Peter IVANYI[1], Amalia IVANYI[1]

Pollack Mihaly Faculty of Engineering and Information Technology, University of Pecs (1)

Hysteresis in semi-rigid steel joints

***Abstract**. In this paper the dynamic behaviour of a completely rigid steel column with a mass on the top, loaded by exponentially increasing cycling force is investigated. The joint at the fixed end of the column is modelled with a semi-rigid rotational spring; its non-linear characteristic is theoretically represented by a Preisach hysteresis model. In the solution of the non-linear dynamic equation of the motion the fix-point technique is inserted into the time marching iteration. The results are plotted in figures.*

***Streszczenie.** W artykule opisano badania reakcji sztywnej, stalowej kolumny z masą na szczycie, na działanie wzrastającej wykładniczo siły skręcającej. W celu zamodelowania zamontowanego na stałe końca kolumny, wykorzystano półsztywną sprężynę wirową, której nieliniowy charakter jest określony poprzez model histerezowy Preisacha. W celu rozwiązania równania ruchu, dokonano jego dyskretyzacji. Przedstawiono wyniki symulacyjne. (**Histereza w półsztywnych połączeniach stalowych**).*

Keywords: dynamical motion, Preisach model, fix-point technique.
Słowa kluczowe: ruch dynamiczny, model Preisacha, technika stało-krokowa.

Introduction

Several engineering research deals with the modelling of the non-linear behaviour of systems. The non-linear properties of structures can be treated by single-valued or multi-valued hysteresis characteristics.

The more detailed description of the non-linear behaviour of structures or materials is the microscopic model, where the elementary scale of material is simulated from the energetic aspect. In this case the non-linearity of material clusters can be analysed by Stoner-Wohlfarth type [1], Jiles-Atherton style [2] models.

The mesoscopic description of the hysteresis behaviour with respect to some physical properties can be handled by the Preisach type models [3]-[10], which is the most popular tool to model the non-linearity of materials and structures.

The simplest representation of the non-linear characteristics is the macroscopic model, where the phenomenological description of the physical process is the purpose. Several empirical models and analytical simulations have been developed [11]-[12] with the different level of the Ramberg-Osgood and the Richard-Abbott models [13]-[19] for the non-linearity of joints in steel frames in mechanical systems.

To assemble the material from elementary clusters and modelling their non-linearity from microscopic aspect can be found in [20]. To represent the hysteresis of steel due to the stress and strain effect the modified versions of Jiles-Athertom models can be found in [21], [22], and with loss separation under different stresses [23]. There are several researches to extend the Preisach type models for the simulation of mechanical properties of materials and to describe the non-linear behaviour of steel structures [24]-[26].

The purpose of this research is to extend the Preisach model for theoretical investigation of semi-rigid joint of steel columns and insert the model into the computation of the dynamic behaviour of the mechanical system. The experimental validation of the results will be evaluated after measurements on an actual structure have been carried out.

Dynamic model of the mechanical system

In Pecs, city of Hungary, in the frame of the preparation to be the Cultural Capital of Europe in 2010 a bell tower has been designed by architects Zoltan Bachman and Balint Bachmann [27] to one of the corner of the mosque, with the statue of St. Bartholomew. The bell tower has a moving telescopic structure hydraulically rising to become a tower during the tolling action (Fig. 1).

Fig. 1. St Bartholomew's bell tower

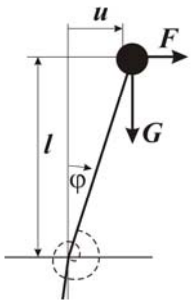
To model the behaviour of the dynamic system first the upper part of one column with semi rigid hinge at the joining point with non-linear hysteretic characteristic is modelled and checked how the hinge behaves during the dynamical action.

The external diameter of the investigated upper part is d_{ex}=0.282 m, the thickness is 8 mm, the length of the column is 12.75 m. The mass of the column m is concentrated to the top of the pillar and it is completed with the mass of the bell (290 kg) and the fly (40 kg). Under the action of the external cycling force and the mass of the columns the column has declination angle φ around the joining point resulting in deflection u (Fig. 2).

