

# Cogging torque minimization of the double stator cup-rotor machine

**Abstract.** This paper discusses the cogging torque minimization of a Double Stator Cup-Rotor (DSCR) Machine. It is an integration of two permanent magnet machines, for use in applications of wind energy generation. By investigation, it is shown that an optimum pole-slot combination can be selected, under design specifications, to achieve the minimum cogging torque. 2-D Finite Element Modelling and Analysis (FEM) is used in this paper.

**Streszczenie.** W artykule opisano zagadnienie minimalizacji momentu zaczepowego w maszynie typu DSCR (ang. Double Stator Cup-Rotor). Maszyna taka ma zastosowanie jako generator do turbin wiatrowych. W analizie wykazano, że wybranie optymalnego stosunku bieguny/żłobki przy projektowaniu pozwala na minimalizację tego momentu zaczepowego. W pracy zastosowano modelowanie 2-D oraz analizę metodą elementów skończonych. (**Minimalizacja momentu zaczepowego w maszynie DSCR**).

**Keywords:** Permanent magnet machine, low- cogging torque.

**Słowa kluczowe:** maszyna z magnesami trwałymi, moment zaczepowy.

## Introduction

There has been an increasing interest in permanent magnet machines, because of their superior energy efficiency, and an increasing number of applications requiring such efficient machines. This paper introduces a Double Stator Cup-Rotor (DSCR) brushless dc machine [1] for high torque and energy efficient applications. This machine produces more torque than a conventional Brushless-DC-Machine (BLDCM) of the same frame size, because torque is applied to both the inner and outer surfaces of the cup-rotor. It is, therefore, very important to design this machine correctly, because otherwise the level of cogging torque may become unacceptably high. There are many techniques to reduce cogging torque such as skewing the stator or the rotor [2], shaping the stator slots or rotor magnets [3], or by selecting an appropriate combination of pole number and slot number [2]. To achieve the Minimum Cogging Torque (MCT) of the DSCR machine, 2-D Finite Element Modelling and Analysis (FEM) was applied, and the results are presented in this paper.

The objective of this paper is to investigate that cogging torque of DSCR can be minimized without skewing by choosing an optimum pole-slot combination and selecting the optimum magnet arc length.

## Cogging torque analysis

The most popular technique used these days for analysing machines is the FEM. The machine is meshed into a finite number of elements to be analysed. Cogging torque analysis in FEM is achieved by calculating the changes in stored energy within the airgap with respect to rotor position as [4]:

$$(1) \quad T_{cog} = -\frac{1}{2} \phi_g^2 \frac{dR}{d\theta_r}$$

Where  $\phi_g$  - flux distributed in the air gap R - the reluctance,  $\theta_r$  - the rotor position in mechanical degrees.

According to [5], the cogging torque can also be analysed by applying the Fourier Series to the numerical combination of magnet poles and stator slots in the PM machine as follows:

$$(2) \quad T_{cog}(\theta_r) = \sum_{K=1,2,3}^{\infty} T_{FK} X_{SK} \sin(KN_{CM} \theta_r)$$

where:  $T_{FK}$  - the Fourier coefficient,  $X_{SK}$  - the skew factor,  $N_{CM}$  - the smallest common multiple between the number of poles, p, and the slot number, Q.  $\theta_r$  - the mechanical rotor position angle. The cogging torque equation (2) can be rewritten without skewing as:

$$(3) \quad T_{cog}(\theta_r) = \sum_{K=1,2,3}^{\infty} T_{FK} \sin(KN_{CM} \theta_r)$$

For machines with symmetrical magnet configurations, the complete cycle of the cogging torque profile can be calculated from  $\theta_r = \pi p / N_{CM}$ .

According to [3], there are many factors can influence the cogging torque such as the magnet pole arc, the pole-slot number combination and the stator slot configuration. These factors are applied in the DSCR machine to minimize the cogging torque.

