

# An algorithm for fault diagnosis in analogue circuits based on correlation

**Abstract.** The paper deals with algorithm for single and multiple catastrophic fault diagnosis in analogue circuits. The algorithm bases on FFT analysis of the circuit response to the rectangular testing signal and uses as a classifier Pearson product-moment correlation coefficient. The algorithm represents SBT technique and requires multiple analyses of circuit under test, which enable us to built a fault dictionary. Each entry of dictionary is assigned with one fault. The numerical example shows the effectiveness of the proposed algorithm.

**Streszczenie.** W artykule przedstawiony został algorytm diagnozowania pojedynczych i wielokrotnych uszkodzeń układów analogowych. Algorytm bazuje na wykorzystującej FFT analizie odpowiedzi badanego układu na prostokątny sygnał wejściowy oraz stosuje jako klasyfikator współczynnik korelacji Pearsona. Algorytm reprezentuje technikę SBT i na etapie przygotowawczym wymaga wielokrotnych analiz diagnozowanego układu pozwalających na zbudowanie słownika uszkodzeń. Jego sygnatury odpowiadają uszkodzeniom, których możliwość występowania została przewidziana na etapie przygotowawczym. Przedstawiony przykład obliczeniowy potwierdza efektywność algorytmu. (**Algorytm diagnostyki błędów w układach analogowych bazujący na korelacji**).

**Keywords:** fault diagnosis, catastrophic faults, correlation, parameter tolerance

**Słowa kluczowe:** diagnostyka uszkodzeń, uszkodzenia katastroficzne, korelacja, tolerancja parametrów

## Introduction

The problem of failure detection in electronic circuits appeared when the first electronic circuits were produced. The problem's severity started to intensify along with the increasing complexity of the circuits and decreasing accessibility to their inside. The rapid development of diagnostics, observed since the early 1980's, has led to the invention of many new methods [1]-[5], which performance in the case of digital circuits can be regarded as satisfactory. The problem with the diagnostics of analog circuits is still on the map, because the effective, universal, quick and sufficiently accurate method was not yet developed, and thus the area of research cannot be regarded as exhausted. The need of finding new diagnostic solutions for analog circuits is confirmed also by the comparison of the number of failures in the current analog and digital circuits. Despite the fact that digital ones make the most of circuits produced worldwide, much more failures occur in the analog circuits. The need for new diagnostic solutions applies to methods for the catastrophic faults as well as for the soft ones.

The major challenges of modern diagnostic problems include problems appearing in the examination of failures in highly-integrated systems. One of the most important problem is to ensure a sufficient number of measuring points, while there is often very limited access to the circuit's inside. The term "measurement point" means both a place of measurement and the measurand which changes are the basis of the diagnostic process. At the design stage the issue involves the question of proper choice of measurement points, i.e. the design of appropriate access to the interior of the circuit. The proposed method uses one measurement location - output of the system, and the number of measurands was increased by measuring the harmonics of the output signal diagnosed.

Very important parameter of diagnostic algorithms is the time of obtaining the result with particular emphasis on the time relationship of the process before the reading, at the pre-test stage, and the time needed to complete the second part of the diagnosis, the post-test stage. The ratio of the two times determines the division of methods for algorithms with the pre-test simulation, known in short as the SBT technique (named after the English: simulation-before-test) and post-test simulation algorithms, known as SAT techniques (from the determination of English: simulation-after-test). The first group consists mainly of dictionary

algorithms. Application of the second group of methods is the system design stage.

During the last years many methods for testing of the analog circuits have been developed [6]-[13] but all of them have not achieved the development level of the method for digital circuits. Due to element tolerances, as well as nonlinearity of circuits and other reasons, the analogue circuits fault diagnosis is much more complicated than digital circuits fault diagnosis. Therefore, the research of analogue circuit fault diagnosis is always a hot and challenging subject.