To model the moment of the electrical motor for forcing the bells to toll a concentrated, horizontal directed, exponentially increased periodically changing force is acting during the tolling process with amplitude F_0=1 kN with cycling periodicity T_p=0.2 s (Fig. 3)

(1) $$F(t) = F_0\left(1 - e^{-t/T_p}\right)\sin\left(\frac{2\pi}{T_p}t\right).$$

The investigated column itself is considered as a semi rigid cantilever with rotational spring at the joining point. The behaviour of rotational springs is modelled with hysteresis characteristics between the spring moment (P) and the declination angle (φ), $\varphi=H\{P\}$. During the investigations three types of characteristics are considered for the rotational springs as soft (H1), medium (H2) and hard (H3) as it can be seen in Fig. 4.



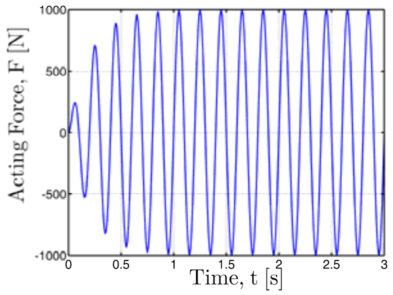


Fig. 2. The column Fig. 3. The cycling load of the column

Although in mechanical systems the characteristics of the spring moment-declination angle is handled, in this case the Preisach hysteresis model is applied to model the behaviour of the rotation spring, so the inverse hysteresis is constructed (Fig. 5).

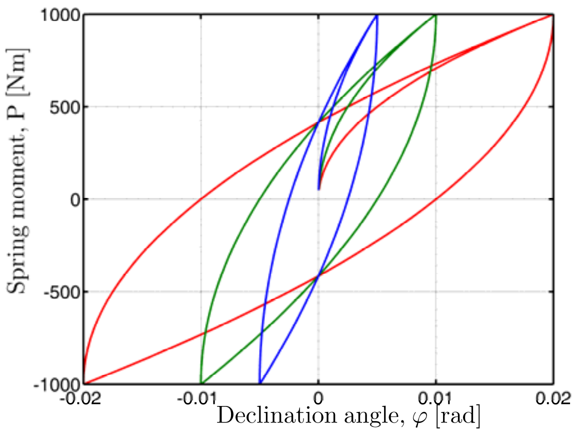


Fig. 4. Direct characteristics of rotational spring

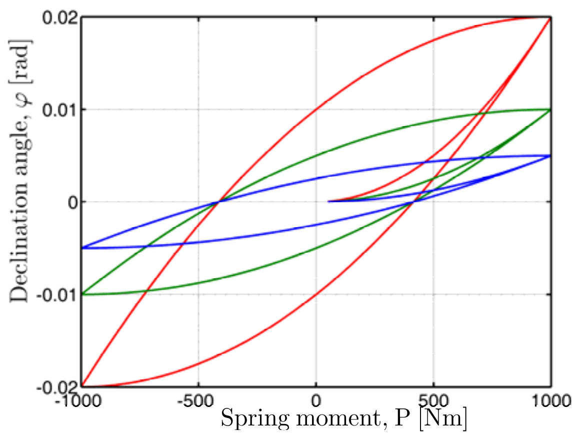


Fig. 5. Invers characteristics of rotational spring

Considering small declination $u \sim \varphi \cdot l$, ($\sin\varphi \sim \tan\varphi \sim \varphi$, $\cos\varphi \sim 1$) the acting bending moment generated by the external force and the mass positioned to the top of the cantilever produce the rotation and influence the rotation spring as

(2) $$I\ddot{\varphi}(t) + P(\varphi(t)) = F(t)l + Gu(\varphi(t)),$$

where I is the inertia moment of the mass m, $I=ml^2$, P is the bending moment acting on the rotational spring, G is the gravity force of the mass [29], [30].

