

## Impact of saturation modelling on the losses of electric drive controlled by QFT

**Abstract.** The aim of this paper is to present the impact of magnetic nonlinearities and saturation on the losses of an electric drive controlled by Quantitative Feedback Theory. For exact analysis of losses three magnetically different types of series wound DC motor dynamic models are presented and used in this work. As its main contribution this study shows that the modelling of magnetic nonlinearities improves control synthesis of electrical drive, which is reflected in lower energy consumption. For the purpose of control design of series wound DC motor Quantitative Feedback Theory was used.

**Streszczenie.** W artykule przedstawiono analizę wpływu nieliniowości magnetycznych oraz saturacji na straty w maszynie elektrycznej, sterowanej metodą QFT. Badaniom poddano modele dynamiczne trzech magnetycznie różnych, szeregowych maszyn DC. Wyniki pracy pokazują, że tego rodzaju analiza uskutecznia proces tworzenia algorytmu sterowania, co przekłada się na redukcję zużycia energii. (Wpływ modelowania nasycenia na wielkość strat w maszynie elektrycznej, sterowanej metodą QFT).

**Keywords:** series wound DC motor, magnetically nonlinearities, static and dynamic inductances, motor control, QFT

**Słowa kluczowe:** szeregową maszyną DC, nieliniowości magnetyczne, indukcyjność statyczna i dynamiczna, sterowanie silnikiem, QFT.

### Introduction

The mathematical model of series wound DC motor (SWM) represents a real physical system written in equations related to system variables. Because of magnetic saturation, magnetic nonlinearities and the complexities of physical systems that we want to model it is most often impossible to avoid different assumptions and simplifications [1]. Because of these simplifications and assumptions different motor models are obtained, which behave differently in the controlled system. This paper presents the differences for three dynamic SWM models: magnetically linear model, magnetically nonlinear model with static inductances and magnetically nonlinear model with dynamic inductances.

For control design we have selected the “Quantitative Feedback Theory” method; also indicated as QFT. QFT is a robust control design method which in comparison to other robust control design methods offers a number of advantages. One of the most important ones is that QFT often results in simple controllers, which are easy to implement in real systems.

This paper presents an analysis of the influence of magnetic nonlinearities and magnetic saturation on the losses of an electric drive controlled by QFT. Electric drive in this case consists of a controller with QFT algorithm and SWM. Analysis of the losses and consequently energy consumption in the electric drive at dynamic changes (for example rotor acceleration) are the central issues of this paper.

### Dynamic models

Dynamic mathematical models of electrical machines from the controller point of view show the connection between their inputs and outputs and describe the dynamics of their operation. These models are useful for analysis of dynamic and static properties of electric drives. Mathematical model is obtained using theoretical and experimental modelling [1,2,3].

Because of magnetic non-linearity electrical machines are used as dynamic models defined for one operating point in which they are ordinarily linearized, so they become magnetically linear models. For a wider range of observations we need magnetically nonlinear models with an appropriate way to model the magnetic saturation of iron.

Mathematical models are convenient for control design purposes, and they are complete when all parameters of the model are defined. Parameters can be determined numerically or experimentally. It is not always possible to carry out an experimental analysis, because of the limitations of the experimental system or dangerous conditions which might arise when electric values are out of range. Experimental analysis also cannot be performed, if the prototype of the electrical machine is not yet produced, i.e. if it is still in the design phase. In such cases mathematical models are convenient for performing dynamic and steady state analyses of the electrical drive.

The model includes nonlinearities and contains some physical parameters. The values of physical parameters are not known precisely and can be subject to some variation. These parameter variations are included as parameter uncertainties [4].

Figures (1) and (2) illustrate the determination of inductance for all three dynamic models.

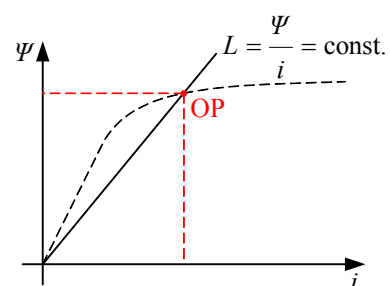


Fig.1. Inductance determination of a linear model

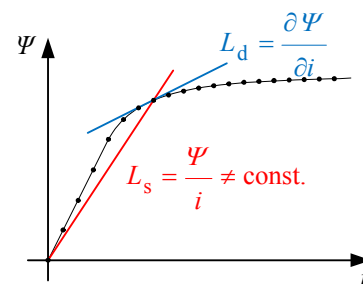


Fig.2. Static and dynamic inductance characteristic

The following figures (3, 4 and 5) shows block diagrams of dynamic models used in the study. The main purpose of these presented diagrams is to show the differences among the models.