## Specifications of the proposed DSCR split machines

Figure 1, shows how the integrated machine is divided into two PM machines to facilitate the modelling. Table I illustrates the specification for both machine parts. The range of design variables such as magnet thickness and magnet arc length are determined by calculation and simulation, respectively. The range for the number of poles is chosen to be 8, 10 and 12 for this particular machine as shown in Table II.

Table I. physical design parameters of the DSCR machine

Stack length:	59 mm	Inner part inner dia.:	44 mm
Magnet material:	NdFeB	Inner part outer dia.:	150 mm
Air-gap thickness:	1 mm	Outer part inner dia.:	153 mm
Rated speed:	1000 rpm	Outer part outer dia.:	204 mm
Rated voltage:	200 v	Turns per coil:	12 turns
Magnet thickness : (4 mm for inner part), (6 mm for outer part)			
Magnet arc angle (deg) $P_8$ : $30^\circ < \alpha < 45^\circ$ , $P_{10}$ : $24^\circ < \alpha < 36^\circ$ and $P_{12}$ : $20^\circ < \alpha < 30^\circ$ .			
Cogging torque limit is $\leq 0.08$ Nm			

Table II. The selected pole-slot combinations

Poles	No. of Slots							
8	9	12	18	21	24	27	30	33
10	12	15	24	27	30	33		
12	9	18	27					

The pole-slot combinations in Table II are preferable according to Hendershot and Miller [2] for an inherently low cogging torque. The MCT can be selected for the DSCR machine amongst these pole-slot combinations. The reason for choosing a large pole number is to provide a high torque at a lower speed, and to reduce the volume of the back iron. However, a large pole number will increase the fundamental frequency for a given speed, which may also increase iron losses.

