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# The Modified Frequency Symbolic Models (FSMs) of Linear Periodically Time-Variable Circuits

***Abstract.*** *The methods of constructing modified FSMs of linear periodically time-variable circuits represent systems of linear equations and not require special action for o account for harmonic signal components is considered. Filed modified FSMs of single-circuit parametric amplifier, which are formed by the discussed methods.*

***Streszczenie.*** *Artykuł omawia metody konstruowania zmodyfikowanych modeli FSMS liniowych obwodów opisanych układami równań liniowych i nie wymagających specjalnych działań dla uwzględnienia składowych harmonicznych sygnału. Zaprezentowano i omówiono przykładowy model zmodyfikowanego FSMS jedno-obwodowego wzmacniacza parametrycznego.* (***Zmodyfikowane modele częstotliwościowe liniowych obwodów okresowych***).

**Keywords:** linear parametric circuits, symbolic analysis, frequency symbolic models .
**Słowa kluczowe:** liniowe obwody parametryczne, analiza symboliczna, częstotliwościowe modele symboliczne

## Introduction

In [1] the formation of FSMs of a linear periodically time-variable (LPTV) circuit is considered. Each of such models are the system of linear algebraic equations (SLAE), and therefore can be analyzed by using analysis programs for the circuits with constant parameters. However, the mentioned FSMs have the general feature which consists that at calculations of time dependences of voltage or currents by such models it is necessary to carry out special actions that take into account the superposition of harmonic components of a signal [1].

In the proposed paper is presented a new, so-called, modified models of LPTV circuits which does not require implementation of specific actions are presented and are more simple in the further usage, for example, at the multivariate analysis.

## Technique

The modified FSMs we build in the form of SLAE, therefore we can mark the following. As active elements of LPTV circuit (the resistors, controlled sources) are described by the algebraic component equations so a problem is creation of the algebraic component equations of reactive elements (capacities and inductances) circuit. This problem is solved using of a frequency symbolic method (FS-method) [2] by using time-dependent $t$ parametric transfer function of circuit $W(s,t)$ with the action of harmonic signal $e^{s\cdot t}$ on its input ($s$ - complex variable of the Laplace transform), determines the proper the voltage or current $X(s,t)$ in the circuit as:

(1) $$X(s,t) = W(s,t)\cdot e^{s\cdot t}\,.$$

Presented in (1) harmonic signal $X(s,t)$ components are modeled by the presence of relevant multipliers $e^{\pm ji\Omega t}$ with fractional rational expressions $W_0(s), W_{-i}(s), W_{+i}(s)$ in trigonometric polynomial that approximates the transfer function $W(s,t)$, where $T = 2\pi/\Omega$ - period of change of transfer function in time ($i = 1,2,...,k$ , $j^2 = 1$).

Thus the essence of the proposed methods and modified models by which they are formed as follows.

***1. The method of additional independent sources*** consists in that the FS-method currents of all reactive elements of circuit in the form of (1) are defined. On the basis of the theorem of substitution [3] these reactive elements of circuit are replaced with sources of the currents which magnitude is accordingly equal to certain currents of these elements. As a result of such actions the circuit turns to a resistive circuit with additional independent sources so by one of known methods, for example, a method of nodal voltages formed SLAE of such resistive circuit

(2) $$Y(s,t)\cdot U(s,t) = J(s)\;,$$

in which the matrix of conductivity $Y(s,t)$ contains only resistive elements (with constants and variable parameters), and the vector of sources $J(s)$ contains both an input signal source, and sources that in frequency domain are models of reactive elements of a circuit. Such SLAE also is FSM of LPTV circuit made by a method of additional independent sources.

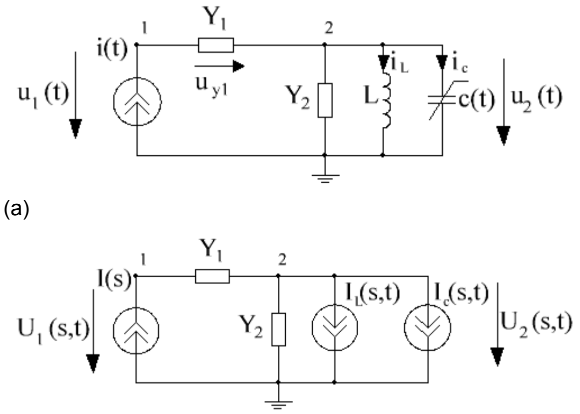



Fig.1. Single-circuit parametric amplifier - (a) equivalent circuit of the amplifier, made by the method of additional independent sources, - (b),