### Magnetically linear dynamic model

Schematic presentation of a magnetically linear dynamic model is presented in Fig. 3.

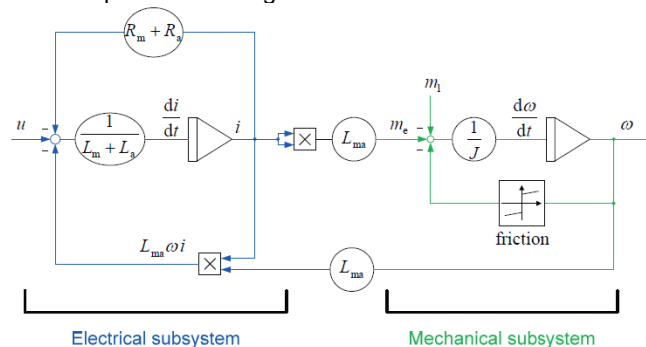


Fig.3. Block diagram of SWM magnetically linear dynamic model [1]

For a magnetically linear dynamic model the inductance is constant and is determined with linearization in the operating point (Fig. 1) as a quotient of flux linkage  $\Psi$  and the corresponding current  $i$ .

As it is known, magnetic material in the stator of the rotor is not as ideal as it is presented in magnetically linear models. To consider magnetic nonlinearities it is necessary to introduce the definition of static  $L_s$  and dynamic  $L_d$  inductances (Fig. 2), which indirectly describe the characteristics of the entire magnetic circuit [5].

### Magnetically nonlinear dynamic model using static inductances $L_s$ and dynamic inductances $L_d$

Magnetically nonlinear magnetic model using static inductances  $L_s$  is presented in Fig. 4.

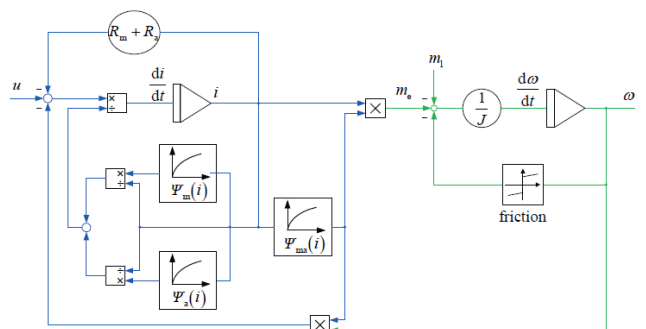


Fig.4. Block diagram of SWM magnetically nonlinear dynamic model using static inductances  $L_s$  [1]

Schematic presentation of magnetically nonlinear magnetic model using dynamic inductances  $L_d$  is given in Fig. 5.

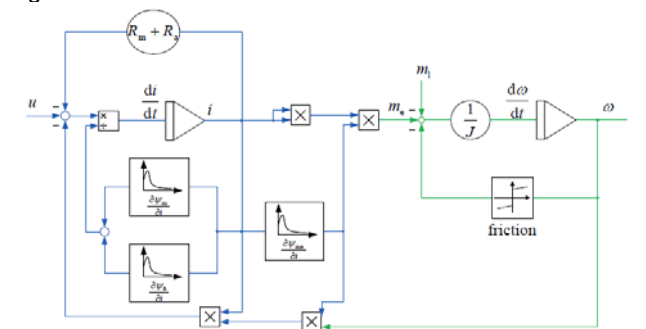


Fig.5. Block diagram of SWM magnetically nonlinear dynamic model using dynamic inductances  $L_d$  [1]

After the presentation of models the next step is the preparation of system transfer function, which is the basis for controller design.

### System model and transfer function

With model linearization and considering the uncertainties on a set of system working points a family of system linear models was obtained. A linearized motor model around a single working point is according to [4] described with equations (1) and (2):

$$(1) \quad u'(s) = [R_1 + R_A + s(L_{11} + L_{AA}) + M_{A1}\omega]i'_A(s) + M_{A1}I_A\dot{\Theta}'(s)$$

$$(2) \quad T'_L(s) = 2M_{A1}I_Ai'_A(s) - (sJ + f)\dot{\Theta}'(s)$$

The block diagram of a linearized motor model is shown in Fig. 6:

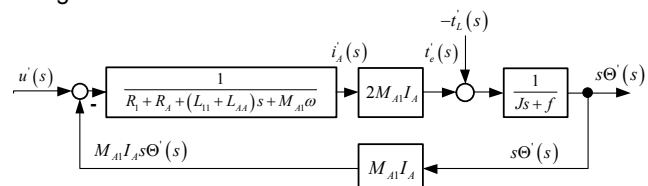


Fig.6. Linearized motor model

Final linearized model (3) has been calculated from equations (1) and (2):

$$(3) \quad \dot{\Theta}'(s) = \frac{2M_{A1}I_A u'(s) - T'_L(s) [(R_1 + R_A) + M_{A1}\omega + (L_{11} + L_{AA})s]}{a_2 s^2 + a_1 s + a_0}$$

$$a_0 = fR_1 + fR_A + fM_{A1}\omega + 2M_{A1}^2 I_A^2$$

$$a_1 = JR_1 + JR_A + JM_{A1}\omega + fL_{11} + fL_{AA}$$

$$a_2 = J(L_{11} + L_{AA})$$

Linearized model of SWM is described by a second order transfer function with four uncertain coefficients:

$$(4) \quad P(s) = \frac{a_3}{a_2 s^2 + a_1 s + a_0}$$

For each model the coefficients of uncertainty intervals are calculated separately:

$$a_0 \in [4,7032; 4,7032] \quad a_0 \in [3,7199; 4,9974] \quad a_0 \in [3,6312; 1,1641]$$

$$a_1 \in [37,0915; 37,0915] \quad a_1 \in [65,0598; 49,3560] \quad a_1 \in [5,4642; 0,6496]$$

$$a_2 \in [0,31746; 0,31746] \quad a_2 \in [0,0449; 0,4290] \quad a_2 \in [0,0465; 0,0152]$$

$$a_3 \in [2,7786; 2,7786] \quad a_3 \in [2,6728; 2,7842] \quad a_3 \in [2,6484; 1,5161]$$

a) b) c)

Where:

- a) is a magnetically linear dynamic model,
- b) is a magnetically nonlinear dynamic model using static inductances  $L_s$ ,
- c) is a magnetically nonlinear dynamic model using dynamic inductances  $L_d$ .

Transfer functions of nominal models around the single working point at  $I_A=10A$  and  $n=1500 \text{ min}^{-1}$ :

$$(5) \quad P_0(s) = \frac{2,779}{0,3175s^2 + 37,09s + 4,703}$$

$$(6) \quad P_0(s) = \frac{2,78}{0,3592s^2 + 39,61s + 4,764}$$

$$(7) \quad P_0(s) = \frac{1,894}{0,01949s^2 + 0,9197s + 1,814}$$

Where transfer function (5) represents magnetically linear dynamic model linearized in nominal operating point, transfer function (6) represents magnetically nonlinear dynamic model using static inductances  $L_s$  and transfer function (7) represents magnetically nonlinear dynamic model using dynamic inductances  $L_d$ .

At this point we have obtained all the needed data for controller design, which is the next step of this study.

## QFT control methodology

QFT control design was used to create a simple low-order robust controller for motor velocity control for all three abovementioned magnetically different dynamic models.

QFT control design has been chosen to achieve reliability and robustness. QFT deals with robust stability margins and robust performance specifications (disturbance rejection, reference tracking, etc.) in the presence of system parameters uncertainty. The QFT approach converts closed-loop system specifications and model uncertainty into a set of constraints or boundaries for every frequency of interest, which need to be fulfilled by the nominal open-loop transfer function [4]. Such an integration of information into a set of simple curves in a Nichols chart enables designing the controller by using only a single-nominal-system model. In the controller design stage (loop shaping), the controller is synthesized by adding controller poles and zeros until the nominal loop lies near its boundaries. The optimal controller is obtained when it meets its boundaries and has the minimum high-frequency gain.

The QFT method demonstrates a general control strategy with two degrees of freedom structure that is presented in Fig. 7. In this block diagram of the system, the transfer function  $P(s)$  belongs to a set  $\{P\}$  of plants with uncertainties;  $C(s)$  and  $F(s)$  denote the controller and the prefilter, which are to be synthesized in order to meet robust stability and closed-loop specification and  $H(s)$  denotes the transfer function of the sensor.

