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Investigation of multicriteria optimal control with time-variable weight coefficients

Abstract. *A new approach for construction of quality functional at multicriteria optimization is proposed. A two-mass dynamical system which consists of two subsystems is considered. A case when in the area of large errors unstable subsystem dominates is considered. Function L is chosen for switching between subsystems and its optimal parameters are found.*

Streszenie. *W artykule zaproponowano nowe podejście do opracowania funkcji jakości w optymalizacji wielokryterialnej. Badania przeprowadzono na systemie dynamicznych dwóch mas, dla przypadku przeważającej niestabilności jednej z nich. Zoptymalizowano parametry funkcji L, wybranej do określania przejść między obiektami. (**Badanie sterowania optimum wielokryterialnym o współczynnikach wagowych zmiennych w czasie**).*

Keywords: optimal control, fuzzy logic, dynamic system.
Slowa kluczowe: optymalna kontrola, logika rozmyta, system dynamiczny.

Introduction

Currently, for optimization of technical systems one traditionally uses approaches that are well known in the linear systems theory. In particular, there are analytical method for controllers designing [1], Pontryagin maximum principle, Bellman dynamic programming [2], [3] and node methods. The disadvantages of these approaches are that they do not take into account changes in the conditions of the system and object's modification.

Application of methods of nonlinear control theory, including feedback linearization [4] has not found widespread use because of difficulties in finding aggregated variables in technical systems. Also, today is not common to apply methods of geometric control theory [5].

An attempt of control influences adaptation to the object's state and conditions of technological process flow by means of the switching systems forming and providing sliding modes along set trajectories leads to possibility of self-oscillations and to so called supercontrol.

New possibilities of multicriterion optimal control problem solving gives an application of fuzzy set theory. In particular, Shin and Chang [6] proposed a global criterion approach on the basis of fuzzy logic to obtain solutions of multicriterion strict and fuzzy control synthesis. Loetamonphong [7] studied optimization problems which has multi-objective functions with fuzzy constraints of equality type. Huang [8] proposed a method of fuzzy multi-objective optimization decision-making method that can be applied to two and more objectives of system functioning. Among methods of optimal control synthesis one should also mention "piecewise Lyapunov function" method. It implies that for a set of subsystems a generalized Lyapunov function is formed from Lyapunov functions for each subsystem taken with some coefficient. Fuzzy control allows synthesizing control influences in the areas where system is functioning and provides the passes between these areas. So, one can speak of control influences synthesis for set of subsystems which form dynamic system. Similar approach is applied here.

One of the possible ways of system optimization is application of Takagi-Sugeno fuzzy controller [9]. Output of this controller is a control influence which is typical for control systems with full state vector. So, for a particular rule, synthesis of control actions based on the classical theory of linear systems is possible.

One uses an object model which is linearized in given area taking into account all imposed restrictions that will operate in this area. In particular, this technique is applied in the works of Mitsuishi T., Shidama Y. [10].

Using, for example, method of Bellman dynamic programming one can consider various restrictions that are necessary for normal functioning of the system such as, for example, limitations on performance that may be useful for systems with backlash, bins and conveyor systems etc. If one changes the workspace then another control signal, which is optimal for a given point of the region of state space system, is synthesized. Thus for its synthesis one can use model obtained by linearization of the nonlinear system at a given point.

Problem statement

In classical control theory generalized functional, at finding the optimal control for the entire system, has the following form

$$F = \sum_i \lambda_i F_i ,$$

where i – index of individual subsystem model, $\lambda_i = const$ is defined basing on peer reviews or on the theory of Pareto optimal solutions. In particular, in Fig. 1 the output signals at different values of the coefficients $\lambda_1 = 1 - \lambda_2$ in the case when the controller of subsystem, which operates in the area of large errors, is set on a standard form of Butterworth $F_1(\overline{x}(t)) = x^2(t) + \omega_0^{-6}\dddot{x}(t)$, and a controller that operates in the vicinity of small errors is set to binomial form $F_2(\overline{x}(t)) = x^2(t) + 3\omega_0^{-2}\dot{x}(t) + 3\omega_0^{-4}\ddot{x}(t) + \omega_0^{-6}\dddot{x}(t)$, is shown.