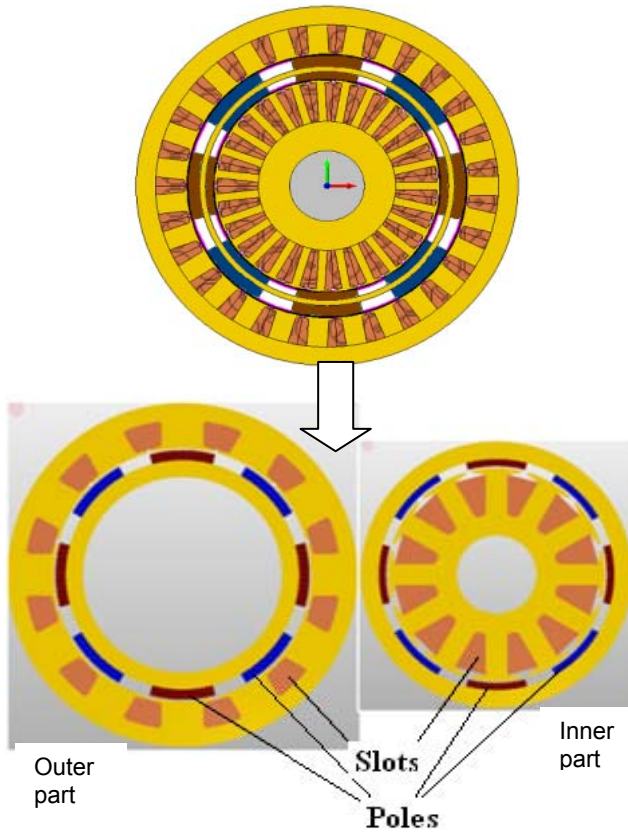


Fig.1. The integrated DSCR machine

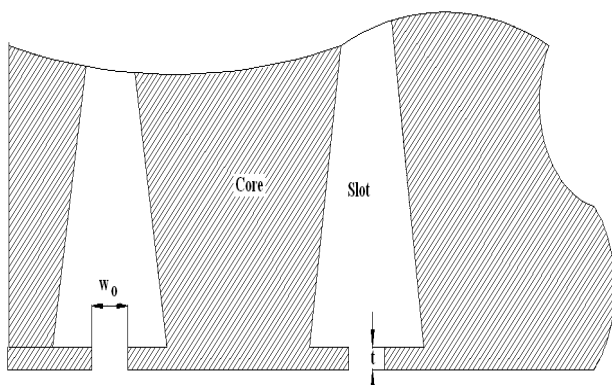


Fig. 2. The slot configuration

### Slot Tang Depth and Gap Width

Investigation of the stator slot shape using FEM analysis shows that it has an effect on the cogging torque. Analysis is performed by varying the slot opening,  $w_0$ , and the tooth tang depth,  $t$ , as shown in Fig. 2.

In order to demonstrate the relationship between the slot configuration and cogging torque, Fig. 3 shows the simulated results of the minimum slot opening. The slot opening should consider the particular conductor gauge. Obviously a lighter gauge conductor requires a narrower slot opening [5]. The minimum slot opening of 2.5 mm can

be considered in practice [6]. On the other hand, the tang depth of the tooth can be fixed to 1mm. However, the thinner the tang depth, the lower cogging torque, but laser cutting or stamping the lamination needs to be considered.

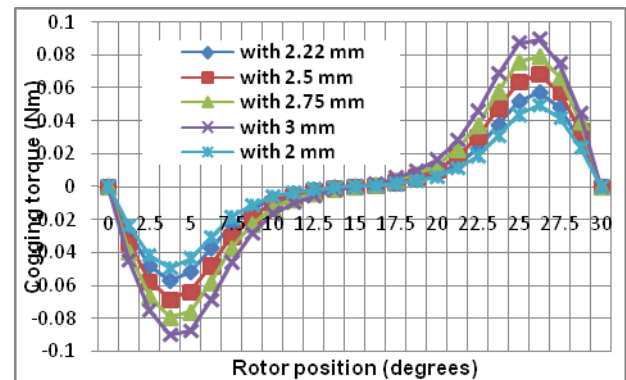


Fig. 3 Cogging torque Vs slot opening

### Magnet Pole Arc to Pole Pitch Ratio

The optimal ratio  $\alpha_K$  of a magnet pole arc to pole pitch  $\beta$  as shown in Fig. 4, is proposed in [3] for any pole-slot combination. In practice, there is a small value (0.01 to 0.03) that can be added to  $\alpha_K$ . This value depends on the air gap length [3].

$$(4) \quad \alpha_K = \frac{N - K}{N}, \quad K = 1, 2, 3, \dots, N - 1$$

where:  $N$  - the least common multiple to pole pair ratio, i.e.  $N = N_{CM}/p$ . Only values of  $K$  between 1 and  $N-1$  are allowed, because  $K = N$  is not a realistic value  $\alpha_K = 0$  and  $K = 0$  (fully pitched magnet) will not be a function of the least common multiple.

Equation (4) is applied for the most commonly used pole-slot combinations as listed in the table II. Although, there are many magnet pole arcs for each pole-slot combination, the first possibility (at  $k = 1$ ) is regarded as the optimum magnet pole arc to pole pitch ratio because this will have the highest Electromagnetic Torque (EMT).

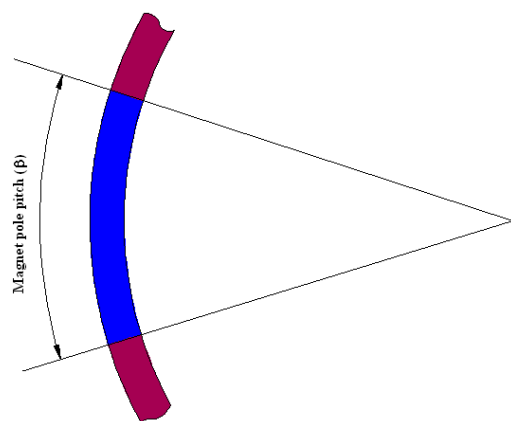


Fig. 4 Magnet fully pole pitch

### Choosing the MCT of the DSCR machine inner part

Amongst the 34 pole-slot combinations, table III shows the minimum cogging torque matrix that was investigated with the optimum magnet arc lengths for both inner and outer machines. The pole-slot combination of the DSCR