$c(t) = c_0(1 + m\cdot\cos(\Omega\cdot t)); c_0 = 0.1F; m = 0.1; \Omega = 20\,rad/s;$
$i(t) = A_m\cdot\cos(\omega\cdot t + \phi); \omega = 10\,rad/s; A_m = 1A; \phi = \pi/4;$
$Y_1 = 0.25S; Y_2 = 0.1S; L = 0.1H.$

**Example 1.** For LPTV circuit of fig. 1,a make a modified FSM by method of additional independent sources. The differential equation that relates the input current $i$ and current of parametric capacity $i_c$ of a circuit in the time domain has the form:

(3) $$(L^2c(t)Y_2c'(t) - Lc(t)^2)i''_c + (L^2Y_2{}^2c'(t) + (-c(t)L -$$
$$-L^2c(t)c''(t) + 2L^2c'(t)^2)Y_2 - Lc(t)c'(t))i'_c +$$
$$+(-L^2Y_2{}^2c''(t) - c(t) + 2LY_2c'(t))i_c = i'(L(-c(t)c'(t) -$$

$$-LY_2c(t)c''(t)+2LY_2c'(t)^2))+i''(L(Lc(t)Y_2c'(t)-c(t)^2).$$

Equation (3) results in equation of L.A. Zadeh [2], that is made relative to function $A_c(s,t)$ of transferring input current $I(s)$ into current of parametric capacity $I_c(s,t)$ in frequency domain:

(4)
$$\begin{aligned}&(L^2c(t)Y_2c'(t)-Lc(t)^2)\cdot 2\cdot A_c''(s,t)\cdot(1/2)+\\&+((L^2c(t)Y_2c'(t)-Lc(t)^2)\cdot s\cdot 2+(L^2\cdot Y_2{}^2c'(t)+\\&+(-c(t)L-L^2c(t)c''(t)+2\cdot L^2c'(t)^2)Y_2-\\&-Lc(t)c'(t))\cdot A_c'(s,t)+((L^2c(t)Y_2c'(t)-\\&-Lc(t)^2)\cdot s^2+(L^2Y_2{}^2c'(t)+(-c(t)L-L^2c(t)c''(t)+\\&+2\cdot L^2c'(t)^2)Y_2-Lc(t)c'(t))\cdot s+(-L^2Y_2{}^2c''(t)-c(t)\\&+2\cdot L\cdot Y_2c'(t))\cdot A_c(s,t)=s\cdot(L(-c(t)c'(t)-LY_2c(t)c''(t)+\\&+2\cdot L\cdot Y_2c'(t)^2))+s^2(L\cdot(Lc(t)Y_2c'(t)-c(t)^2).\end{aligned}$$

The symbolic solution of equation (4) with the help of frequency symbolic method when $k=1$ is the following:

(5) $$A_c(s,t)=\frac{h_0(s)}{\Delta_c(s)}+\frac{h_{c1}(s)}{\Delta_c(s)}\cos(\Omega t)+\frac{h_{s1}(s)}{\Delta_c(s)}\sin(\Omega t),$$

where $h_0(s)=$ 0.10*s*(0.12e52*s^5+0.11e57*s+ +0.12e55*s^2+0.24e52*s^4+0.12e55*s^3-0.90e53);
$h_{c1}(s)=$ 0.50e3*(0.25e47*s^4+0.25e49*s^3+0.25e50*s^2+ +0.13e52*s+0.39e52)*s; $h_{s1}(s)=$ 0.50e3*(0.12e44*s^4+ +0.50e46*s^3+0.50e50*s^2-0.97e50*s+0.15e53)*s;
$\Delta_c(s)=$ 0.12e51*s^6+0.37e51*s^5+0.24e56*s^2+ +0.14e54*s^4 +0.23e56*s + +0.27e54*s^3 +0.11e58.