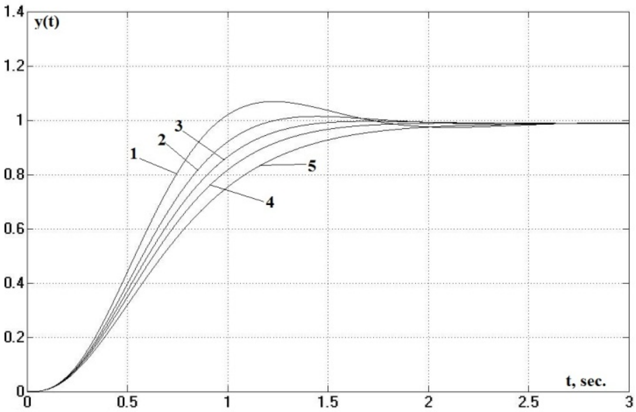

Fig. 1. Output signal of the system in the case of configuration to 1) standard form of Butterworth; 2) $\lambda_1 = 0.7$; 3) $\lambda_1 = 0.5$; 4) $\lambda_1 = 0.3$; 5) binomial form.

One can see that the trajectory of motion of the system lies between the lines formed by output signal "1" and "5".

At present, there is no general approach to the fuzzy controller synthesis. However, for systems with Takagi-Sugeno fuzzy controller, control signal for a particular rule, based on methods of synthesis of control systems by the full state vector, can be synthesized. Synthesizing these regulators, one comes to the structure, which can be represented as

$$u(t) = \sum_i \lambda_i(\overline{x}) u_i ,$$

where $u_i = u(\overline{x}(t), \overline{k})$ is vector-function, $\overline{k}$ is vector of coefficients.

Optimal control $u(t) = u^*(t)$, synthesized for each subsystem, provides a single functional F_i minimization formed for i-th point of state space.

At application of the fuzzy sets theory one actually comes to the functional of the following form

$$F = \sum_i \lambda_i(t) F_i, \quad \sum_i \lambda_i(t) = 1,$$

where $\lambda_i(t)$ are time-variable coefficients, which are defined by membership function type, their location, accepted defuzzyfication method and by formed basis of rules.

Thus, we do not form a trajectory which is optimal for all subsystems of the family, but realize the transition from one optimal trajectory to another specified for a particular subsystem or region of state space. This approach allows us to improve the quality of the system characteristics.

At application of the above approach, the criterion is formed as follows

$$J = \int_0^\infty \left(\sum_i \lambda_i(\overline{x}(t)) F_i^*(\overline{x}(t)) + u^2(t) \right) dt,$$

$$\sum_i \lambda_i(\overline{x}(t)) = 1,$$

which is typical for systems with fuzzy control using Takagi-Sugeno controller, and the trajectory of system's motion (see Fig. 2), unlike the classical case, consists of a section of paths "1" and "3".

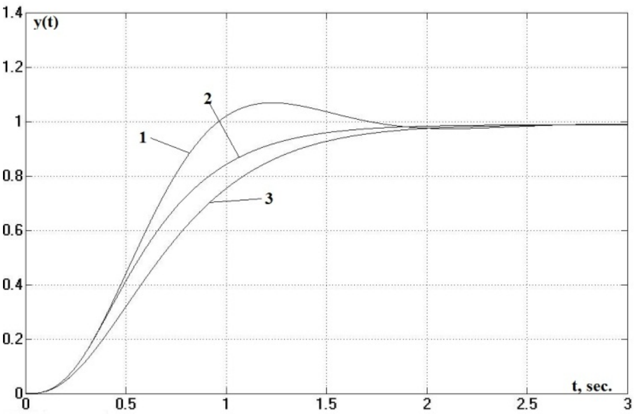

Fig. 2. Output signal of the system in the case of configuration to 1) standard form of Butterworth;
2) $\lambda_1 = 1 - \lambda_2 = L(x_{inp} - x(t); 0.8, 0.9)$ see.(3); 3) binomial form

Taking into account that the weight multiplier depends on the point of state space where the system is now, we come to the system's trajectory formation as a set of optimal trajectories for specific regions.

Each subsystem can generate different types of transitions with different speed. It is possible to form different trajectories of transition to a given point in output signals space using the control in which there is a transition between the control actions.

Formed, thus control influences can synthesize a system that, according to classical theory, can be unstable in the area of large deviations. But due to the fact that, during operation, control influences change in the vicinity of the working point the system is stable.

Results of investigation

Let us consider an example of this approach for the case of two-mass system (see e.g. [11]), which is typical for speed control in many mechanisms.

