

# Comparison of experimentally and by the finite element method determined magnetically nonlinear iron core characteristics applied in the dynamic model of a single phase transformer

**Abstract.** This paper deals with comparison of experimentally and by the finite element method determined magnetically nonlinear iron core characteristics. The obtained characteristic were used in the dynamic model of a single phase transformer. The comparison of the measured and by the dynamic model calculated results is given for the case of transformer steady state operation at rated load and for the case of switch-on of unloaded transformer.

**Streszczenie.** W artykule przedstawiono porównanie podejścia eksperymentalnego i metody elementów skończonych w wyznaczaniu nieliniowej charakterystyki magnetycznej rdzenia żelaznego. Analizy dokonano poprzez wykorzystanie wyznaczonych charakterystyk w modelu dynamicznym transformatora jednofazowego i ich porównaniu dla pracy w stanie ustalonym z obciążeniem oraz przy załączaniu nieobciążonego transformatora. (Porównanie podejścia eksperymentalnego i metody elementów skończonych w wyznaczaniu nieliniowej charakterystyki magnetycznej rdzenia żelaznego w transformatorze jednofazowym z wykorzystaniem modelu dynamicznego urządzenia).

**Keywords:** dynamic model, magnetic nonlinearities, experimental methods, FEM, power transformer.

**Słowa kluczowe:** model dynamiczny, nieliniowości magnetyczne, metody eksperymentalne, FEM, transformator energetyczny.

## Introduction

This work deals with magnetically nonlinear dynamic model of a single phase transformer. In order to achieve the best possible agreement between the measured and calculated responses the dynamic model is completed by the magnetically nonlinear characteristic of the tested transformer. It is given as current-dependent flux linkage characteristic, which is determine experimentally and by applying the finite element computations.

Nowadays, there exist many electromagnetic devices with magnetically nonlinear iron core. For analysis of these devices and for their control design dynamic models are required. When the magnetically nonlinear properties of the device iron core are neglected, then we have to do with magnetically linear models. Such models cannot provide a good agreement between the calculated responses and those measured on the real device. In order to improve agreement between the measured and calculated responses, the magnetically nonlinear iron core behaviour can be accounted for in the dynamic model [1]-[2] of the electromagnetic device in different ways. The first one is experiment based [3]-[4], while the second one is based on finite element method (FEM) computations. Both approaches are evaluated in this work. The evaluation is based on the case study performed on a single phase transformer. The results presented show that the use experimentally determined iron core characteristic in the dynamic model of a single phase transformer provides much better agreement between the measure and calculated results.

## Description of applied methods

**Experimental methods:** The experimental methods based on numerical integration of measured currents and voltages [3], and [5]. A power grid and linear amplifier were used as a voltage source. The linear amplifier was used to generate sinusoidal and stepwise changing voltage waveforms. The sinusoidal and stepwise changing voltage and current are measured. The magnetically nonlinear characteristic  $\psi(i)$  can be determined after the numerical integration by (1)

$$(1) \quad \psi(t) = \psi(0) + \int_0^t [u(\tau) - Ri(\tau)] d\tau$$

where  $\psi(0)$  is the initial condition due to the remanent flux, while  $u$  and  $i$  are the voltage and current, while  $R$  is the ohmic resistance. The experimental methods are described in detail [5].

**Finet element method:** A detailed 3D finite element model of a tested single phase transformer was built using the commercial software Vector Fields. The 3D model of a single phase transformer is presented in Fig. 1.