The equation that relates the input current $i$ and $i_L$ current of inductance of circuit in the time domain is:

(6) $$i_L''\cdot Lc(t)+i_L'\cdot(LY_2+Lc'(t))+i_L=i$$

Equation (6) results in equation of L.A. Zadeh [2], that is made relative to function $A_L(s,t)$ of transferring input current $I(s)$ into current of inductance $I_L(s,t)$ in frequency domain:

(7)
$$\begin{aligned}&Lc(t)\cdot A_L''(s,t)+(2\cdot s\cdot Lc(t)+(LY_2+Lc'(t)))\cdot A_L'(s,t)+\\&+(s^2\cdot Lc(t)+s\cdot(LY_2+Lc'(t))+1)\cdot A_L(s,t)=1.\end{aligned}$$

The solution of equation (7) with the help of frequency symbolic method when $k=1$ is the following:

(8) $$A_L(s,t)=\frac{w_0(s)}{\Delta_L(s)}+\frac{w_{c1}(s)}{\Delta_L(s)}\cos(\Omega t)+\frac{w_{s1}(s)}{\Delta_L(s)}\sin(\Omega t),$$

where $w_0(s)=$ 0.50e29*s^4+0.45e34+0.10e30*s^3+ 0.50e32*s+0.50e32*s^2; $w_{c1}(s)=$ -0.50e28*s*(0.50e3*s+ 0.40e3+s^3++s^2); $w_{s1}(s)=$ -0.10e30*s*(s^2+0.30e3);
$\Delta_L(s)=$ 0.55e30*s^4+0.50e27*s^6+0.15e28*s^5+ +0.11e31*s^3 +0.95e32* *s^2+0.95e32*s+0.45e34.

According to (1), expression (5) defines the current of parametric capacity $I_c(s,t)$ in the frequency domain:

(9) $$I_c(s,t)=A_c(s,t)\cdot I(s),$$

and the expression (8) – current of inductance $I_L(s,t)$:

(10) $$I_L(s,t)=A_L(s,t)\cdot I(s)$$

Considering defined currents of inductance $I_L(s,t)$ with constant parameter and parametric capacity $I_c(s,t)$ as independent sources of a current, we build the equivalent circuit of the amplifier (fig. 1,b) for frequency domain of which by a method of nodal voltages in the form of SLAE it is formed the required modified frequency symbolic model of the amplifier with additional independent sources:

(11) $$\begin{bmatrix}Y_1 & -Y_1\\ -Y_1 & Y_1+Y_2\end{bmatrix}\times\begin{bmatrix}U_1\\ U_2\end{bmatrix}=\begin{bmatrix}I(s)\\ -[A_L(s,t)+A_c(s,t)]\cdot I(s)\end{bmatrix}.$$

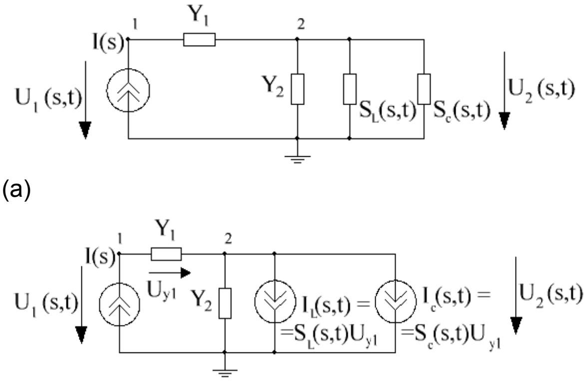
The modified model (11) is algebraic and symbolic, so it can be the basis for further multivariate research of circuit. Thus, from (11) complex nodal voltage $U_1$ determined in symbolic form:

(12) $$U_1=\frac{Y_1+Y_2-Y_1\cdot A_c(s,t)-Y_1\cdot A_L(s,t)}{Y_1+Y_2}\cdot I(s),$$

that allows quite easily repeatedly calculate $U_1$ for different values of $Y_1, Y_2, L, m, c_0, \Omega, \omega,\ A_m, \phi$.

The results of computing time values of voltage $u_1(t)=\mathrm{Re}[U_1]$ for $k$ = 4 in the parametric functions $A_c(s,t)$ and $A_L(s,t)$ for values of time $t$ 400s, 401s, 402s, 403s are equal -0.3498V, -5.8695V, 10.1122V, -10.3496V, respectively. These calculations are completely consistent with the corresponding computations for program of the numerical calculations Micro-Cap 7.0.

***2. The method of frequency reactive two-terminal.*** Behind a FS- method we find voltage and current of all reactive elements of circuit (with constant and variable parameters). For the $m$ such element, we have:




(a)

(b)

Fig.2. Equivalent circuits of single-circuit parametric amplifier from Fig.1, and are composed by the method: frequency reactive two-terminal - (a); controlled sources - (b).