machine outer part was chosen to be 10 poles-27 slots. This combination has the minimum cogging torque, and a high (but not the highest) electromagnetic torque; this is because it has a magnet arc length 96.3% of the fully pitched pole. It also has a smooth back EMF with a near trapezoidal waveform as can be seen in Fig. 5. Since the outer part has 10 poles, the inner part then must have the same number of poles as the outer part, i.e. 10 poles. This is because the flux should pass through the magnet carrier between the inner and outer parts to maximize the flux utilisation. In ensuring the MTC for the inner machine, the obvious choice for the magnet arc angle is also 34.67°. However, as can be seen from table III column 4, this may not be the optimum solution, because the cogging torque is greater; 0.011 Nm peak compared with 0.005 Nm peak. So, 27 and 33 slots were investigated with further iterations of equation (4) for optimum magnet arc ratios, as can be seen in table IV. The iterations have been carried out with only  $k = 2, 3, 4$  and 5, in order to maintain the electromagnetic torque as high as possible.

Table III. The minimum CT at high electromagnetic torque

P = 8				
Q	$\alpha_k$ at k=1	$\beta$ (deg)	$T_{cog}$ [Nm] peak value	
			Inner part	Outer part
9	0.889	40	0.18	0.66
12	0.667	30	3.27	1.79
18	0.889	40	0.48	0.62
21	0.952	42.86	0.036	0.12
24	0.667	30	3.57	1.66
27	0.963	43.33	0.03	0.06
30	0.933	42	0.24	0.12
33	0.970	43.64	0.011	0.021
p = 10				
Q	$\alpha_k$ at k=1	$\beta$ (deg)	$T_{cog}$ [Nm] peak value	
			Inner part	Outer part
12	0.833	30	0.27	0.114
15	0.667	24	4.06	0.38
24	0.917	33	0.44	0.282
27	0.963	34.67	0.011	0.005
30	0.667	24	5.3	2.22
P = 12				
Q	$\alpha_k$ at k=1	$\beta$ (deg)	$T_{cog}$ [Nm] peak value	
			Inner part	Outer part
9	0.667	20	1.17	0.06
18	0.667	20	4.16	0.34
27	0.889	26.67	0.299	0.097

Table IV. The inner part pole-slot combination

$\alpha_k$	10p-27s		10p-33s	
	$\beta$ (deg)	CT	$\beta$ (deg)	CT
At k=2	33.33	0.023	33.82	0.021
At k=3	32	0.004	32.73	0.024
At k=4	30.67	0.006	31.64	0.025
At k=5	29.33	0.014	30.55	0.018

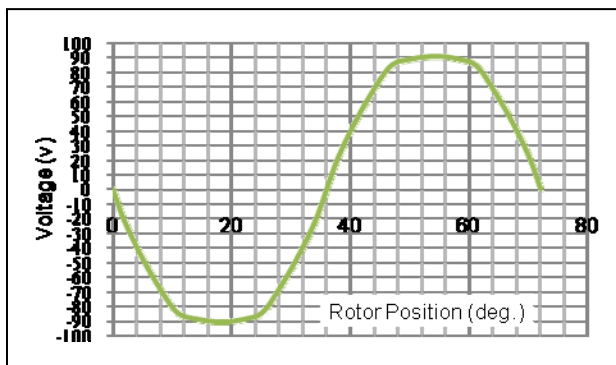


Fig. 5 Back EMF wave form of the outer part

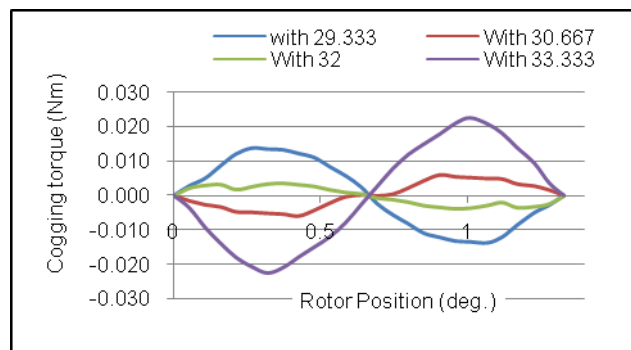


Fig. 6 The CT of the 10 poles 27 slots (inner part)