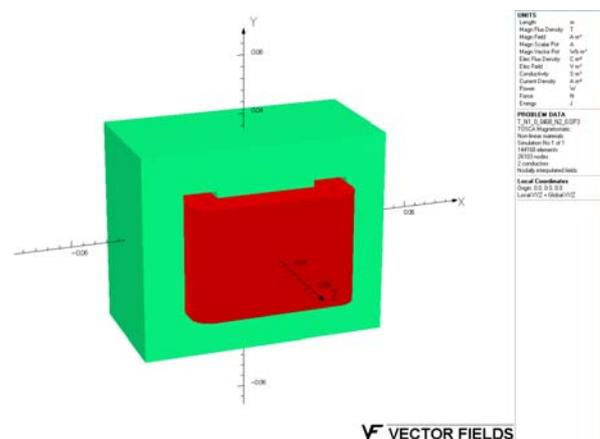


Fig.1. 3D model of a single phase transformer

The post-processor provides the indispensable interface between the user and the finite element solution for a field problem. The solution of the FEM equations results to computation of the magnetic scalar potential at each node of the mesh. From this data, other quantities need to be extracted, in a way that the software user can exploit the results of the FEM analysis. The handling of large amount of data generated by the finite element solver and their efficient visualization according to the user requirements constitute the final step to the accomplishment of a functional and powerful FEM software [6]. Figs. 2 and 3 show the post-processing of results and calculation of magnetic flux of a tested single phase transformer (2).

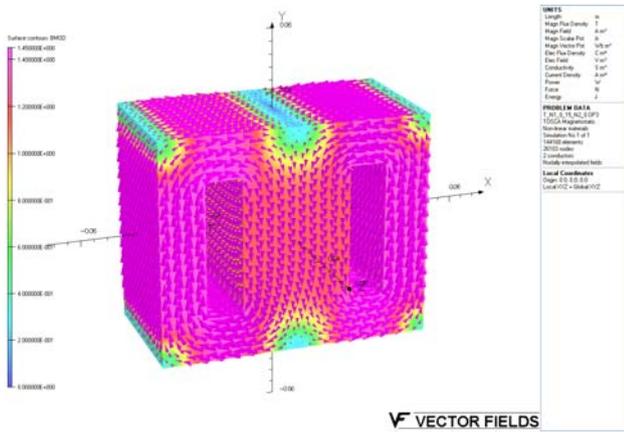


Fig.2. Post processing of results of a single phase transformer

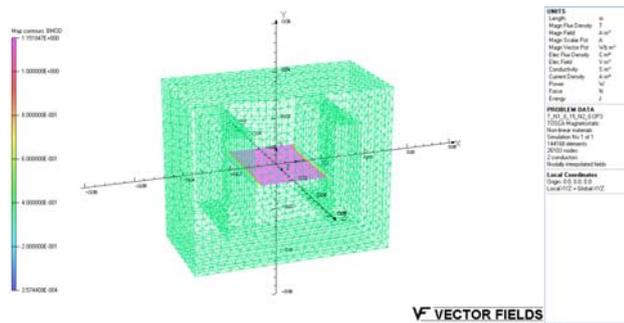


Fig.3. Calculation of magnetic flux of a single phase transformer

$$(2) \quad \Delta\phi \rightarrow \psi = N\Delta\phi$$

**Magnetically nonlinear characteristics:** The magnetically nonlinear behaviour of material is normally given by the  $B(H)$  characteristics, where  $B$  denotes the flux density while  $H$  is the magnetic field strength. When this material is built in an electromagnetic device, the magnetically nonlinear behaviour of the entire device can be described by the flux linkage versus magnetomotive force characteristic  $\psi(i)$ . Fig. 4 shows experimentally and by the FEM determined magnetically nonlinear iron core characteristic of tested single phase transformer.

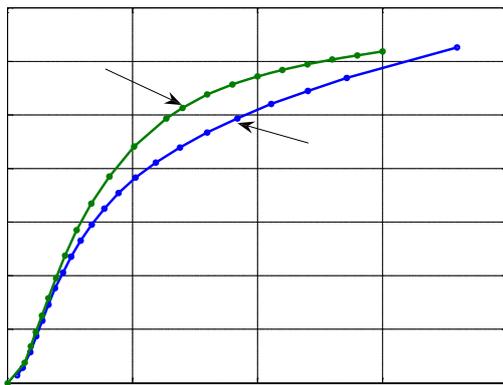


Fig.4. Magnetically nonlinear characteristic  $\psi(i)$  of the tested single phase transformer