The presented paper deals with an algorithm for single and multiple catastrophic fault diagnosis in analogue circuits. The algorithm represents SBT technique and requires multiple analyses of circuit under test, which enable us to built a fault dictionary. Signatures of the dictionary constructed on the basis of pre-test analyzes of the studied system include the relative differences between measured harmonics of the nodal voltages of the system intact and the system with failure. Each signature of the dictionary are assigned with one fault. The algorithm bases on harmonic analysis of the circuit response to the rectangular testing signal [2],[3]. This testing signal and the analysis are easy to reach because both, a rectangular signal and FFT analysis, are offered in all contemporary oscilloscopes. The execution of one test and the FFT analysis is less time-consuming as the performance of a few test with the sinusoidal signals having different frequencies.

## Description of the proposed method

Method described below has much in common with the one proposed in [14] and [15]. The basis of both methods is the analysis of the system in a function of frequency. The important difference is the kind of faults and the kind of classifier used in the process of fault diagnosis. The algorithm presented in this paper is provided to the diagnose of catastrophic defects in dynamic parametric analog circuits with using the correlation classifier.

Values measured in the proposed method are the harmonic of the signals in the available measurement nodes of the investigated system (only one point: output of the circuit in the presented example). The effectiveness of the method depends on a change in function of frequency of a characteristic of system's dependence on stimulation. To minimize the time of system's testing, it is expected to

use the square wave signal for stimulation, which provides the smallest decrease in the effective value of the harmonic increases. Harmonic analysis is advantageous for two reasons: it allows to increase the number of measured values (greater number of measurement points) and is easily feasible in practice by the use of Fast Fourier Transform function, which is available in commercial measuring instruments.

Analysed quantities are the relative differences of amplitudes of harmonics at the test point in the healthy and faulty circuit:

$$(1) \quad \delta_i^j = \frac{A_i^j - A_i^1}{A_i^0}; \quad i = 1 \dots n; \quad j = 1 \dots k$$

where:  $n$  is the number of analysed harmonics,  $k$  is the number of investigated circuit states (including the healthy circuit,  $j=1$ ).

The proposed method is the dictionary method. Each signature of the dictionary are assigned with one state of the circuit under test. It consists of the  $n$  amplitude values of harmonics. The amplitude values in the dictionary are calculated during the pre-test simulations. The state of the circuit under test is associated with the harmonics of the measured signal. The set of harmonics measured in the circuit under test needs to be dependent on one of the sets of harmonics associated with the signatures of dictionary. The presented algorithm uses the Pearson product-moment correlation coefficient to determine the linear dependence of the two set of harmonics. Pearson's correlation coefficient  $P_{xy}$  between two variables  $x$  and  $y$  is defined as the covariance of the two variables divided by the product of their standard deviations [16]. The formula for the Pearson's coefficient applied to a sample is as follow:

$$(2) \quad P_{xy} = \frac{\sum_{i=1}^{i=n} (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{i=n} (x_i - \bar{x})^2} \sqrt{\sum_{i=1}^{i=n} (y_i - \bar{y})^2}}$$

where:  $\bar{x} = \frac{1}{n} \sum_{i=1}^{i=n} x_i$ ;  $\bar{y} = \frac{1}{n} \sum_{i=1}^{i=n} y_i$ ;  $n$  is the number of analysed samples. This coefficient is a measure of the correlation (linear dependence) between two variables, giving a value in a range from -1 to 1.

The algorithm calculates the Pearson's coefficients for the harmonics of the measured at the test point signal and all sets of harmonics in dictionary. The best value of calculated coefficients indicates the state of circuit under test. Usually it is not equal to 1 because of the element tolerances.

The values of harmonics in the dictionary are calculated with the nominal values of all circuit elements. The element tolerances cause that the harmonics measured in the circuit under test are different from the calculated ones. Fortunately, the differences between the calculated and measured values are usually relative small. So the calculated and measured differences of the harmonics are fast the same as they have the same character. Thus the element tolerances cause that the best value of calculated coefficients is not equal 1 but usually they are not reason of the error.

### The numerical example

The analysis of the benchmark circuit [17] shown at the figure 1 is presented as a numerical example. It illustrates the proposed algorithm.