Taking into account the small declination, the deflection can be represented as $u=\varphi l$, the second order non-linear differential equation has the form

(3) $$I\ddot{\varphi}(t) + P(\varphi(t)) = F(t)l + G\varphi(t)l,$$

(4) $$\varphi(t) = H\{P(\varphi(t),t)\}.$$

Numerical treatment

The above non-linear dynamic problem (3), (4) can be solved as it is proved in [31]. For the numerical approximation of the above problem the time discretisation is evaluated by the double application of the Crank-Nicolson iteration schema [32]. The non-linear iteration is realised by the fix-point technique [33], [34] to have a contract transformation of the direct characteristics

(5) $$P(\varphi) = k_{FP}\varphi + R\,,$$

where k_{FP} is the fix-point constant to represent the linear part of the connection stiffness and R is the residual non-linearity. Substituting (4) into (3) at a fixed time moment n, ($t_n = n \cdot dt$)

(6) $$I\ddot{\varphi}_n^{i+1} + k_{FP}\varphi_n^{i+1} = F_n l + G\varphi_n^i l - R_n^i,$$

the iteration steps are as follows:

Step 1, At the new n-th time step the initial value of the declination angle is equal to the last value of the previous time step, $\varphi_n^i = \varphi_{n-1}$, the initial value of the residual part of the hysteresis is equal to the last value of the previous time step, $R_n^i = R_{n-1}$;

Step 2, The value of the acting bending moments are known from the right side of (5) with respect to the iteration of declination φ_n^i;

Step 3, With the solution of (6) a new iteration for the declination φ_n^{i+1} can be determined;

Step 4, An estimation for the bending moment acting on the rotation spring can be determined as $P_n^{i+1} \approx k_{FP}\varphi_n^{i+1} + R_n^i$;

Step 5, With the hysteresis (4) the remaining non-linear part can be calculated as $R_n^{i+1} = P_n^{i+1} - k_{FP} \cdot H\{P_n^{i+1}\}$;

Step 6, The iteration continues on while $\left\|R_n^{i+1} - R_n^i\right\| > \varepsilon\left\|R_n^{i+1}\right\|$, $x_n^i = x_n^{i+1}$, $R_n^i = R_n^{i+1}$, go to Step 2.

Results

The numerical realization of the above theory has been developed within a MATLAB code and the hysteresis has been inserted into the numerical iteration. For all cases the time interval of investigation is 9 s. The cycling time was selected as 0.2 s, in one period 20 time steps have been investigated; the time discretisation was 0.01 s. Throughout the approximation 900 points have been calculated in total.

The different hysteresis characteristics result in different behaviour for the column.

It is assumed that the hinge at the joining point of the column has in order hard (H3), medium (H2) and soft (h1) hysteresis characteristic (see Fig. 5). During the time interval of acting force the variation of the declination angle versus the spring moment, the arising hysteresis during the motion is plotted in Fig. 6, while the deflection u of the end point of the cantilever can be seen in Fig. 7.

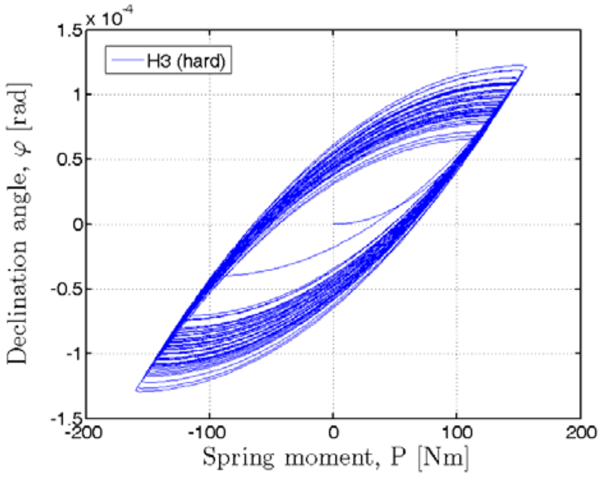


Fig. 6. The hysteresis described by the behaviour of the rotation spring with hard characteristics at the joining point