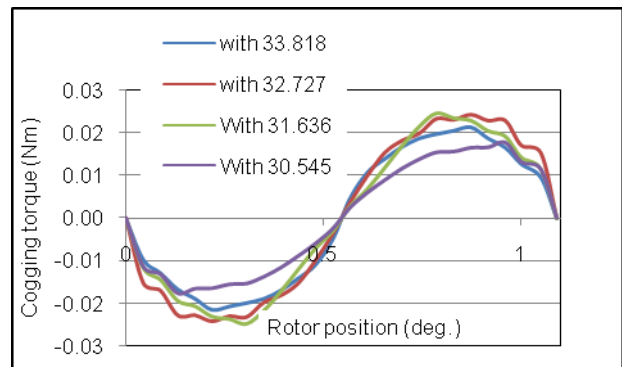


Fig. 7 The CT of the 10 poles 33 slots (inner part)

According to table IV, the minimum cogging torque is produced by the 10 poles 27 slots combination with a magnet arc angle of 32 degrees, with  $k = 2$  in equation (4). Fig. 6 shows a comparison of the cogging torque waveforms for the optimum magnet arc angles for  $k = 2$  to  $k = 5$  of the 10 pole-27 slot combination, and similarly Fig. 7 illustrates the cogging torque waveforms of the 10 pole-33 slot combination.

Choosing 10 poles 27 slots with 32 degrees magnet arc angle is a good solution for minimum cogging torque and smooth back EMF, compared with the 10 poles 27 slots 34.67 degrees as can be seen in Fig. 8

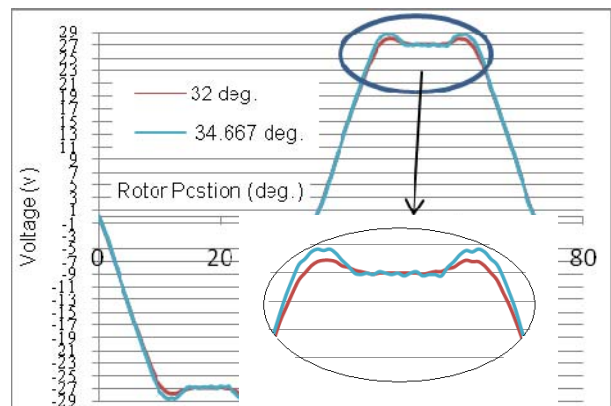


Fig. 8 Back EMF of the inner part comparison

The simulation of the integrated DSCR machine (shown in Fig. 9) by using FEM to predict the resultant cogging torque has been carried out, shown in Fig. 10, and the flux distribution patterns at various mechanical rotor positions are presented in Fig. 11. Due to the inner and outer magnets having different arc lengths; the resultant cogging torque produced is asymmetrical as well.

