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Steady-state analysis of synchronous machines loaded by an angle depended torque

Streszczenie, *Reakcja maszyny synchronicznej na moment obciążenia zależny od kąta obrotu jest ważna dla napędów tłokowych. W celu jej określenia należy równania elektryczne maszyny rozwiązywać łącznie z równaniem ruchu, co prowadzi do nieliniowego układu równań różniczkowych. Najczęściej do ich rozwiązywania stosuje się procedury numeryczne, a stan ustalony otrzymuje się po zaniku procesów przejściowych. W pracy, do wyznaczenia takiego stanu ustalonego zastosowano metodę bilansu harmonicznych. Wówczas rozwiązanie przewiduje się w postaci szeregu Fouriera. Bilans harmonicznych prowadzi do nieskończenie wielu nieliniowych równań algebraicznych wiążących współczynniki Fouriera tych szeregów. W celu rozwiązania tego układu, ograniczonego do wymiarów skończonych, użyto algorytmu Newtona-Raphsona. W rezultacie obliczono bezpośrednio widma prądów maszyny oraz widma zmienności kąta mocy i wahań prędkości obrotowej.* ***(Analiza pracy maszyny synchronicznej przy zaburzeniach momentu mechanicznego zależnych od kąta obrotu)***

Abstract, *Steady-state response of a synchronous machine to the torque with angle depended pulsating component is important for the piston type drives. To find such response, electrical equations have to be solved together with the equation of motion, what leads to nonlinear differential equation set. The numerical integration is often used, and steady-states are obtained when transients disappear. In this paper the method of harmonic balance is used for determining such steady-states. The solution is predicted in the form of the Fourier series, and harmonic balance method leads to an infinite set of nonlinear algebraic equations for coefficients of that series. The Newton–Raphson scheme is used to solve these equations when limited to the finite dimensions. As a result the Fourier spectra of all machine currents, as well as of the rotor speed and variation of the power angle, are directly determined.*

Słowa kluczowe: maszyna synchroniczna, stan ustalony, analiza spektralna, metoda bilansu harmonicznych.
Keywords: synchronous machine, steady-state, spectral analysis, harmonic balance method.

Introduction

Synchronous machines loaded by piston compressors or driven by Diesel engines work at mechanical torque, which contains an alternating component. The torque oscillations appear due to the pulsation character of forces acting on the pistons, and are related to the angular position of the machine shaft. It follows that synchronous machine is loaded by an angle depended torque, which generates the speed ripples, even if the machine is running synchronously. Electromechanical interactions in the machine generate additional alternating components in the currents of all windings. In order to determine all those additional effects in mechanical and electrical variables, it is necessary to solve full set of machine equations, which is nonlinear. For synchronous machine it is, at least, a set of six differential equations.

It is a difficult problem to find directly the steady-state solution for those equations. That is why the equations are usually solved numerically and steady-state is achieved after the transients. Simplified approach relies on linearization of equations, and steady state analysis is provided similar to linear systems [1][2][4]. In [3] a methodology is presented for direct determination of steady-state solution for nonlinear differential equations when periodic solution can be predicted. In [6] it has been adapted to the equations of synchronous machines forced by the angle depended torque. The harmonic balance is the base of that methodology [5].

This paper presents the harmonic balance approach to determine the steady-states of a synchronous motor loaded by mechanical torque with two components: a constant one and a mono-harmonic angle depended component. It has been assumed that a motor under such load is running synchronously and the angle dependent component generates only additional components in motor currents as well as in the rotor speed and the power angle. Further, it has been assumed that a motor is described by commonly used ‘d-q’ model of synchronous machines and the stator windings are supplied by the balanced set of AC voltages and the field winding from DC source. Under those assumptions the harmonic balance method can be used for direct steady-state prediction.

Harmonic balance method for equations of synchronous motor

To apply harmonic balance method, the equations of synchronous machine have been described in coordinates (0,+,–) defined as

$$\begin{bmatrix} x^0(t) \\ x^+(t) \\ x^-(t) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \mathrm{e}^{-j\Omega_\mathrm{s}t} & 0 \\ 0 & 0 & \mathrm{e}^{j\Omega_\mathrm{s}t} \end{bmatrix} \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \mathrm{a} & \mathrm{a}^2 \\ 1 & \mathrm{a}^2 & \mathrm{a} \end{bmatrix} \begin{bmatrix} x_\mathrm{a}(t) \\ x_\mathrm{b}(t) \\ x_\mathrm{c}(t) \end{bmatrix}$$

