equation has universal character. In its general form, the momentary arc conductance [$g \leftrightarrow R = R_a$, $R = R_l$] is a function of the parameter of the channel with plasma, from which energy is transmitted to surrounding at the same time by convection and radiation.

$$(1)\quad g = F\left(P_{in}, P_{our}, t\right) = \frac{i_{arc}}{u_{arc}} = \frac{1}{R};\Rightarrow R = R_a = R_l = \frac{u_a}{i_a}$$

where: P_{in} – the power supplied to the arc plasma channel, P_{our} – the power transmitted through, t – time, i_{arc} – the instantaneous value of arc current, u_{arc} – the instantaneous value of arc voltage, R = R_a, R = R_l – index (l) translate in Serbian language word electrical arc = luk = arc(english)] – the instantaneous value of arc resistance.

Numerical procedure according to Van der Sluis method

Before the calculation, the Simulink system is initialized, then calculation is performed according to the model of eclectic circuit condition. The sample RLC circuit, which shall be used in this paper as the power system blockset (PSB) is presented on Fig. 1.a.

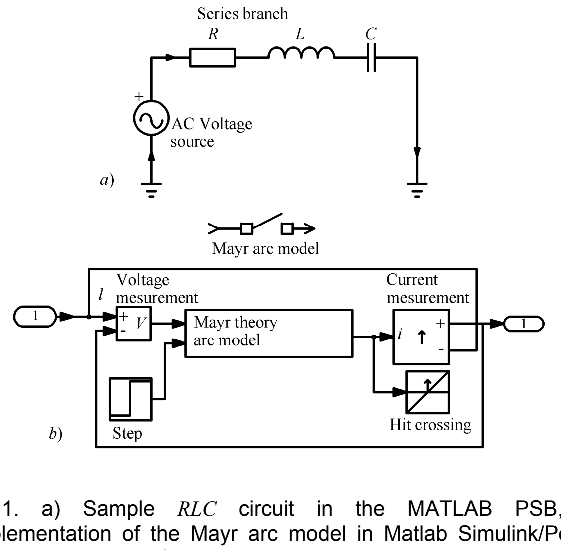

Fig.1. a) Sample RLC circuit in the MATLAB PSB, b) Implementation of the Mayr arc model in Matlab Simulink/Power System Block-set (PSB), [2]

It is important to note that the arrows on Fig. 1.b. do not indicate causes and consequences of creation of value in Simulink block diagram. The arc impedance can be modeled through voltage control of current resources. This approach is presented on Fig. 1.b, where both Mayr block models of the arc in the subsystem are implemented and presented in one block model of the arc which can be freely used.

Another approach was established by Van der Sluis [3], by implementation of the Mayr model in software. In that case, arc model behaves as voltage resource. This technique is presented on Fig. 1.a and 1.b. Network with linear parameters which is connected to arc model, replaces the place in equivalent circuit with corresponding impedance of line loading. Then, the arc resistance R_l is calculated from the differential equation model. Beside the loading impedance and arc resistance, the interesting values are voltage u_s and current i_s. If two points of the load line, u_1, i_1 and u_2, i_2, are known, the voltage u_s and current i_s can be calculated from the following equations:

$$u_s = u_1 + \frac{(R_l i_1 - u_1)(u_2 - u_1)}{u_2 - u_1 + R_l(i_1 - i_2)}, \Leftrightarrow i_s = \frac{u_s}{R_l} \quad (2)$$

After the introduction into the power system block-set, for modeling and simulation of transient process the MATLAB Simulink surroundings has been used, within the mathematical program MATLAB [2, 3]. The Power System Block-set is developed at TEQSIM Inc. and Hydro-Québec. Simulin and its software are designed for modeling, simulation and analysis of dynamic systems [2, 3]. There is also a graphical part which is used for drawing of diagrams of current and voltage flows. In SImulink block are presented mutual components and devices which belong to electrical network. The measurement blocks and control of parameters resources are obtained as electrical signals (voltage's resources as cross elements and current's in line with line- duct) and an Simulink block (transfer functions) [4].