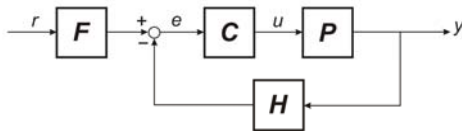


Fig.7. Two degree of freedom QFT controller structure

## QFT control design

For the purpose of performance specifications, the selected upper and lower closed loop requirements in the time domain need to be translated to the frequency domain and so used in the controller design [6,7].

$$\alpha(\omega) = \left| \frac{1100}{(j\omega)^2 + 12 \cdot (j\omega) + 1100} \right| \quad \beta(\omega) = \left| \frac{500}{(j\omega)^2 + 110 \cdot (j\omega) + 500} \right|$$

(8) Upper bound

(9) Lower bound

Controller design method and requirements were the same for all three types of motor models. As a result we obtain three speed controller transfer functions:

Speed controller of the linear model:

$$(10) \quad C(s) = \frac{1075s^3 + 26540s^2 + 157600s + 5,452}{s^3 + 19,02s^2 + 5,384s}$$

Speed controller of the dynamic model using static inductances  $L_s$ :

$$(11) \quad C(s) = \frac{3407s^3 + 58980s^2 + 246100s + 41,76}{s^3 + 34,97s^2 + 23,39s}$$

Speed controller of the dynamic model using dynamic inductances  $L_d$ :

$$(12) \quad C(s) = \frac{391,2s^3 + 5937s^2 + 22050s + 47,12}{s^3 + 61,8s^2 + 17,09s}$$

## Test system

Paper [1] shows that the nearest model to the real motor was magnetically nonlinear dynamic model using dynamic inductances. Therefore, we performed simulations of all three designed controllers on this type of dynamic model.

A complete test system is presented in Fig. 8. Input references and dynamic models (Fig. 5) were the same, only controllers and associated prefilters were different.

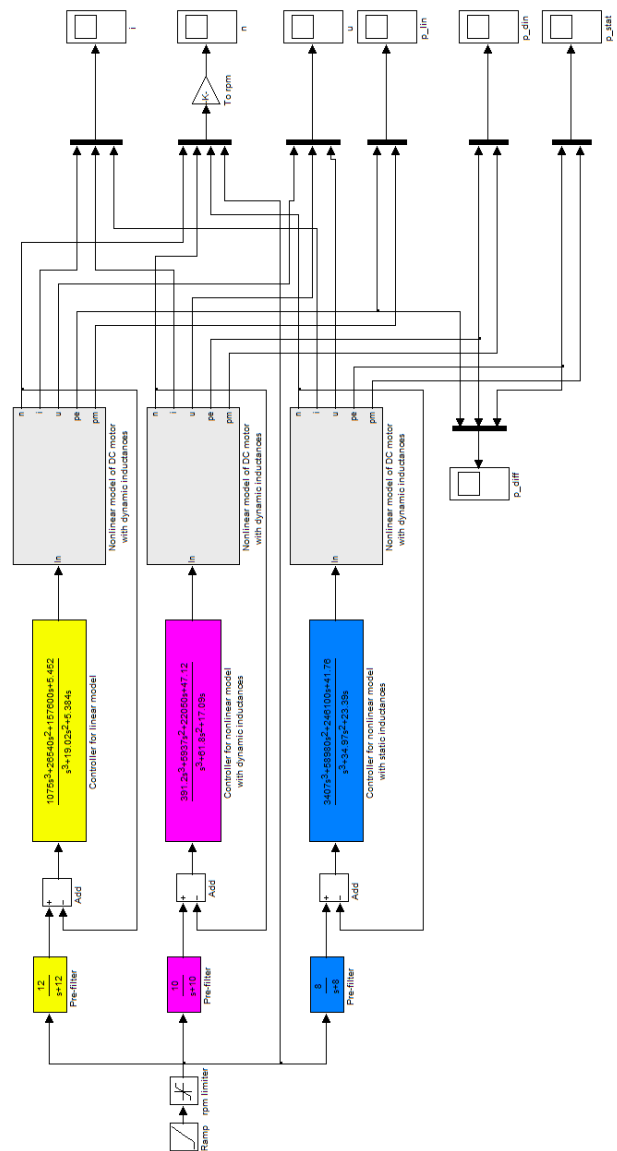


Fig.8. Test system

## Simulation results

Colours in the responses are:

- Yellow: linear model
- Blue: dynamic model using static inductances
- Purple: dynamic model using dynamic inductances

### 1. Step response from 0 to 1500 RPM with added load 12 Nm at 1.2 seconds.

In Fig. 9 red represents the reference value of revolutions with a step response in the SWM electric drive control. Non-linear dynamic model with included dynamic inductances (purple colour) achieves the revolutions value the fastest and with the lowest stationary error. This is confirmed in Fig. 10 and 11, where time lines of corresponding voltages and currents for all three controllers are presented. Both Figures show that the variation in the voltage and consequently in the current is the lowest in case of a limited non-linear dynamic model with included dynamic inductances. Consequently this also means lower energy consumption (Fig. 12 and 13).