(13) $$I_m(s,t)=\mathrm{A}_m(s,t)\cdot I(s),$$

(14) $$U_m(s,t)=Z_m(s,t)\cdot I(s).$$

The ratio of expressions (13) and (14) forms an algebraic equation component (symbolic model) in the frequency domain of $m$ reactive bipolar element of circuit and defines its conductivity $S_m(s,t)$ as:

(15) $$\frac{I_m(s,t)}{U_m(s,t)}=\frac{\mathrm{A}_m(s,t)\cdot I(s)}{Z_m(s,t)\cdot I(s)}=\frac{\mathrm{A}_m(s,t)}{Z_m(s,t)}=S_m(s,t).$$

As the component equations (symbolic models) resistors with constants and variable parameters are certainly algebraic we consider that the frequency character models of all elements of a circuit is defined. They also form (together with the equations of Kirchhoff) modified FSM of parametric circle in general. Thus, using the method of nodal voltages by frequency models of element got the desired modified FSM of a circuit in a type:

(16) $$Y(s,t)\cdot U(s,t)=J(s).$$

In the modified FSM (16) form of matrix and vector sources different than (2), but, as in (2), provides the computation time values of voltages and currents at the predefined from (16) the transfer function of the input signal without performing additional special action [1].The specified singularity of model (16) is reached by double application to each reactive element (its voltage and a current) FS-method that demands additional expenses of computer time. However, it can be compensated by convenience of such FSM at the subsequent design stages of the parametric device.

**Example 2.** For LPTV circuit of fig. 1,a make a modified FSM by method of frequency reactive two-terminal. From the differential equation that relates the input current $i$ and voltage $u_2$

(17) $u_2'' \cdot c(t)L + u_2' \cdot (LY_2 + 2c'(t)L) + u_2 \cdot (Lc''(t)+1) = i' \cdot L$ ,

follows the equation of L.A. Zadeh, that is made relative to function $Z(s,t)$ of transferring input current $I(s)$ into voltage $U_2(s,t)$ in frequency domain:

(18) $$\begin{aligned} &Lc(t)\cdot Z''(s,t) + (Lc(t)\cdot 2\cdot s + LY_2 + 2Lc'(t))\cdot Z'(s,t) + \\ &+ (Lc(t)s^2 + s(LY_2 + 2Lc'(t)) + Lc''(t)Y_2 + 1)\cdot Z(s,t) = sL \end{aligned}.$$

The solution of last with the help of frequency symbolic method when $k=1$ is the following:

(19) $$Z(s,t) = \frac{z_0(s)}{\Delta_z(s)} + \frac{z_{c1}(s)}{\Delta_z(s)}\cos(\Omega t) + \frac{z_{s1}(s)}{\Delta_z(s)}\sin(\Omega t) ,$$

where $z_0(s)$ = 0.50e28*s*(s^4+0.90e5+2*s^3+0.10e4*s+ +0.10e4*s^2); $z_{c1}(s)$ = -0.50e27*s*(0.40e3*s+0.90e3*s^2+ +0.12e6+s^4+s^3); $z_{s1}(s)$ = 0.10e29*s*(0.40e3+0.20e3*s+ +s^2); $\Delta_z(s)$ = 0.15e28*s^5+0.50e27*s^6+0.95e32*s^2+ +0.55e30*s^4+0.95e32*s+0.11e31*s^3+0.45e34.

On the basis of expressions (5), (8) and (19) it is formed the frequency symbolic conductivities of parametric capacity and inductance of a circuit from fig.1,a, accordingly:

(20) $S_c(s,t) = A_c(s,t)/Z(s,t)$ and $S_L(s,t) = A_L(s,t)/Z(s,t)$ .

