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# Sensitivity of Cogging Torque to Permanent Magnet Imperfections in Mass-produced PM Synchronous Motors

***Abstract**. A detailed analysis of cogging torque components reveals that besides well-known native cogging torque components exist also additional cogging torque components which are provoked by assembly tolerances in mass–production. A finite element method and Fast Fourier transform were used to study and analyse a model of PMSM which enables simulation of manufacturing tolerances and assembly imperfections of rotor permanent magnets. Calculations confirmed that imprecisions in mass–production cause the phenomena of additional cogging torque harmonic components, which are not present in the case of a perfectly assembled motor.*

***Streszczenie.** W artykule przestawiono analizę czynników wpływających na pojawienie się momentu zaczepowego w maszynie PMSM. Analizy metodą elementów skończonych oraz szybką transformatą Fouriera, przeprowadzone na modelu maszyny pozwoliły na wykonanie symulacji momentu zaczepowego i wpływu jaki mają na jego składniki stopnie tolerancji na etapie produkcji maszyny. (**Wpływ niedokładności w produkcji masowej na wartość momentu zaczepowego w maszynie synchronicznej z magnesami trwałymi**).*

**Keywords:** cogging torque, finite-element method, harmonic components, PM synchronous motor.
**Słowa kluczowe:** moment zaczepowy, metoda elementów skończonych, składowe harmoniczne, maszyna synchroniczna z magnesami trwałymi.

## Introduction

High-performance drives in automotive applications require permanent magnet synchronous motors (PMSM) that produce smooth torque with a very low component of cogging torque. This is not easy to attain as improper designs of PMSMs result in cogging torque that may be inadmissibly high regarding the rated torque. In lower cost machines, it has typically value around 5 % of the rated torque [1, 2]. As nowadays number of high-performance applications necessitate cogging torque not to exceed 1 % of the rated torque, effective methods for its analysis and computation are required to design machines that meet such specifications [3, 4].

Minimization of cogging torque frequently becomes a challenging task, since it is composed of numerous harmonic components, which originate in combination of many design parameters and material or assembly imperfections [5-7]. Besides applying appropriate combination of stator and rotor design techniques [8-10], the other option for cogging torque reduction is to introduce advanced control methods of the drive [11]. Regularly observed substantial differences between calculated (simulated) and measured cogging torque values was a cogent motive for the presented research of cogging torque sensitivity to manufacturing imperfections and tolerances, which are unavoidable facts in mass-production [6].

## Cogging torque harmonic components

Total cogging torque $T_{cog}$ of PMSM is composed of several harmonic components, which can be classified into two groups [6, 12]. The first group represents an array of native harmonic components $T_{NHC}$ which originate in a combination of design parameters, mostly the number of stator slots $Q$ and the number of rotor magnetic poles $P$. The second group is an array of additional harmonic components $T_{AHC}$, which are the consequence of material and assembly imperfections and are not present in the case of a perfectly manufactured motor [13-15]. Each harmonic component could be described by the following parameters: an amplitude $A$, an order $N$ (number of repetitions in mechanical revolution), and $\varphi$ stands for a phase shift.

Native harmonic components $T_{NHC}$ have orders defined by expression (4). Additional harmonic components $T_{AHC}$ can be further classified as contributions of imperfections which originate either on the stator or rotor side. In this paper only the case of rotor permanent magnet (PM) imperfections will be presented, where the orders of such $T_{AHC}$ are defined by expressions (5).

$$(1)\qquad T_{cog}(\alpha) = T_{NHC}(\alpha) + T_{AHC}(\alpha)$$

$$(2)\qquad T_{NHC}(\alpha) = \sum_{i=1}^{\infty} A_{NHC\,i} \cdot \sin(N_{NHC\,i} \cdot \alpha + \varphi_{NHC\,i})$$

$$(3)\qquad T_{AHC}(\alpha) = \sum_{i=1}^{\infty} A_{AHC\,i} \cdot \sin(N_{AHC\,i} \cdot \alpha + \varphi_{AHC\,i})$$

$$(4)\qquad N_{NHC\,i} = \mathrm{LCM}(Q, P) \cdot i$$

$$(5)\qquad N_{AHC\,i} = Q \cdot i$$

where: $\alpha$ - rotor angular position, LCM - least common multiple, $i$ - integer.