The structure of such system is given in Fig. 3. Consider the synthesis of various control influences in such a system and thus assume that each of the synthesized systems can be considered as a separate subsystem of the system shown in Fig. 3.

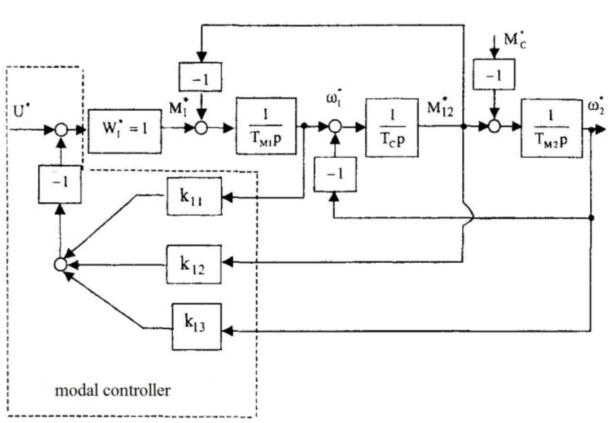

Fig. 3. The scheme of modal speed control

On its basis, let us build a model of two-mass system which consists of two subsystems. To ensure a smooth switching between the subsystems and eliminate sliding modes we used membership function as influence's weight function for each of the subsystems on the motion trajectory of the whole system. To analyze the functioning effectiveness of the system which is formed by a combination of several subsystems, we apply integrated quality scores

$$I_1 = \int_0^T e^2(t) dt; \qquad I_2 = \int_0^T te^2(t) dt;$$
(1)
$$I_3 = \int_0^T |e(t)| dt; \qquad I_4 = \int_0^T t|e(t)| dt.$$

To estimate the quality of the system in Fig. 3 based on multicriteria optimization we compute generalized integral index of quality

(2) $$I = \sum_{j=1}^4 \gamma_j I_j ,$$

where the coefficients γ_j can be determined either by the Pareto-optimal solutions or defined by peer reviews. We set $\gamma_j = 0.25,\ j = \overline{1..4}$.

The coefficients of controller corresponding to binomial form configuration or standard Butterworth form are well known and are given e.g. in [12]. At research of the case when in the area of large errors controller is configured to unstable subsystem (in the case of one and two roots in the right half-plane) the coefficients given in [13] were used.

To switch between subsystems L function (see e.g. [14]) was used

(3) $$L(x;\alpha,\beta)=\begin{cases}1, & x<\alpha\\ (x-\beta)/(\alpha-\beta), & \alpha\le x\le\beta\\ 0, & x>\beta,\end{cases}$$

where the parameters α and β determine the boundaries of polar error values corresponding to the cases of subsystems functioning in the areas of either large or small errors, respectively.

Let the synthesis problem is to create optimal trajectory of the system with the maximum possible, at adopted conditions, speed, at the absence of overshoot. In this case if the error is large, a determining influence will have control received by setting up the system to a standard Butterworth form or control configured to one of the unstable subsystems (depending on the selected coefficients of controller).

On the basis of the calculations quantitative benefits of the investigational approach (see Table. 1-3) were determined and compared with the case when the system is configured only to binomial form, or only to a standard Butterworth form.

Table 1. Comparison of the quantitative effect of using the system with stable subsystems with corresponding optimal parameters

	Function L (L)	Binomial form (Bin)	Butter-worth (But)	L/Bin	L/Bat
I_1	0.419	0.522	0.419	0,803	1,000
I_2	0.105	0.176	0.106	0,597	0,991
I_3	0.580	0.789	0.604	0,735	0,960
I_4	0.254	0.465	0.299	0,546	0,849
I	0.339	0.488	0.357	0,695	0,950
t,c	0.970	1.662	0.915	0,584	1,060
$t_{5\%}$ c	0.970	1.662	1.420	0,584	0,683
max	1.017	0.988	1.069	1,029	0,951

Table 2. Comparison of the quantitative effect of using the system with unstable subsystem in the case of one root in the right half-plane at the corresponding optimal parameters