By expressions (20) we build the equivalent circuit (fig. 2,a) from which by a method of nodal voltages is defined required frequency symbolic model of the circuit containing the frequency symbolic conductivities of the coil of inductance $S_L(s,t)$ and parametric capacity $S_c(s,t)$ :

(21) $$\begin{bmatrix} Y_1 & -Y_1 \\ -Y_1 & Y_1 + Y_2 + S_L(s,t) + S_c(s,t) \end{bmatrix} \times \begin{bmatrix} U_1 \\ U_2 \end{bmatrix} = \begin{bmatrix} I(s) \\ 0 \end{bmatrix}.$$

Thus, from the model (21) is determined the symbolic nodal voltage $U_1$ as

(22) $$U_1 = \frac{Y_1 + Y_2 + S_c(s,t) + S_L(s,t)}{Y_1Y_2 + Y_1S_c(s,t) + Y_1S_L(s,t)} \cdot I(s)$$

The results of computing time values of this voltage $u_1(t) = \mathrm{Re}[U_1]$ for *k* = 4 in the parametric functions $A_c(s,t)$, $A_L(s,t)$ and $Z(s,t)$ for values of time *t* 400s, 401s, 402s, 403s are equal -0.3498V, -5.8695V, 10.1122V, -10.3496V respectively. These calculations are completely consistent with the corresponding computations for program of the numerical calculations Micro-Cap 7.0. and by the method of additional independent sources.

***3. The method of controlled sources.*** Number of parametric transfer functions required to build a frequency model of circuit by the method of frequency reactive two-terminal may be reduced as follows. Thus in each expression (15), by which are formed frequency model of reactive elements can be selected not voltage of corresponding reactive element, but any other voltage of circuit. In this case, the reactive element is modeled not by two-terminal but by controlled source. Enough convenient by such voltage in all models of reactive elements of circuit choose the same voltage, for example, for the circuit from fig.1,a - voltage on element $Y_1$. In this case the relationship of input current $i$ with a voltage $u_{Y_1}$ is algebraic: $u_{Y_1} = (1/Y_1)\cdot i,$ the corresponding equation of L.A.Zadeh will also algebraic: $Z(s,t) = Z = 1/Y_1$. From this it follows that the denominator in models of all reactive elements will be the same and equal $1/Y_1$. In other words, the denominator in the expression (15) and his similar disappears. If you need a given circuit series with the input source should introduce a single resistance that most simplify the models of reactive elements. This is the essence of the method of controlled sources.

**Example 3.** For a given LPTV circuit of fig.1,a make a modified FSM by the method of controlled sources. Frequency models of reactive elements of circuit in this case are the expressions

(23) $$\begin{aligned} S_c(s,t) &= A_c(s,t)/Z(s,t) = A_c(s,t)\cdot Y_1, \\ S_L(s,t) &= A_L(s,t)/Z(s,t) = A_L(s,t)\cdot Y_1. \end{aligned}$$

On this basis, we build the equivalent circuit (Fig. 2, b) and by the method of nodal voltages - the required frequency symbolic model of circuit:

(24) $$\begin{bmatrix} Y_1 & -Y_1 \\ -Y_1 + S_L(s,t) + S_c(s,t) & Y_1 + Y_2 - S_L(s,t) - S_c(s,t) \end{bmatrix} \times \begin{bmatrix} U_1 \\ U_2 \end{bmatrix} = \begin{bmatrix} I(s) \\ 0 \end{bmatrix}$$

Thus, the model (24) defines the nodal voltage $U_1$ of circuit in the frequency domain using the expression, which coincides with (12). Obviously, the results of calculations of time values of voltage $u_1(t) = \mathrm{Re}[U_1]$ at k = 4 in the parametric functions $A_c(s,t)$ and $A_L(s,t)$ also coincide with the results obtained by the method of additional independent sources.

## Conclusions

1. The methods of constructing of modified frequency symbolic models of LPTV circuits that do not require additional special actions to account for harmonic signal components is proposed.
2. These models have the form of SLAE, and can be analyzed by programs that are designed for symbolic analysis of linear circuits with constant parameters.
3. Select a specific (one of three) method of constructing of modified model of circuit, is dictated by the terms of designing the given parametric device as a whole.

REFERENCES


[1] Shapovalov Yu. Frequency symbolic models of linear parametric circuits / Yu. Shapovalov, B. Mandziy, D. Smal, J. Lipinski //Przeglad Elektrotechniczny (Electrical Review). – 2012. – R. 88 NR, № 3a. – P.52–54.

[2] Shapovalov Yu. The peculiarities of frequency symbolic method applied to parametric circuits analysis / Yu. Shapovalov, B. Mandziy, S. Mankovsky // Przeglad Elektrotechniczny. – 2011. – Vol.87, № 5. – P.155–159.

[3] Desoer A., Kuh S. Basic Circuit Theory/ A. Desoer, Ernest S. Kuh. //Hardcover, McGraw-Hill Companies, 1969 – 477.



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