## FEA model of PMSM

PMSM with $Q$ = 9 and $P$ = 6, which are manufactured in mass-production with demand for extremely low level of cogging torque $T_{cog}$ (less than 9 mNm), have been parametrically modelled. Rotor consists of 6 PMs (Fig. 1 left), which in general have slightly diverse dimensions (thickness, width, radii) due to production and assembly tolerances [6, 13-15]. Simulation FEA model has been developed in a way that for each individual PM all 7 dimensions can be varied independently with the following parameters (Fig. 1b right):

- $a$, $f$ ... PM width,
- $b$, $e$ ... left and right PM edge thickness,
- $c$ ... thickness at the centre of PM,
- $d$ ... misalignment of outer PM radius,
- $g$ ... PM position on rotor core.

In one rotor group there are 6 PMs, which gives a total amount of 42 parameters. To reduce native components of $T_{cog}$ two options of well-known design technique have been considered [8]:

- 2 step-skew with 2 groups of 6 PMs, which gives the total of 12 PMs on the rotor core (Fig. 2),
- 3 step-skew with 3 groups of 6 PMs, which gives the total of 18 PM on the rotor core,

thus additional parameter that can deviate from exact value is also a step-skew angle $\theta$.

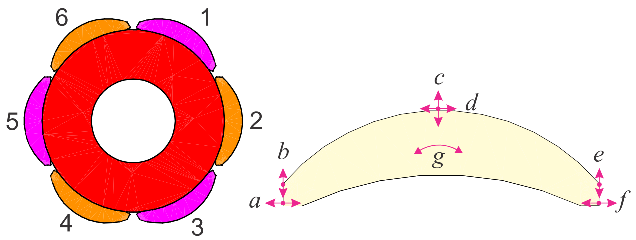


Fig. 1. Rotor with 6 PMs and variable parameters of FEA model.

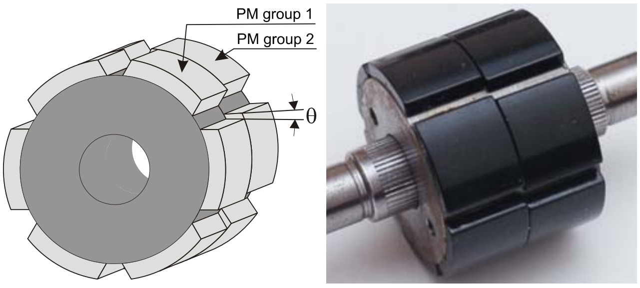


Fig. 2. Step-skew of 2 PM groups applied for cogging torque reduction.

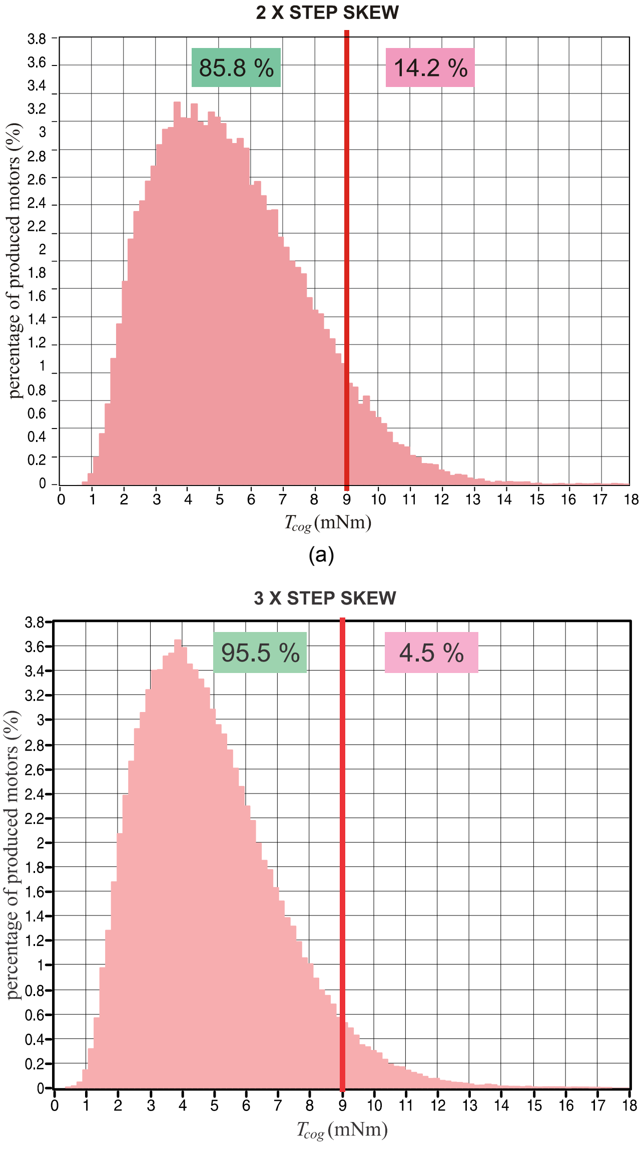
## Modeling of PM imperfections and assembly tolerances