In the case of magnetically isotropic material, the material magnetically nonlinear properties are normally described by the permeability  $\mu$ , which is defined as a ratio between the magnetic flux density  $B$  and the magnetic field

strength  $H$ . Similar role as characteristic  $B(H)$  in the case of material, has the characteristic  $\psi(i)$  in the case of an electromagnetic device. For material, the dynamic permeability  $\mu_d$  is defined by the partial derivative (3), while for an electromagnetic device, the dynamic inductance  $L_d$  can be defined by the partial derivative (4) and it is shown in Fig. 5.

$$(3) \quad \mu_d = \frac{\partial B}{\partial H}$$

$$(4) \quad L_d = \frac{\partial \psi}{\partial i}$$

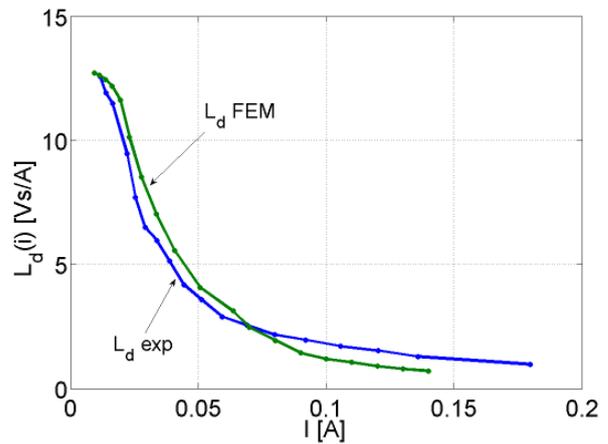


Fig.5. Dynamic inductance  $L_d$  of the tested single phase transformer

#### Dynamic model of a single phase transformer

This section deals with the magnetically nonlinear dynamic model of a single phase transformer [5]. When the eddy current losses and the hysteresis losses are neglected, the voltage balances in the primary and the secondary winding of the single phase transformer can be described by (5) and (6):

$$(5) \quad u_1 = i_1 R_1 + \frac{d}{dt} [\psi_{\sigma 1} + \psi_1]$$

$$(6) \quad u_2 = i_2 R_2 + \frac{d}{dt} [\psi_{\sigma 2} + \psi_2]$$

where  $u_1$ ,  $u_2$  and  $i_1$ ,  $i_2$  are the primary and the secondary voltages and currents, while  $R_1$  and  $R_2$  are the primary and secondary resistances,  $\psi_{\sigma 1}$  and  $\psi_{\sigma 2}$  are the primary and secondary leakage flux linkages.  $\psi_1$  and  $\psi_2$  are the primary and secondary current-dependent flux linkages. The obtained expressions (7) and (8) are appropriate to be solved by the explicit integration methods.

$$(7) \quad \frac{di_1}{dt} = \frac{L_{22}}{L_{11}L_{22} - L_{12}^2} \left( u_1 - \frac{L_{12}}{L_{22}} u_2 - i_1 R_1 + \frac{L_{12}}{L_{22}} i_2 R_2 \right)$$

$$(8) \quad \frac{di_2}{dt} = \frac{L_{11}}{L_{11}L_{22} - L_{12}^2} \left( u_2 - \frac{L_{12}}{L_{11}} u_1 - i_2 R_2 + \frac{L_{12}}{L_{11}} i_1 R_1 \right)$$

where  $L_{11}$  is the self inductance of the primary winding,  $L_{22}$  is the self inductance of the secondary winding, while  $L_{12}$  is the mutual (magnetizing) inductance.

## Results

The test object is a small single phase laboratory transformer shown in Fig. 6. Its data are shown in the Table 1. All simulations are performed in the program package Matlab/Simulink using the dynamic model of a single-phase transformer given by equations (7) and (8) and the magnetically nonlinear iron core characteristics shown in Fig. 4. The magnetically nonlinear behaviour of the transformers iron core is accounted for by the dynamic inductances (4) shown in Fig 5.



Fig.6. The tested single phase transformer

Table 1: Testing transformer data

$N_1$	The number of primary turns	425
$N_2$	The number of secondary turns	1722
$R_1$	The primary resistance	11 $\Omega$
$R_2$	The secondary resistance	141.8 $\Omega$
$L_{\sigma 1}$	The primary leakage inductance	33 mH
$L_{\sigma 2}$	The secondary leakage inductance	33 mH

Figs. 7 and 10 show the primary voltage measured during the no-load test and the primary voltage measured during the test performed at loaded transformer, respectively.