Short circuit on parallel lines

Fig. 2 shows the distribute of currents I_I and I_{II} in the case of a fault-short circuit on line-duct 1 and when breakers $B1$ and $B2$ are closed/open.

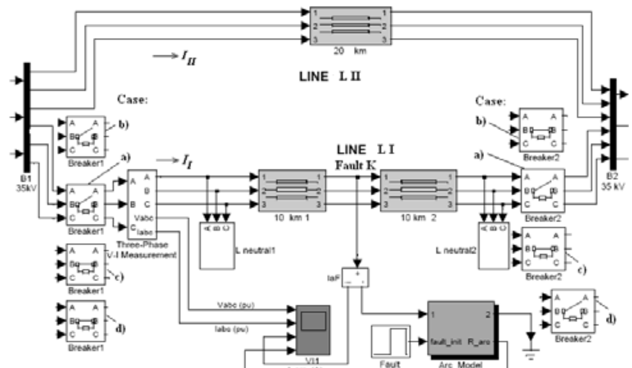

Fig.2. Short circuit on parallel lines (L_{I} i L_{II}). Fault in line L_{I} [5, 6, 7, 8]

On bus $B1$ through which lines are fed, in the case of fault on lines L_{I} or L_{II} currents have the same directions. Currents I_I and I_{II} differ in value; the current in the faulty line is always larger than in the healthy line, because the impedance value is lesser from bus $B1$ to fault point K through which flows the current of faulty line, then the healthy line impedance.

Therefore, the value of difference ($I_I - I_{II} \geq 0$) is grater than zero, and in fault at any line through buses $B1$ and $B2$ flow currents I_{B1}, I_{B2}. Directions and polarity of currents depend on whether line has a fault. At short circuit on line L_{I}, Fig. 2, ($I_I > I_{II}$) current through bus $B1$ I_{B1} is directed towards the larger current in fault line. Unlike from the previous case, in possible short circuit on line L_{II} when ($I_{B1} = I_I - I_{II}$), the current has negative sign and is directed towards higher current, that is, towards current of faulty line. This case is not presented on Fig. 2.

At a receiving end (bus of substation $B2$)-currents I_I and I_{II} have opposite direction. In the faulty line, the current flows from bus of substation $B2$ while in the healthy line to bus $B2$.

In accordance with this, the current in the breaker $B2$ at the bus $B2$ shall have equal values $I_{B2} = I_I + I_{II}$), as it follows from the current sequencing in lines according to Fig. 2.a. The polarity of current (I_{B2}) depends on which of the lines is faulted (L_{I} or L_{II}). At short circuit on line L_{I}, the short circuit current flows through breaker $B2$ towards fault point (point $K = F$) from line L_{II} to line L_{I}. In the case of a fault on line L_{II} the direction of short circuits currents will change, and they will flow from line L_{I} to line L_{II}.