One group of 6 PMs results in 42 variables, thus to make one rotor with step-skew of 2 groups there are 84 variables and additional one for a step skew misalignment. For the case of 3 step-skew PM groups, which more intensively reduce $T_{NHC}$ there is the total of 128 variables. Each variable has a recommended tolerance, which depends on applied mass-production technology, thus analysing every single one combination is practically unfeasible.

In order to found out how prescribed tolerances effect the $T_{cog}$ and consequently which designs of PMSM are less sensitive to them and therefore more appropriate for mass-production, we first randomly distribute dimensions inside tolerance boundaries and after that repeat computations applying normal (Gaussian) distribution, which is more likely in mass-produced PMSM. Cogging torque was initially calculated for a single PM group and then analytically combined for the case of 2 and 3 step-skew groups with corresponding cogging torque signals, respectively. In both cases, step-skew variations $\theta$ of PM groups has also been considered.

Fig. 3 shows FEA calculation results for random distribution of all possible 7 PM parameters inside prescribed tolerances, which are in tune with applied mass-production technology. The $y$-axis value in Fig. 3 presents the percentage of all produced motors having $T_{cog}$ appurtenant to $x$-axis value. Population of rotors in analysis for the 2 and 3 step-skew PMs was over 50000, which is adequate high number of samples for reliable statistical analysis. Total percentage of rotors having $T_{cog}$ over admissible 9 mNm is 14.2 % for the 2 step-skew PM groups (Fig. 3a), whereas for the 3 step-skew groups the 4.5 % of all PMSM reveal too high $T_{cog}$ (Fig. 3b). There is obvious benefit in reducing $T_{cog}$ caused by PM production tolerances by applying the 3 step-skew technique. Nevertheless, a scrap ratio of 4.5 % is still not acceptable for required quality of PMSM mass-production, so further investigations are in progress and only design with 3 step-skew PMs has been considered in the following examples.

To additionally improve the knowledge how PM dimensional tolerances impact the cogging torque, two supplementary simulations with tightened dimensional tolerances were carried out. In the first case we reduced PM thickness (parameter $c$ in Fig. 1) tolerances to a half with random distribution. Percentage of produced motors, which would have $T_{cog}$ over admissible 9 mNm is 2.77 % (Fig. 4a). This is significant improvement compared to prior 4.5%. In the second case we have tightened both, PM thickness and width (parameters $a$, $f$ in Fig. 1) tolerances to a half, again with random distribution. The results show that in this instance $T_{cog}$ over admissible 9 mNm have 1.52 % of all produced motors (Fig. 4b). As expected, with contraction of production tolerances it is possible to reduce scrap due to exceeding cogging torque, but one should also take into account technological ability how to produce PM with so narrow tolerances at a reasonable price.