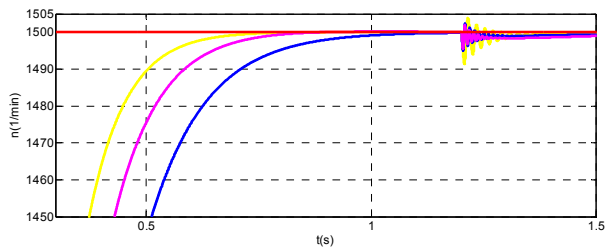


Fig.9. Step response with load at 1.2 seconds

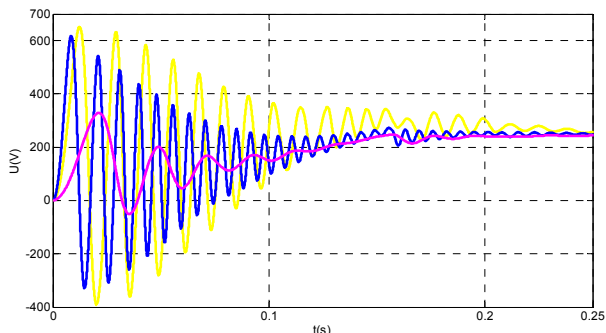


Fig.10. Time waveforms of voltages at step response 0-1500 rpm

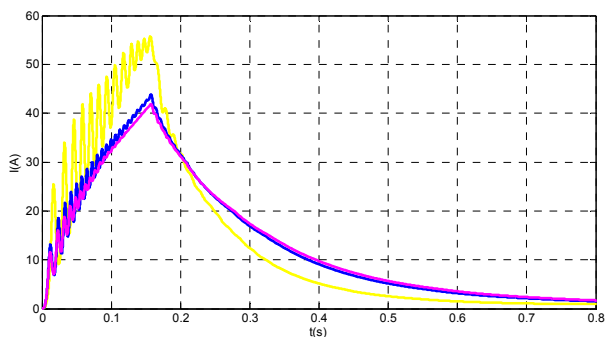


Fig.11. Time waveforms of currents 0-1500 rpm

**2. Power consumption graph at step response from 0 to 1500 RPM, without load:**

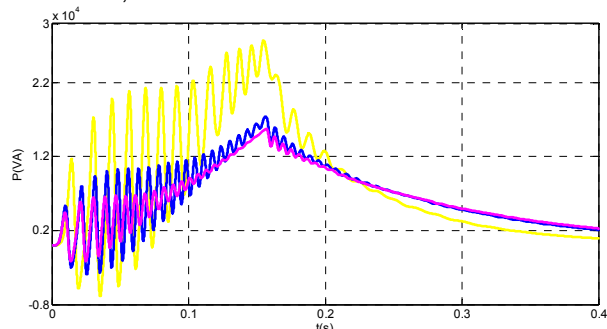


Fig.12. Power consumption differences - without load

**3. Power consumption graph after step response, added load at 1.2 seconds:**

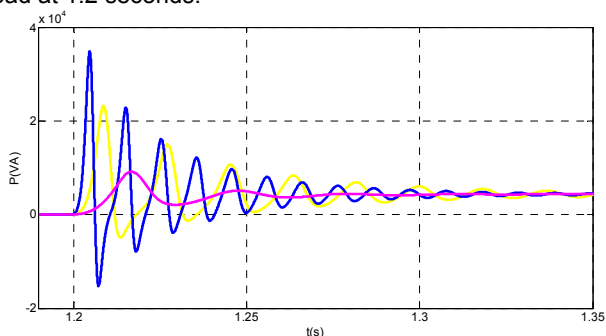


Fig.13. Power consumption differences - with load

As we can see, from all the responses the purple curve has the lowest value and it is smoother than others. This represents the lowest power consumption among all three controllers.

Additional confirmation of models' applicability is obtained by adding current limiter, where only the magnetically nonlinear dynamic model using dynamic inductances passes the test.

**Conclusion**

Three magnetically different types of SWM dynamic models were presented. Magnetically nonlinear dynamic model using dynamic inductances has provided the most accurate calculated results; therefore it can be used for steady state analysis and dynamic operation analysis.

For control design the "Quantitative Feedback Theory" method was used. Controller design with QFT technique leads to a very low order controller (the obtained controller is simple, robust and of low order).

For all three motor models the comparison of current, voltage and power time waveforms were presented. Magnetically nonlinear dynamic model with included dynamic inductances as parameters had the lowest power consumption.

The used approach is suitable for controller design to optimize the energy/power consumption in an electric vehicle.

The main focus of future research will be additional system energy optimizations, adding different loads and disturbances to the nonlinear model, testing all three designed controllers on a real SWM and comparison with the results presented in this paper.

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