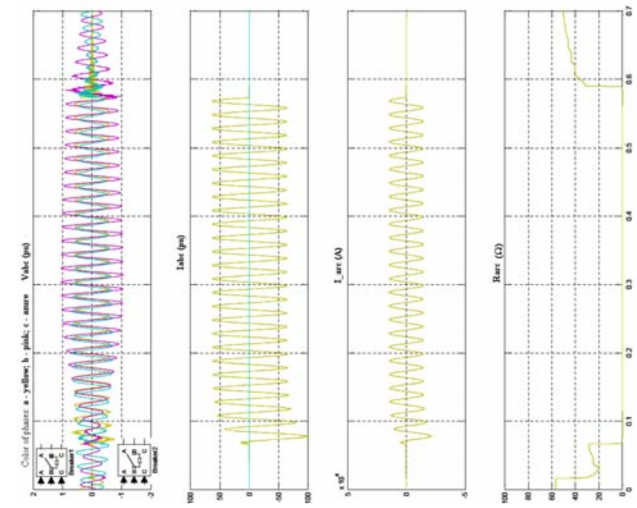

Fig.3. Case: Open Breaker 1, Open Breaker 2, $Rarc(0)$=56Ω

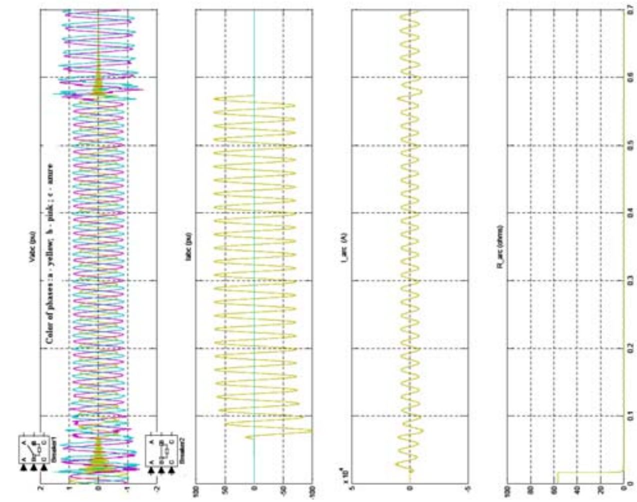

Fig.4. Case Closed Breaker 1, Closed Breaker 2, $Rarc(0)$=56Ω

Analytical procedure of calculation of values of electric arc transient resistances

Analytical approach is very important because it comprises the following important questions.

The influence of electric network parameters values of selected structure and voltage, with arbitrary number of electric generators (n) has been presented by simple electric scheme on Fig. 5, where all impedances are derived to the place where short circuit has occurred, Z_e (R_e, X_e) and voltage U (kV).

The value of derived nominal voltage, which feeds place of short circuit is U and imitance (impedance) of equivalent circuit is Z_e (R_e, X_e), where the impedance character was defined with the factor $\alpha = R_e/X_e$. Transient resistance in the place of short circuit, according to all well-known researches, depends on electric arc resistance value and is nonlinear value.

Previous IEC standard [1, 9], has defined formulae for calculation of arc electric resistance:

(3) $$R_l = 1.05 \cdot \frac{l}{I} [\Omega]$$

where: $l[m]$ – length of arc, $I[A]$ – intensity of electrical current (I_2 – for two phase short circuit, I_3 – for three phase short circuit).

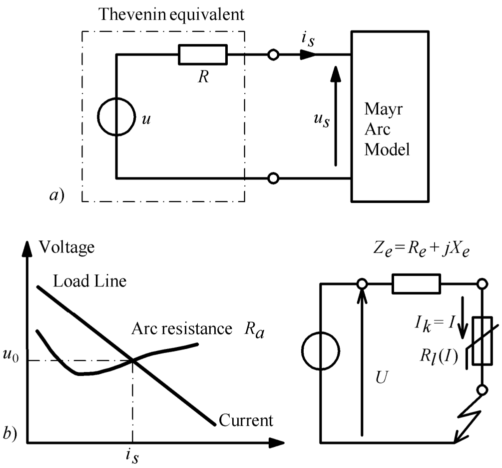

Fig.5. Equivalent circuit: a) numerical method [2], b) analytical approach

Electrical arc at two-phase and three-phase short circuit

During calculation of value R_l, the different arc lengths of breaker in certain phases have been often neglected, which leads to the case that arc current at three-phase short circuit is $\sqrt{3}$ times lesser, and electrical resistance by phase is three times lesser then inter-phase arc resistances. This means that arc resistance by phase, at three-phase short circuits is calculated according to semi empirical method, and is presented with expression obtained at:

(4) $$R_{l(3)} = 1.05 \cdot l \cdot \sqrt{3} / 3 I_3$$

In case when two-phase short circuit occurs, the full current of fault goes through arc, and electric resistance of arc by phase is equal to half value of full arc resistance, [2]:

(5) $$R_{l(2)} = 1.05 \cdot l / 2 I_2$$

Next step in calculation, at equivalent scheme, is to investigate whether values of impedances of direct and inverse order are equal, and if this is the case, the procedure can be simplified according to equality:

(6) $$I_2 = \frac{\sqrt{3}}{2} I_3$$

From the previous formulas follows:

(7) $$R_{l(3)} = R_{l(2)}$$

According to previous assumptions, equation (5) shows that electric arc resistances by phase, at two-phase and three-phase short circuits, in the given regime and at the same point, the electric networks have the same values. Till now the proceeding was from the assumption where were calculated faults at the symmetric three-phase short circuits, and was neglected that at different short circuits the values of burning lengths of electric arc have stochastic nature and that all values depend on immediate structure of the network.

The review of influence of two-phase and three-phase short circuits on the resistance of electric arc

In big electric networks which dispose of sufficient electric power, mainly the condition (6) is satisfied but not the conditions (4) and (5), because the lengths of electric arc in two-phase (l_2) and three-phase (l_3) short circuits are the values of stochastic character and are not mutually equal. This hypothesis influences that expression (7) becomes inequality, which means that the variables $L_i = 1.05 \cdot l_{(i)} / \sqrt{3}$, which indicates the type of short circuit, must be introduced into further calculation:

a) for two-phase short circuit:

Introducing the of shift $L'' = 1.05 \cdot l_{(2)} / \sqrt{3}$, is obtained:

(8) $$R_{l2} = L'' / I_2 = L'' \cdot \sqrt{X_e^2 + (R_e + R_{l3})^2} / U$$

b) for three-phase short circuit:

Introducing the shift $L' = 1.05 \cdot l_{(3)} / \sqrt{3}$, is obtained:

(9) $$R_{l3} = L' / I_3 = L' \cdot \sqrt{X_e^2 + (R_e + R_{l3})^2} / U$$

Using impedances $Z_e = \sqrt{X_e^2 + R_e^2}$ and factor $\alpha = R_e / X_e$ is obtained:

(10) $$\frac{R_{l2}}{Z_e} = \frac{\left(\alpha L''^2 \pm L'' \cdot \sqrt{\alpha^2 L''^2 + U^2 - L''^2}\right)}{\left(U^2 - L''^2\right)}$$

(11) $$\frac{R_{l3}}{Z_e} = \frac{\left(\alpha L'^2 \pm L' \cdot \sqrt{\alpha^2 L'^2 + U^2 - L'^2}\right)}{\left(U^2 - L'^2\right)}$$

It is clear that value of parameter (L) cam occur in many forms (L', L''…), that is, this value becomes variable and indicates the type of short circuit and the length of the arc (l). For complete analysis regarding the possible variable values of voltage (U), a new variable value $\beta = \frac{U}{L}$ must be introduced, where β = (β, β'…).

The verification of the analytical method

Values (L, β) depend on geometric parameters of electrical arc (length and diameter), but closely looking, arc is shaped by plasma current.

Photo shootings of electric arc pillars between the contacts of breaker provide the construction of dependency of plasma current length, which from the moment of circuit separation I_0 do 0. Fig. 6.a shows dependence $lp = f(i)$, where I_0 is a parameter.

This has been verified by the experiments of S.B. Braun, which relate to plasma fluxes, that is, experimentally obtained dependency of the length of plasma current l_p, its average diameter dp and the speed of extension of plasma fluxes v, which is presented on Fig. 6.b. This data relate to investigation conditions with currents to 15kA.

Table 1. gives significant values of company "Minel" Belgrade for breakers of the same class of current I = 630A.