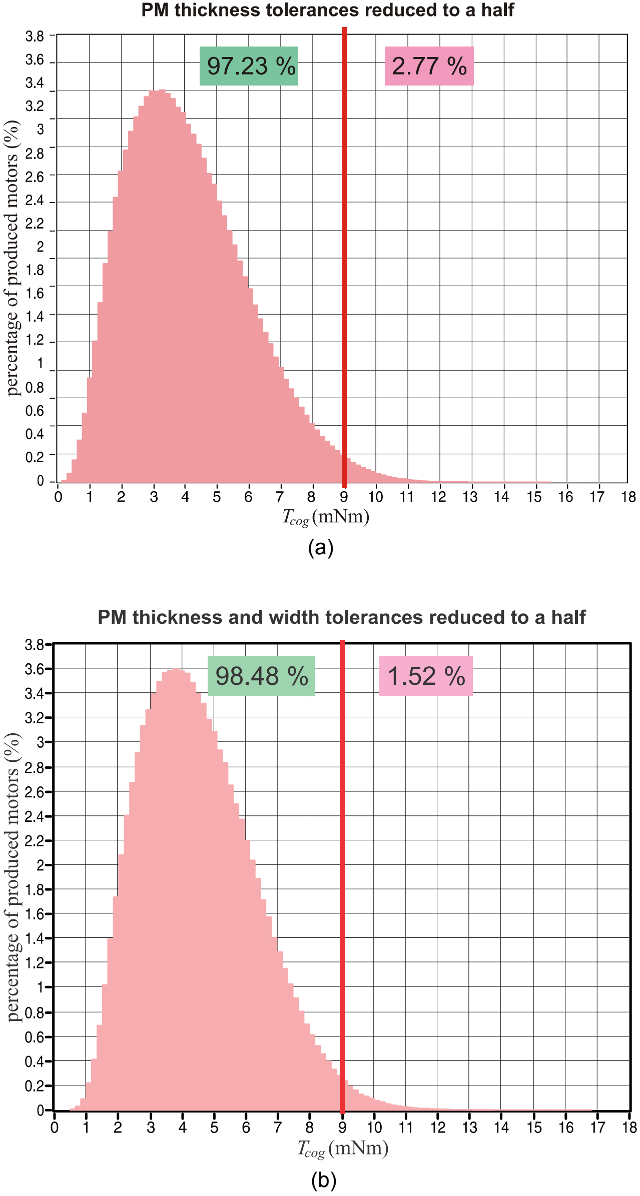


(a)

(b)

Fig. 3. Calculated histograms for the case of: (a) 2 step-skew; (b) 3 step-skew of PM groups on the rotor.

To properly define PM dimensional tolerances our goal is to make a PMSM model as close as possible to real fabrication circumstances. In mass-production is unlikely to expect that PM dimensions will be distributed randomly. For a stable manufacturing process PM dimensions are distributed inside normal (Gaussian) distribution, as shown in Fig. 5 for one PM variable parameter.



(a)

(b)

Fig. 4. Calculated histograms for the case of: (a) PM thickness tolerances reduced to a half; (b) PM thickness and width tolerances reduced to a half.

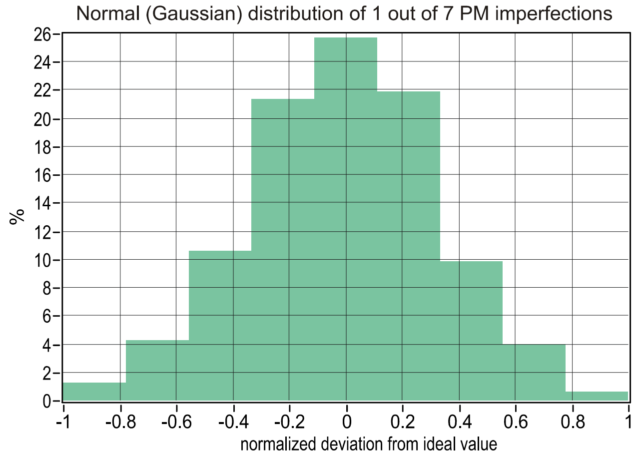


Fig. 5. Modeled normal (Gaussian) distribution of PM imperfection.

Apart from already explained PM parameters, magnetic remanence $B_r$ also varies among PM samples [14]. By measurements of numerous PMs, it was found out 2 % discrepancy from ideal magnetization value. Furthermore, imperfections on stator also contribute to $T_{cog}$ with additional harmonic components $T_{AHC}$ [12]. With presented FEA analysis has been calculated $T_{cog}$ for 530 rotors having 6 PMs. As one rotor consists of 3 groups of 6 PMs which are step-skewed for θ = 6.66 °, calculations were performed for 149 million ($530^3$) combinations. We observed a 2.02 % of motors with $T_{cog}$ > 9 mNm.

## Experimental results

How important is in mass-production of PMSM keeping the PM dimensions inside prescribed (easily defined by presented simulation procedure) tolerance boundaries can be noticed in the following measurements of PMSM from a production line. Fig. 6 shows a distribution of measured $T_{cog}$ for motors made up of PMs which dimensional parameters defined in Fig. 1 are outside prescribed tolerances. The percentage of motors exceeding permitted level of $T_{cog}$ is 13.2 %, which is inadmissible high.