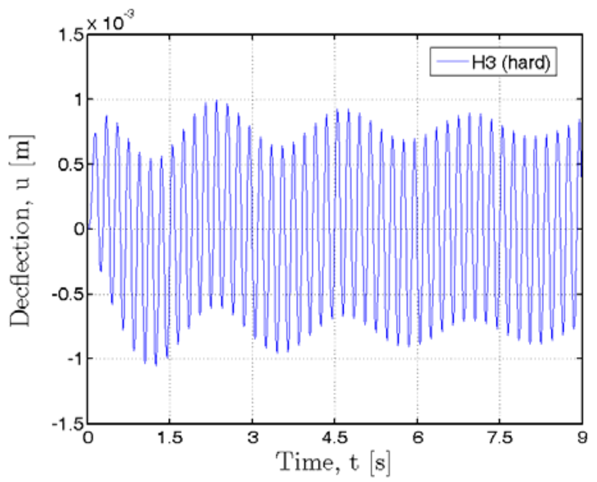


Fig. 7. The deflection at the joining point generated by the rotational spring of hard characteristics

For the rotational spring with medium characteristics (H2) the arising hysteresis and the time variation of the deflection is plotted in Fig. 8 and Fig. 9.

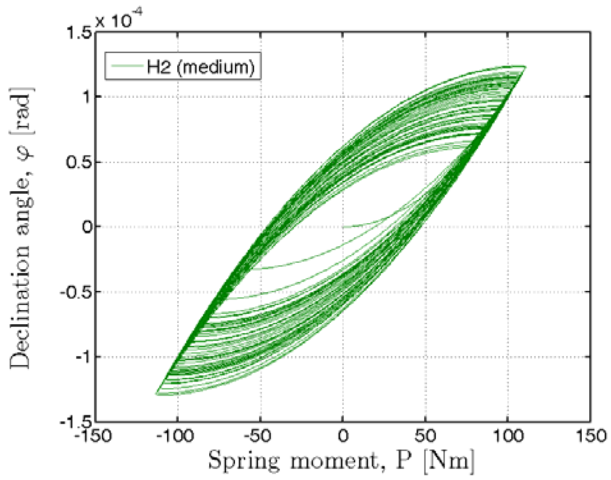


Fig. 8. The hysteresis described by the behaviour of the rotational spring with medium characteristics at the joining point

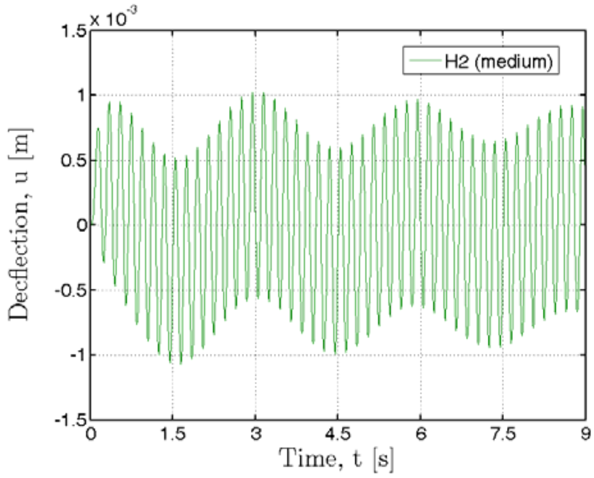


Fig. 9. The deflection at the joining point generated by the rotational spring of medium characteristics

Finally the behaviour of the soft rotation spring (H1) is investigated. The behaviour of soft spring can be seen in Fig. 10, while the deflection of the end point of the cantilever is plotted in Fig. 11.

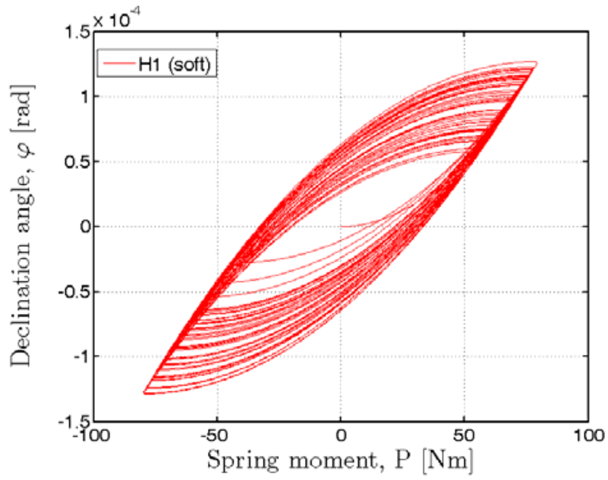


Fig. 10. The hysteresis descried by the behaviour of the rotational spring with soft characteristics at the joining point