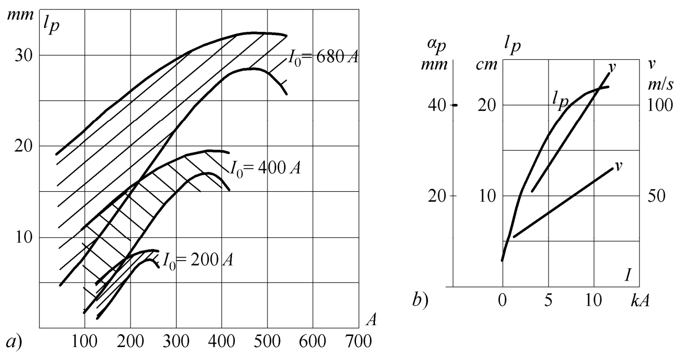

Fig.6. a) Length of plasma current in function of immediate values of currents when circuit is turned off, b) Length of plasma current l_p, diameter d_p and speed of extension v of plasma fluxes

Table 1. Indicated values of breaker of "MINEL" Belgrade, (same current classes for different voltages)

Number	Voltage kV	Current A	Power P_i MVA	Permissible current in 1 s
1	10	630	250	14,5 kA
2	35	630	750	14,5 kA
3	110	630	1500	18,3 kA (for 4 s)

Investigations on breakers for 100A, 200A i 400A [3], which are presented on Fig. 6.a and 6.b, have shown that the lengths arc plasma currents- the lengths of electric arcs- have domain-stochastic character, and therefore according to stochastic criteria from analyzed domains 100A, 200A i 400A can be assumed that this also applies for current class 630A of breakers of MINEL Belgrade, for which are calculated parameters values in Table 2.

Table 2. Results of parameters of electric arc depending on network parameters

Rated current 630 A				U = 10 kV		U = 35 kV		U = 110 kV	
Number	l_l (m)	L' (m)	L'' (m)	β' (kV/m)	β'' (kV/m)	β' (kV/m)	β'' (kV/m)	β' (kV/m)	β'' (kV/m)
1	0	0	0	Infinite value					
2	0,05	0,02625	0,0288	381	346,41	1333,5	1212,4	4187,5	3807,2
3	0,10	0,0525	0,0577	190,5	173,22	666,75	606,2	2093,6	1903,5
4	0,15	0,07825	0,0866	127	115,46	444,5	404,11	1395,7	1269,1
5	0,20	0,105	0,1155	95,2	86,60	333,2	303,11	1046,3	951,7
6	0,25	0,13125	0,1444	76,2	69,28	266,2	242,5	837,4	761,4
7	0,30	0,1575	0,1732	63,5	57,80	222,47	202,3	697,9	635,2
8	0,35	0,18375	0,2021	54,4	49,50	190,25	173,25	598,1	544,1
9	0,40	0,210	0,231	47,6	43,30	166,67	151,55	523,4	475,9

According to suggested method, the values in Table 2 are calculated and arranges, and for calculation are used expressions (12) and (13) regarding the type of short circuit switched off by the breaker:

$$\frac{R_{l2}}{Z_e}=\frac{\left(\alpha\pm\sqrt{\alpha^2+\beta''^2-1}\right)}{\left(\beta''^2-1\right)} \tag{12}$$

$$\frac{R_{l3}}{Z_e}=\frac{\left(\alpha\pm\sqrt{\alpha^2+\beta'^2-1}\right)}{\left(\beta'^2-1\right)} \tag{13}$$

Since according to relations (3), (8) and (9) the value β>1, radicands in relations (12) and (13) are always greater then α^2, therefore because (R_e/Z_e>0) doesn't have to include negative values. The following solutions are obtained:

$$\frac{R_{l2}}{Z_e}=\frac{\left(\alpha+\sqrt{\alpha^2+\beta''^2-1}\right)}{\left(\beta''^2-1\right)} \tag{14}$$

$$\frac{R_{l3}}{Z_e}=\frac{\left(\alpha+\sqrt{\alpha^2+\beta'^2-1}\right)}{\left(\beta'^2-1\right)} \tag{15}$$

Introducing universal variable β into calculation the relation becomes universal:

$$\frac{R_l}{Z_e}=\frac{\left(\alpha\pm\sqrt{\alpha^2+\beta^2-1}\right)}{\left(\beta^2-1\right)} \tag{16}$$

With the increasing of factor value α, the value of arc resistance at two-phase and three-phase short circuit also increases. This can be explained with the fact that with the increase of proportion of active resistance R_e the value of arc current decreases.