After careful selection and sorting of equal PMs by width and thickness dimensions, the percentage of motors not satisfying the demand for $T_{cog} \leq 9$ mNm was reduced to 7.1 % (Fig. 7).

Following the request for manufacturing PMSM with as low scrap ratio as possible, the producer of PMs has supplied better samples which dimensions were inside prescribes tolerances. Measurements of such PMSM (Fig. 8) provided much better results of just 0.8 % of motors exceeding the $T_{cog}$ > 9 mNm.

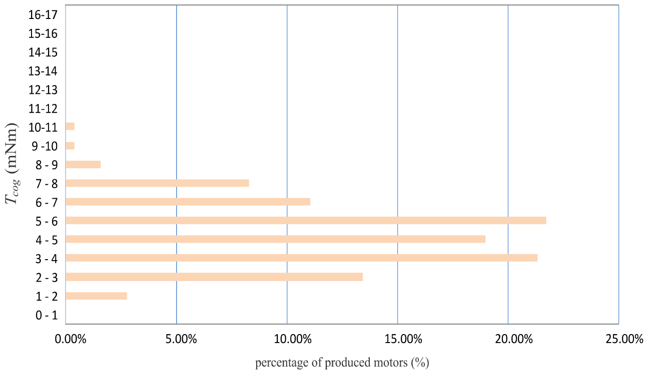


Fig. 6. Statistics of $T_{cog}$ measured on 281 produced motors assembled with PMs exceeding prescribed tolerances of dimensions defined in Fig. 1

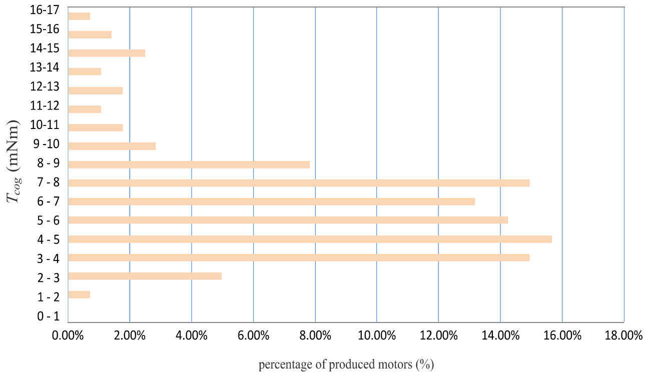


Fig. 7. Statistics of $T_{cog}$ measured on 129 produced motors assembled with PMs which were selected and sorted by width and thickness.

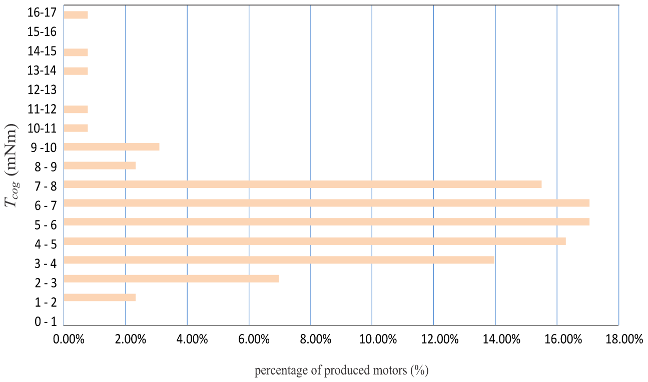


Fig. 8. Statistics of $T_{cog}$ measured on 253 produced motors assembled with PMs inside prescribed tolerances of dimensions defined in Fig. 1.

## Conclusions

Manufacturing assembly imperfections and tolerances like variations of PM thickness, width, radii, and misplacements cause the phenomena of $T_{AHC}$, which are not present in the case of an absolutely symmetrical rotor and mostly result in an increased level of total $T_{cog}$. To reduce unavoidable percentage of scrap in mass-production of PMSM due to increased level of $T_{cog}$, tolerance boundaries have to be tightened, which increases the cost of permanent magnets and feasibility of their production. The other possibility is to develop more robust PMSM design that is less sensitive to production tolerances of PM [8]. Considering the introduced analysis motor designers are able to predict the entire cogging torque harmonic spectrum for any PMSM in advance, thus predetermine required manufacturing tolerances to minimize cogging torque and fulfill the stringent market demands.

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