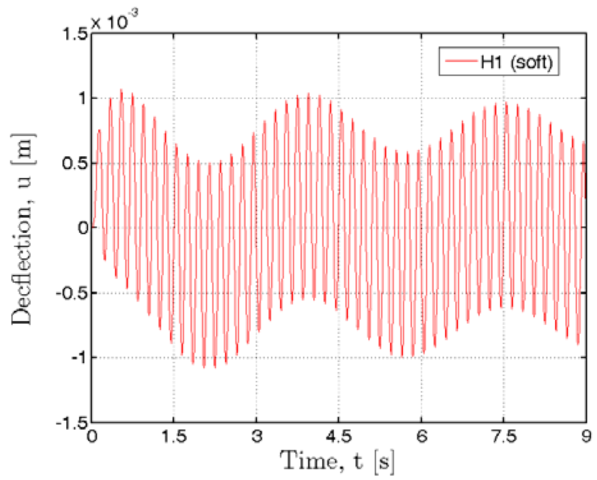


Fig. 11. The deflection at the joining point generated by the rotational spring of hard characteristics

From the figures it can be seen, that after the initial transient behaviour, the motion of the system can be stabilised during the tolling action. The small offset of the hysteresis curves originates from the mass of the column, the bell and the fly positioned at the top of the cantilever. The bending moment produced by the gravity force of the mass acts as a constant load on the system, which is added to the cycling load of the operating external force.

From the figures it can be seen as well, that by changing the property of the rotational spring (H3, H2, H1) the self-oscillation of the system is changing. Under hard property of the rotational spring (H3) the self oscillation time is 2.2 s, for medium property of the rotational spring this self oscillation time became 2.8 s, while for soft rotational spring this time period increases up to 3.6 s.

Conclusion

In the research the behaviour of the upper part of one of the columns of a bell tower has been investigated. The semi rigid joint point of the cantilever has been modelled by rotational spring of different properties of hysteresis. The mass of the column, the bell and the fly has been concentrated to the completely rigid top of the cantilever. The tolling process is modelled by a cycling load. The non-linear equation of motion has been solved by the double application of the Crank-Nicolson schema; the iteration along the non-linear hysteresis characteristics is evaluated by the fix-point technique.

The numerical simulation has been evaluated under theoretically assumed soft, medium and hard hysteresis property of the hinge at joining point. The results prove, that after an initial transient behaviour the motion of the system is stabilised.

REFERENCES

[1] Stoner E.C., Wohlfarth E.P., A mechanism of magnetic hysteresis in heterogeneous alloys, Reprintes in *IEEE Trans. on Magn.* 27 (1991) 3475-3518

[2] Jiles D.C., Atherton D.L., Theory of ferromagnetic hysteresis, *J. of Magn. and Mag. Materials,* 61 (1986) 48–60

[3] Chikazumi S., Charap S.H., *Physics of Magnetism*, J. Wiley (1964)

[4] Krasnosel'skii M.A., Pokrovskii A.V., *System with Hysteresis*, (in Russian) Nauka, Moscow (1983)

[5] Mayergoyz I.D., *Mathematical Model of Hysteresis*, Springer (1991)

[6] Visintin A., Differential Models of Hysteresis, Springer (1994)

[7] Brokatte M., Sprekels J., *Hysteresis and Phase Transitions*, Springer (1996)

[8] Ivanyi A., Hysteresis Models in Electromagnetic Computation, Akademiai Kiado, Budapest (1997)

[9] Bertotti G., Hysteresis in Magnetism, for Physicists, Material Scientists, and Engineers, Academic Press, NY (1998)

[10] Della Torre E., *Magnetic Hysteresis*, IEEE Press, NY (1999)

[11] Díaz C., Martí P., Victoria M., Querin O. M., Review on the modeling of joint behavior in steel frames, *J. of Constructional and Steel Research*, 67 (2011) 741–758

[12] Kwofie S., Description of cycling hysteresis behavior based on one-parameter model, *Material and Engineering*, A357, (2003) 86–93

[13] Ramberg W., Osgood W.R., Description of stress-strain curves by there parameters, Technical Note, No. 902, *National Advisory Committee for Aeronautics*, Washington DC (1943)