The presented circuit is a filter with three outputs: HPO – high - pass output, BPO – band - pass output and LPO –

low – pass output. Nominal values of all elements are shown in the figure.

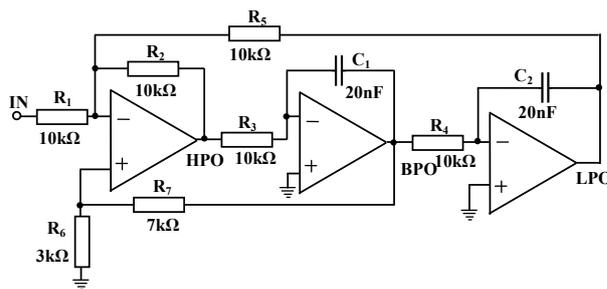


Fig.1. The benchmark circuit analysed in the paper

The rectangular signal with the amplitude 5V and the frequency 100Hz is closed at input node, marked IN (fig.1). An output signal is acquired at only one measurement node marked in the figure 1 as LPO. The constant term of Fourier series, 0-th harmonic, and all odd harmonics from 1-th to 15-th (9 harmonics) are used in diagnostic process. Simple catastrophic faults of all resistors with the exception of 4 (in this 4 cases the output signal is not periodic) and 7 selected double faults are considered. The single faults are short circuits of  $R_1, R_3, R_4, R_5, R_6, R_7$  and open circuits of  $R_2, R_5, R_6, R_7$ . Double faults are:  $R_1$ -short+ $R_2$ -open,  $R_1$ -short+ $R_4$ -short,  $R_1$ -short+ $R_5$ -open,  $R_1$ -short+ $R_5$ -short,  $R_2$ -open+ $R_4$ -short,  $R_2$ -open+ $R_5$ -open,  $R_2$ -open+ $R_5$ -short. The 18. state is the unfaulty circuit.

The analyses for 18 states of the circuit under test are performed. 9 harmonics for each state and then all relative differences of amplitudes of harmonics at the test point in the healthy and faulty circuits, according to equation (1), are calculated. The relative differences enable us to calculate correlation coefficient for all faulty states of the circuits using the formula (2). The results are presented in the table 1. The results of the analyse of each column are as follows. In the circuit with nominal values of all unfaulty elements the diagnose for 11 states of the faulty circuit is unambiguous (in each column associated with this states is single 1). For 6 states of faulty circuit the diagnostic decision is ambiguous. It is impossible to distinguish 2 states:  $R_6$ o (open circuit of  $R_6$ ) and  $R_7$ s (short circuit of  $R_7$ ), they create the ambiguous group in this algorithm (in each column associated with this states is double 1).

Table 1. The correlation coefficients for all analysed faulty states

	R1s	R2o	R3s	R4s	R5o	R5s
R1s	1.0000	0.5880	0.0609	-0.2196	0.9636	0.5441
R2o	0.5880	1.0000	0.8197	0.6578	0.3637	0.8877
R3s	0.0609	0.8197	1.0000	0.9452	-0.1746	0.7192
R4s	-0.2196	0.6578	0.9452	1.0000	-0.4505	0.5771
R5o	0.9636	0.3637	-0.1746	-0.4505	1.0000	0.3854
R5s	0.5441	0.8877	0.7192	0.5771	0.3854	1.0000
R6o	0.9495	0.3460	-0.1569	-0.4525	0.9894	0.3446
R6s	-0.5388	-0.3728	-0.2602	0.0109	-0.5148	-0.3182
R7o	-0.5493	-0.3767	-0.2569	0.0162	-0.5255	-0.3223
R7s	0.9482	0.3450	-0.1557	-0.4521	0.9885	0.3437
R1s R2o	1.0000	0.5876	0.0603	-0.2201	0.9638	0.5437
R1s R4s	0.8721	0.9047	0.5295	0.2838	0.7249	0.8408
R1s R5o	1.0000	0.5901	0.0635	-0.2170	0.9630	0.5459
R1s R5s	-0.2402	0.6417	0.9399	0.9997	-0.4695	0.5595
R2o R4s	0.9596	0.7865	0.3181	0.0565	0.8583	0.7307
R2o R5o	1.0000	0.5901	0.0636	-0.2170	0.9630	0.5459
R2o R5s	0.9759	0.4140	-0.1528	-0.4125	0.9861	0.3873