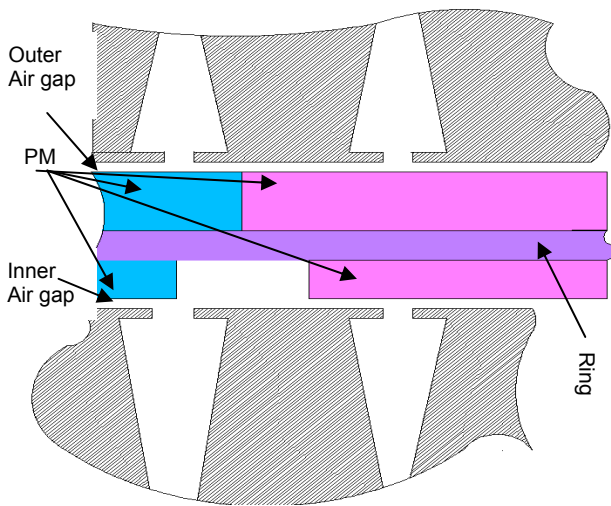


Fig.9 Integrated DSCR machine topology

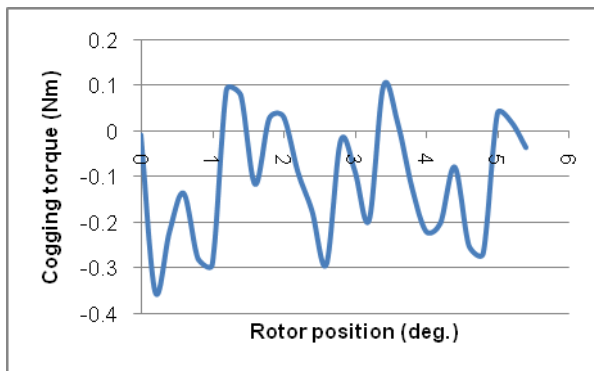


Fig. 10 The resultant cogging torque produced in the integrated DSCR machine

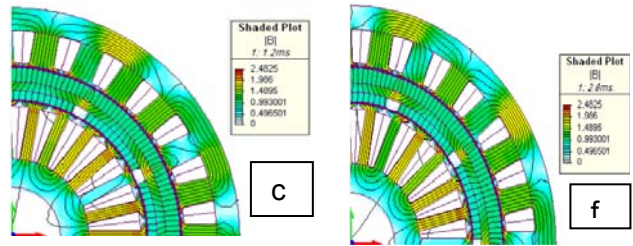
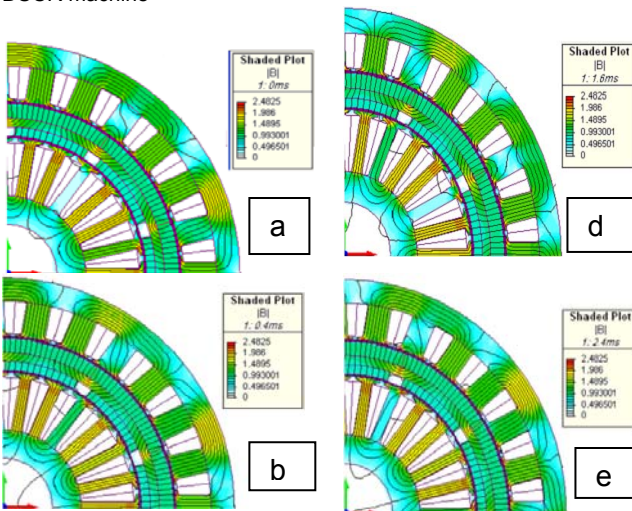


Fig. 11 Flux distribution at (a) 0 deg., (b) 10 deg., (c) 20 deg., (d) 30 deg., (e) 40 deg., and (f) 45 deg.

## Conclusions

This paper presents the results of the cogging torque minimization of the DSCR machine. It was modelled as two separate brushless DC machines with surface-mounted permanent magnets. Each machine part was designed and examined with 34 possible pole-slot combinations as shown in table III in order to find the minimum cogging torque. The outer machine part has been chosen with almost a fully pitched magnet arc ( $34.67^\circ$ ). This will give a high electromagnetic torque, minimum cogging torque and a flat trapezoidal back EMF waveform.

However, it was found that the inner machine part required a different solution for the MTC and so a  $32^\circ$  magnet arc was chosen. The reason for this difference can be seen from the flux plots in Fig. 11, where the inner stator iron is close to being magnetically saturated compared with the outer stator iron.

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