	Function L (L)	Binomial form (Bin)	Butterworth (But)	L/Bin	L/Bat
I_1	0.339	0.522	0.419	0.649	0.809
I_2	0.069	0.176	0.106	0.392	0.651
I_3	0.478	0.789	0.604	0.606	0.791
I_4	0.165	0.465	0.299	0.355	0.552
I	0.263	0.488	0.357	0.539	0.737
t, c	0.879	1.662	0.915	0.529	0.961
$t_{5\%}$,c	1.987	1.662	1.420	1.196	1.399
max	1.026	0.988	1.069	1.038	0.960

Table 3. Comparison of the quantitative effect of use the system with unstable subsystem in the case of two roots in the right half-plane at the corresponding optimal parameters

	Function L (L)	Binomial form (Bin)	Butter-worth (But)	L/Bin	L/Bat
I_1	0,306	0,522	0,419	0,586	0,730
I_2	0,057	0,176	0,106	0,324	0,538
I_3	0,444	0,789	0,604	0,563	0,735
I_4	0,153	0,465	0,299	0,329	0,512
I	0,239	0,488	0,357	0,490	0,669
t, c	0,805	1,662	0,915	0,484	0,880
$t_{5\%}$, c	0,731	1,662	1,42	0,440	0,515
max	1,035	0,988	1,069	1,048	0,968

Research has shown that in the case of two stable subsystems (configured to a standard Butterworth form and binomial form), when switching from first to second, it is optimal to use function L with the parameters $\alpha=0.48, \beta=0.5$ since the value of the integral quality index is minimal.

Studies have shown that in the case of subsystem with one root in right-hand plane, the membership function L with the parameters $\alpha=0.87, \beta=0.88$ provides a benefit in comparison with the configuration to the binomial or Butterworth filter. At this maximum overshoot as applying this function does not exceed the 5% area.

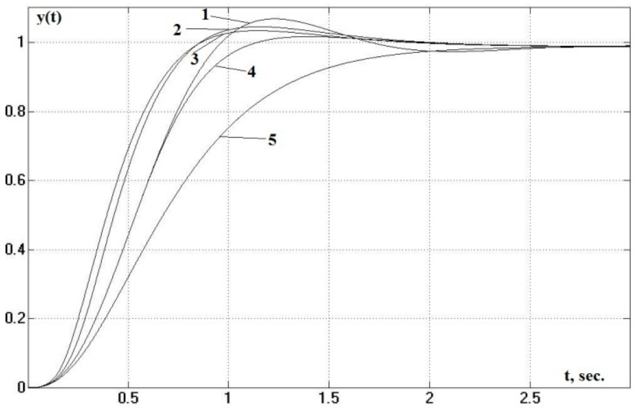

Fig.4 Output signal of the system in the case of configuration to 1) standard form of Butterworth; 2) subsystem with one root in the right half-plane $L(u;0.87,0.88)$; 3) subsystem with two roots in the right half-plane $L(u;0.9,0.96)$; 4) stable subsystems synthesized according to standard forms of Butterworth and binomial $L(u;0.48,0.5)$; 5) binomial form

Research has shown that in the case of subsystem with two roots in right-hand plane, it is optimal to use membership function L with parameters $\alpha=0.9, \beta=0.96$. The results of system simulation for all studied cases are shown in Fig. 4

Conclusion

Thus, in this paper criterion, which unlike classical, will be formed as a set of specific criteria, weight of which will vary depending on the facility and process requirements is proposed.

Research held on the example of two-mass systems which consists of two subsystems. The transition between the subsystems occurs due to the use of membership function that depends on parameters. For each case, the same method of defuzzyfication is applied namely simplified method of center gravity. Qualitative indices of the system are investigated on the basis of the generalized integral index of quality. In each case, for a range of parameter values of integral indexes of quality, maximum overshoot, time of entry into the 5% area and time of first achievement of a given performance level were calculated.

According to the research optimum parameter, values for the investigated membership function for each of the cases were found. If one chooses between the subsystems to use (with one root in the right half-plane, with two or no roots in the right half-plane), the survey results show that one should choose an unstable subsystem with two roots in the right half-plane, as its general integral index of quality is smaller while maximum overshoot does not exceed 5% area. For transition between the subsystems, in this case, one should use membership function L with parameters $\alpha=0.9, \beta=0.96$.

If on the intermediate coordinates some restrictions be imposed, these restrictions will be included in the generalized integral index of quality (2) and in values of the investigated membership function parameters, and hence output signal of the system will differ from those set forth here. It should also be added that the criteria formed ensure stability of the whole system, despite the presence of the root in the right half-plane in one of the subsystems [15–17].

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