	R6o	R6s	R7o	R7s	R1sR2o	R1sR4s
R1s	0.9495	-0.5388	-0.5493	0.9482	1.0000	0.8721
R2o	0.3460	-0.3728	-0.3767	0.3450	0.5876	0.9047
R3s	-0.1569	-0.2602	-0.2569	-0.1557	0.0603	0.5295
R4s	-0.4525	0.0109	0.0162	-0.4521	-0.2201	0.2838
R5o	0.9894	-0.5148	-0.5255	0.9885	0.9638	0.7249
R5s	0.3446	-0.3182	-0.3223	0.3437	0.5437	0.8408
R6o	1.0000	-0.5892	-0.5991	1.0000	0.9496	0.7065
R6s	-0.5892	1.0000	0.9999	-0.5925	-0.5387	-0.5136
R7o	-0.5991	0.9999	1.0000	-0.6023	-0.5492	-0.5213
R7s	1.0000	-0.5925	-0.6023	1.0000	0.9483	0.7054
R1s R2o	0.9496	-0.5387	-0.5492	0.9483	1.0000	0.8718
R1s R4s	0.7065	-0.5136	-0.5213	0.7054	0.8718	1.0000
R1s R5o	0.9489	-0.5396	-0.5500	0.9476	1.0000	0.8733
R1s R5s	-0.4704	0.0197	0.0252	-0.4699	-0.2407	0.2631
R2o R4s	0.8351	-0.5080	-0.5175	0.8336	0.9594	0.9723
R2o R5o	0.9489	-0.5395	-0.5500	0.9476	1.0000	0.8733
R2o R5s	0.9641	-0.4505	-0.4618	0.9623	0.9760	0.7524

	R1sR5o	R1sR5s	R2oR4s	R2oR5o	R2oR5s
R1s	1.0000	-0.2402	0.9596	1.0000	0.9759
R2o	0.5901	0.6417	0.7865	0.5901	0.4140
R3s	0.0635	0.9399	0.3181	0.0636	-0.1528
R4s	-0.2170	0.9997	0.0565	-0.2170	-0.4125
R5o	0.9630	-0.4695	0.8583	0.9630	0.9861
R5s	0.5459	0.5595	0.7307	0.5459	0.3873
R6o	0.9489	-0.4704	0.8351	0.9489	0.9641
R6s	-0.5396	0.0197	-0.5080	-0.5395	-0.4505
R7o	-0.5500	0.0252	-0.5175	-0.5500	-0.4618
R7s	0.9476	-0.4699	0.8336	0.9476	0.9623
R1s R2o	1.0000	-0.2407	0.9594	1.0000	0.9760
R1s R4s	0.8733	0.2631	0.9723	0.8733	0.7524
R1s R5o	1.0000	-0.2376	0.9602	1.0000	0.9753
R1s R5s	-0.2376	1.0000	0.0349	-0.2376	-0.4318
R2o R4s	0.9602	0.0349	1.0000	0.9603	0.8843
R2o R5o	1.0000	-0.2376	0.9603	1.0000	0.9753
R2o R5s	0.9753	-0.4318	0.8843	0.9753	1.0000

Also 4 states: R1s, R1s+R2o, R1s+R5o and R2o+R5o create another ambiguous group (in each column associated with this states are 4 values equal to 1).

### Results of tolerances of unfaulty elements

In order to determine, how tolerances of unfaulty elements influence results of fault diagnosis, the analyses of a faulty circuit with the values of unfaulty elements different from nominal, are performed. The selected fault is R1s+R5s. 20 Monte Carlo analyses for the tolerance of unfaulty elements equal to 1% and then 20 for the tolerance equal to 5% are executed.

In the first case the changes of harmonics are very small and the changes of calculated correlation coefficients are the changes to pass over. The tolerances do not affect the result of fault diagnosis.