Figs. 8, 9 and 11, 12 show the comparison of measured and calculated transformer currents in different operating conditions. In all figures presented, the measured currents are marked with  $i$ , the dynamic model calculated ones using the dynamic inductances  $L_{exp}(i)$  marked with  $i_{exp}$ , while the dynamic model calculated ones using the dynamic inductance  $L_{FEM}(i)$  are marked with  $i_{FEM}$ .

Fig. 7 shows the measured primary voltage  $u_1$  applied during the no-load test. The same voltage is used in the dynamic model. Its amplitude is 136.7 V at the frequency of 50 Hz.

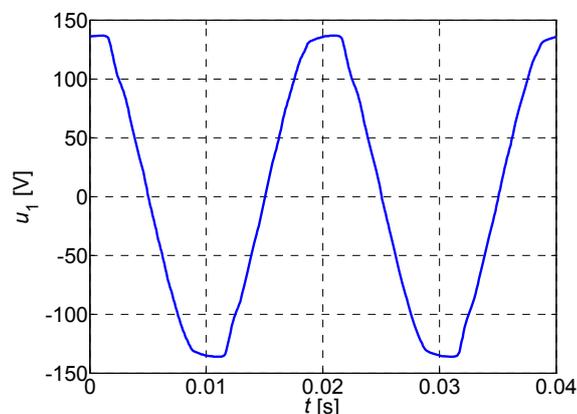


Fig.7. Primary voltage  $u_1$  measured during the no-load test

Fig. 8 shows the comparison of measured and calculated current for the steady state operation at no load. The agreement between the measured and the calculated results is very good when the experimentally determined iron core characteristic is used in the dynamic model.

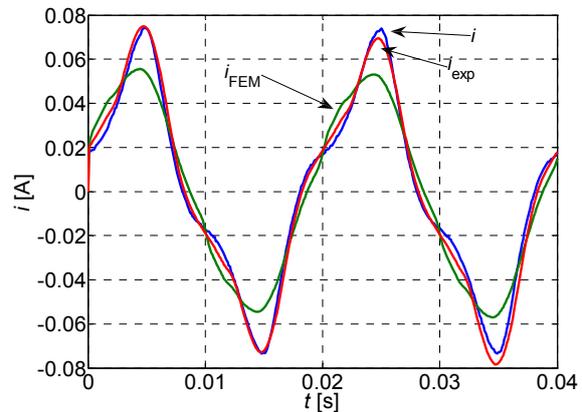


Fig.8. Steady state at no load: measured current  $i$ , current calculated with dynamic inductances  $L_{exp}(i)$  marked with  $i_{exp}$  and current calculated with dynamic inductance  $L_{FEM}(i)$  marked with  $i_{FEM}$

The comparison of measured and calculated currents during switch-on of the unloaded testing transformer is shown in Fig. 9. The agreement between measured and calculated results is very good if the experimentally determined iron core characteristic is used. However, in the case when the FEM determined iron core characteristic is used in the model the agreement between measured and calculated results is relatively good.

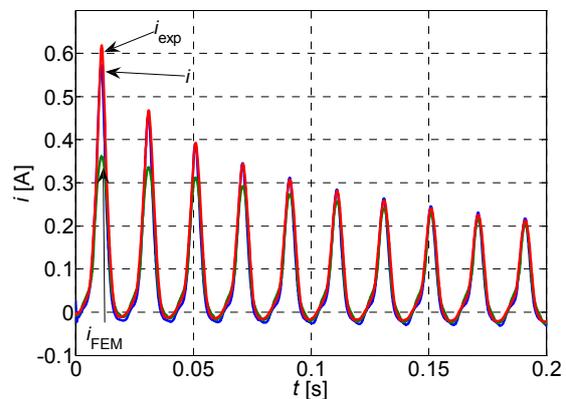


Fig.9. Inrush: measured current  $i$ , current calculated with dynamic inductances  $L_{exp}(i)$  marked with  $i_{exp}$  and current calculated with dynamic inductance  $L_{FEM}(i)$  marked with  $i_{FEM}$

Fig. 10 shows the measured primary voltage  $u_1$  applied during the test performed at the loaded transformer. The same voltage is used in the dynamic model. Its amplitude is 137.8 V at the frequency of 50 Hz.