In the case of inductive character of equivalent resistance (R_e = 0, α = 0), the simpler relation is obtained:

$$\frac{R_l}{Z_e}=\frac{1}{\sqrt{\left(\beta^2-1\right)}} \tag{17}$$

The largest value of resist arc would be attained if equivalent resistance of ohm character is α = 1. With increase of value (β) relative value of transient arc resistance decreases, this can be seen from the relation (17).

The Analysis of Results of Analytical procedure

Fig. 7 shows the family of analytically obtained functions $\frac{R_l}{Z_e}$ □□□□□□□ for different values of factor α. From previous description it can be concluded that with increase of factor α the value of electric arcs increases in the presence of two-phase and three-phase short circuit, and if variable β increases then relative value of electric arc decreases:

$$\varphi(\beta) = \frac{R_l}{Z_e} = \frac{\alpha + \sqrt{\alpha^2 + \beta^2 - 1}}{\beta^2 - 1} = \frac{1.05 \cdot l_l}{Z_e} \cdot \frac{1}{I_l} \tag{18}$$

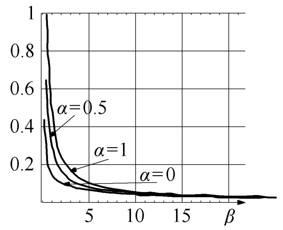

Fig.7. Family of analytical mathematical functions $\frac{R_l}{Z_e} = \varphi(\beta)$, parameter $\alpha = \frac{R_e}{X_e}$, $\beta = \frac{U}{L}$

The presented analytical procedure for solving of transient process of electric arc and obtained expressions from transient voltages in the case of interphase short circuits, presents the contribution by which in modern way, with semi empirical and stochastic approach and with simulation, the most accurate solution is obtained. Obtained results confirm possibility of application of analytical procedure for solving of problems which are not comprised with modern numerical methods.

Conclusion

In this paper a new analytical procedure was developed for analysis of processes in equivalent circuit at the fault of electric arc as nonlinear element, and analytical equations have been derived. Analytical expressions are based on determination of variable parameters in electric networks and are variable for precise adjustment in calculation of electric arcs resistances.

In electric networks exists numerous connections with nodes and their elements. Elements and nodes are connected with special relations with known quantities and parameters.

On lines which are linked in nodes or on buses of distribution plants due to given stability conditions, it is necessary to disconnect short circuits followed by electric arcs in given domain without time delay. Transient process occurs and lasts in time from a few μs (in the case of overvoltage because of short circuits on lines) to a few ms (when transient processes are followed by commutating overvoltages), and also can be of s order at ferroresonance processes.

Arc currents are usually interrupted by current breakers in time μs. Since current at that time interval maintains the constant value, the voltage of electric arc achieves the peak value-extinction value, then decreases to zero, which can be seen from the diagrams of voltage flows du/dt, currents di/dt and values of arc resistance on Fig. 3 and 4. Since arc impedance has the resistance character, voltages and currents simultaneously pass through zero.

In analytical procedure processes in circuit with electric arc are presented on schemes on Fig. 5. Breakers interrupt short circuits (three-phase, two-phase and earth faults) in the moment when current passes through zero value. Impedances that are proportional to lines length to the fault place decrease the value of current fault.

Developed transient program XTrans was used for analysis of transient process and for comparisons in analytical procedure. This program was developed by Bijl and Van der Sluis and contains MNA (the modified nodal analysis) method of application of equation of state of example of breaker influence in voltage fault with parallel capacitances and current through inductivities.

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