In the second case the changes of harmonics are greater than in the first case. The changes of correlation coefficients are smaller than 1% and the results of the fault diagnosis are the same as in the circuit with nominal value of unfaulty elements.

### Conclusion

The presented algorithm is destined for catastrophic fault diagnosis in analog dynamic circuits. An example of a calculation confirms the effectiveness of the proposed algorithm. The important advantage of this method is the

weak influence of the tolerances of elements on the diagnosis results. The effectiveness of algorithm depends on a change in function of frequency of a characteristic of system's dependence on stimulation. The efficiency of the algorithm can be improved with increasing number of measurement points. It can be achieved with the increasing number of calculated harmonics or number of output signals.

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### REFERENCES

- [1] Sen N., Saeks R., Fault diagnosis for linear systems via multifrequency measurement, *IEEE Transaction on Circuits and Systems*, vol.26 (1979), 457-465
- [2] Chen H., Saeks R., A search algorithm for the solution of multifrequency fault diagnosis equations, *IEEE Transaction on Circuits and Systems*, vol.26 (1979), 589-594
- [3] Bandler J. W., Salama A. E., Fault diagnosis of analog circuits, *Proceedings IEEE*, vol.73 (1981), 1279-1325
- [4] Lin P.M., Elcherif Y.S., Analogue circuits fault dictionary - New approaches and implementation, *International Journal of Circuit Theory and Applications*; vol. 13 (1985), 149-172
- [5] Ozawa T., Analog Methods for Computer Aided Analysis and Diagnosis, New York, USA: Marcel Dekker, 1988
- [6] Tadeusiewicz M., Halgas S., An Algorithm for Multiple Fault Diagnosis in Analogue Circuits, *International Journal of Circuit Theory and Applications*, vol. 34, pp. 607-615, 2006.
- [7] Liu D., Starzyk J.A., A generalized fault diagnosis method in dynamic analog circuits, *International Journal of Circuit Theory and Applications*, vol.30 (2002), 487-510
- [8] Rutkowski J., Słownikowe metody diagnostyczne analogowych układów elektronicznych, Warszawa, Polska: Wydawnictwa Komunikacji i Łączności, 2003
- [9] El-Gamal M., Mohamed M.D.A., Ensembles of neural networks for fault diagnosis in analog circuits, *Journal for Electronic Testing*, vol.23 (2007), 323-339
- [10] Tadeusiewicz M., Korzybski M., A method for fault diagnosis in linear electronic circuits, *International Journal of Circuit Theory and Applications*, vol.28 (2000), 245-262
- [11] Cannas B., Fanni A., Manetki S., Montisci A., Piccirilli M.C., Neural network-based analog fault diagnosis using testability analysis, *Neural Comput & Applic*, 13 (2004); 288-298
- [12] Tadeusiewicz M., Halgas S., Korzybski M., An algorithm for soft-fault diagnosis of linear and nonlinear circuits, *IEEE Transaction on Circuits and Systems-I*, vol.49 (2002), 1648-1653
- [13] Golonek T., Grzechca D., Rutkowski J., Evolutionary method for test frequencies selection based on entropy index and ambiguity sets, *International Conference on Signals and Electronic Systems ICSES'06* (2006), 511-514
- [14] Korzybski M., Diagnostowanie uszkodzeń parametrycznych wykorzystujące FFT oraz metodę programowania wyrażań genetycznych, *Proceedings of IX KKE*, CD, pp.181-186, 2010.
- [15] Korzybski M., Two-stage algorithm for soft fault diagnosis in analog dynamic circuits, *Przegląd Elektrotechniczny*, 86, nr 5, pp.195-198, 2011.
- [16] Golonek T., Grzechca D., Rutkowski J., Optymalizacja pobudzenia PWL testującego analogowe układy elektroniczne, *Proceedings of VII KKE*, pp.311-316, 2008.
- [17] Kamińska B., Arabi K., Bell I., Goteti P., Huertas J.L., Kim B., Rueda A., Soma M., Analog and Mixed-Signal Benchmark Circuits - First Release, *Proceedings International Test Conference*, 1997

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