Figs. 11 and 12 show the comparison of measured and calculated for the case of transformer loaded with the nominal load. Fig. 11 shows steady state operation, while Fig. 12 shows the loaded transformer switch-on. In the case of loaded transformer, there is only a small difference between the currents calculated with experimentally and by the FEM determined magnetically nonlinear iron core characteristic. This could be explained by the relatively small values of the magnetizing current in the case of loaded transformer. The agreement with the measured results is relatively good.

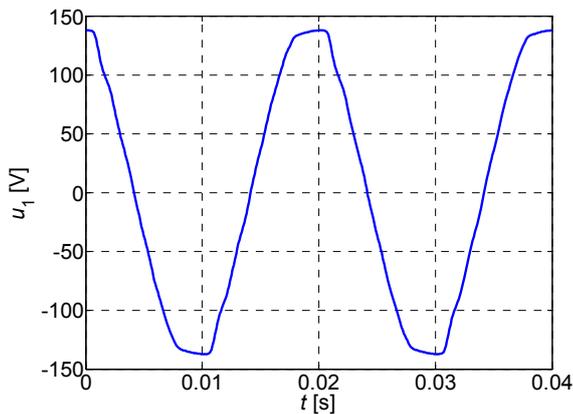


Fig.10. Primary voltage  $u_1$  measured during the test at loaded transformer

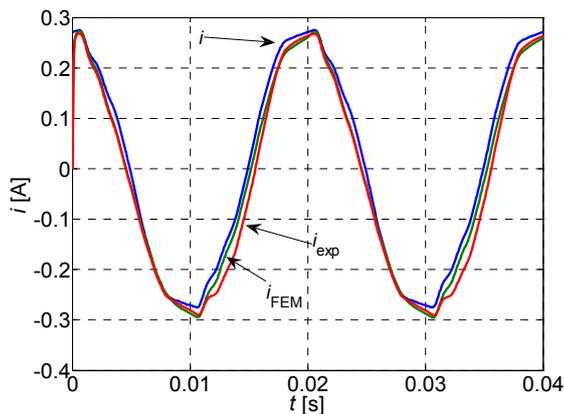


Fig.11. Steady state at nominal load: measured current  $i$ , current calculated with dynamic inductances  $L_{exp}(i)$  marked with  $i_{exp}$  and current calculated with dynamic inductance  $L_{FEM}(i)$  marked with  $i_{FEM}$

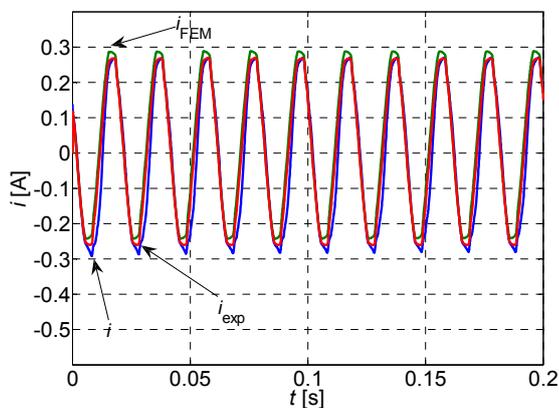


Fig.12. Loaded transformer switch-on: measured current  $i$ , current calculated with dynamic inductances  $L_{exp}(i)$  marked with  $i_{exp}$  and current calculated with dynamic inductance  $L_{FEM}(i)$  marked with  $i_{FEM}$

## Conclusions

This work discusses the use of experimentally and FEM determined magnetically nonlinear iron core characteristics in the single phase transformer dynamic model. The model derivation and the presented results clearly show that the experimentally determined iron core characteristic should be used in the model. However, the results of simulations given for the case of loaded transformer clearly show that even the use of FEM determined iron core characteristic is acceptable in such operating conditions.

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