[14] Demartino A., Landolfo R., Mazzolani F.M., The use of Ramberg-Osgoos law for materials of round-house type, *Materails and Structures*, 23 (1990) 59–67

[15] Skelton R.P., Maier H.J., Christ H.J., The Baushinger effect, Masing model and the Ramberg-Osgood relation for cycling deformation in materials, *Material Science in Engineering*, A238 (1997) 377-390

[16] Mostaghel N., Byrd R.A., Inversion of Ramberg-Osgood equation and description of hysteresis loop, *Int. J. of Non-Linear Mechanics*, 37 (2002) 1319-1335

[17] Niesłony A., el Dsoki C., Kaufmann H., Krug P., New method for evaluation of the Masson-Coffin-Basquin and Ramberg-Osgood equations with respect to compatibility, *Int. J. of Fatigue*, 30 (2008) 1967–1977

[18] Ni Y.Q., Wang J.Y., Ko J.M., Advanced method for modeling hysteretic behaviour of semi-rigid joints, in *Advances in Steel Structures*, Ed. by S.L. Chan, J.G. Teng, Proceedings of the Second Int. Conf. on Advances in Steel Structures, Hong Kong, China, 15-17 December 1999, Elsevier, I (1999) 331–338.

[19] Richard R.M., Abbott B.J., Versatil elastic-plastic stress-strain formula, *J. Eng. Mech. Div. ASCE*, 101, 4 (1975) 511–515

[20] Smith R.C., Seelecke S., Dapino M., Ounaies Z., A unifies framework for modeling hysteresis in ferroic materials, *J. of the Mechanics and Physics in Solids*, 54 (2006) 46–85

[21] Anglada-Rivera J., Padovese L.R., Capó-Sánchez J., Magnetic Barkhausen noise in hysteresis loop in commercial carbon steel: influence of applied tensile stress and grain size, *J. of Magn. and Mag. Materials*, 231 (2001) 299–306

[22] Steven K.J., Stress dependence of ferromagnetic hysteresis loop for two grades of steel, *NDT&E International*, 33 (2000) 111–121

[23] Permiakov V., Dupré L., Pulnikov A., Melkebeek J., Loss separation and parameters for hysteresis modeling under compressive and tensile stresses, *J. of Magn. and Mag. Materials*, 272–276 (2004) e2553–e2554

[24] Aleshin V., Van Den Abeele K., Micro-potential model for stress-strain hysteresis of micro-cracked materials, *J. of the Mechanics and Physics of Solids*, 5553 (2005) 795–824

[25] Bolshakov G.V., Lapokov A.J., A Preisach model for magnetoelastic hysteresis, *J. of Magn. and Mag. Materials*, 162 (1996) 112–116

[26] Berquist A., Engdahl G., A stress-dependent magnetic Preisach hysteresis model, *IEEE Trans. on Magn*. 27 (1996) 112–116

[27] Bachmann B., Bachman Z., St. Bartholomew's bell tower, *Pollack Periodica*, 5, 3, (2010) 19–26

[28] Kuczmann M., Ivanyi A., *The Finite Element Method in Magnetics*, Akademiai Kiado, Budapest (2008)

[29] Chan S.L., Chui P.P.T., Nonlinear Static and Cycling Analysis of Steel Frames with Semi-rigid Connections, Elsevier (2000)

[30] Maroti Gy., Finding closed-form solutions of beam vibration, *Pollack Periodica*, 6, 1, (2011) 141–154

[31] Krejci P., Forced periodic vibrations of an elastic system with elasto-plastic damping, *Application of Mathematics*, 33 (1988) 145–153

[32] Ivanyi A., Continuous and Disctere Simulations in Electrodynamics, Akademiai Kiado, Budapest (2003)

[33] Bottachio O., Chiampi M., Ragusa C., Transient analysis of hysteretic field problems using fixed point technique, *IEEE Trans. on Magn*, 34 (2003) 1179–1182

[34] Kuczmann M., Vector Preisach hysteresis modeling, measurement, identification and application, *Physica B*, Condensed Matter, 406 (2011) 1403-1409