($\mathrm{a} = \mathrm{e}^{j\frac{2\pi}{3}}$) both for stator currents ($x_n(t) \to i_n(t)$) and voltages ($x_n(t) \to u_n(t)$). Balanced stator voltages with pulsation Ω_s are represented for those coordinates by constant values $u^+ = u^- = \sqrt{\tfrac{3}{2}}U$. The machine equations, using commonly accepted notation, takes the form

$$\text{(1a)} \qquad \begin{bmatrix} u^+ \\ u^- \\ \mathrm{U}_\mathrm{f}' \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \mathrm{R}_\mathrm{s} \cdot i^+ \\ \mathrm{R}_\mathrm{s} \cdot i^- \\ \mathrm{R}_\mathrm{f}' \cdot i_\mathrm{f}' \\ \mathrm{R}_\mathrm{D}' \cdot i_\mathrm{D}' \\ \mathrm{R}_\mathrm{Q}' \cdot i_\mathrm{Q}' \end{bmatrix} + \frac{\mathrm{d}}{\mathrm{d}t} \begin{bmatrix} \psi^+ \\ \psi^- \\ \psi_\mathrm{f}' \\ \psi_\mathrm{D}' \\ \psi_\mathrm{Q}' \end{bmatrix} + \mathrm{j} \cdot \Omega_\mathrm{s} \begin{bmatrix} \psi^+ \\ -\psi^- \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\text{(1b)} \qquad \mathrm{J}\frac{\mathrm{d}^2\varphi}{\mathrm{d}t^2} + \mathrm{D}\frac{\mathrm{d}\varphi}{\mathrm{d}t} = \mathrm{j} \cdot \mathrm{p} \cdot \left(i^- \cdot \psi^+ - i^+ \cdot \psi^-\right) + T_\mathrm{m}(\varphi)$$

Considering a steady-state performance at the synchronous rotor speed with the angle depended torque periodically repeated $T_\mathrm{m}(\varphi) = T_\mathrm{m}(\varphi + 2\pi)$, all currents can be seen as the periodic time functions, except the rotation angle $\varphi(t)$ because it is permanently increasing. Introducing the angle deviation $\Delta\varphi(t)$ with respect to linearly growing term due to the synchronous speed $(\Omega_\mathrm{s}/\mathrm{p}) \cdot t$, where ‘p’ is a pole-pair number of a machine, the rotational angle is given by the formula

$\varphi(t)=(\Omega_s/p)\cdot t+\Delta\varphi(t)$. Using the angle deviation as the variable, the motion equation can be rewritten in the form

(2) $$J\frac{d^2\Delta\varphi}{dt^2}+D\frac{d\Delta\varphi}{dt}=j\cdot p\cdot\left(i^-\cdot\psi^+-i^+\cdot\psi^-\right)+T_m(\varphi)-D\frac{\Omega_s}{p}$$

Now, the angle deviation $\Delta\varphi(t)$ can be described as periodic at the steady-state.

The equations (1a) and (2) can be combined to the one nonlinear set in the form

(3) $$\frac{d^2}{dt^2}\mathbf{F}_2(\mathbf{x})+\frac{d}{dt}\mathbf{F}_1(\mathbf{x})+\mathbf{F}_0(\mathbf{x})=0$$

for the vector $\mathbf{x}=\left[i^+\quad i^-\quad i_f^{'}\quad i_D^{'}\quad i_Q^{'}\quad \Delta\varphi\right]^T$ of unknown functions, which can be predicted in the form of the Fourier series

(4) $$\mathbf{x}=\sum_{k=-\infty}^{\infty}\mathbf{X}_k\cdot e^{jk\Omega_x t}\ ,\qquad \Omega_x=2\pi/T_x$$

with the period T_x. Also, the vectors $\mathbf{F}_0$, $\mathbf{F}_1$ and $\mathbf{F}_2$ can be expressed as the Fourier series

(5) $$\mathbf{F}_n=\sum_{k=-\infty}^{\infty}\mathbf{F}_{n,k}\cdot e^{jk\Omega_x t}\ \text{ for }\ n\in\{0,1,2\}$$

because their elements are nonlinear functions of the unknown functions in the vector $\mathbf{x}$.

The period T_x of predicted solution can be found from the following interpretation of physical phenomena. Generally, the angle dependent load torque can be expanded into the Fourier series

$$T_m(\varphi)=\sum_{k=-\infty}^{\infty}T_k\cdot e^{jk\varphi}=T_m(\varphi+2\pi)$$

and in the steady-state it should repeated with the period T_x, i.e. $T_m(t)=T_m(t+T_x)$. This is possible when

(6) $$T_x=p\cdot T_s$$

where T_s is the period of supply voltages. Individual torque harmonics are repeated with the periods $T_{k,x}=(p/k)\cdot T_s$.

Harmonic balance method compares the Fourier series (5) on both sides of equations (3). It leads to an infinite set of algebraic equations (7) with respect to the unknown coefficients $\mathbf{X}_k$ of the series (4), The equation set (7) is nonlinear because the Fourier coefficients $\mathbf{F}_{n,k}$ of the series (5) in those equations depend nonlinearly on the coefficients $\mathbf{X}_k$.

(7) $$-\mathrm{diag}\begin{bmatrix}\vdots\\ \mathbf{\Omega}^2\\ \mathbf{0}\\ \mathbf{\Omega}^2\\ \vdots\end{bmatrix}\cdot\begin{bmatrix}\vdots\\ \mathbf{F}_{2,1}\\ \mathbf{F}_{2,0}\\ \mathbf{F}_{2,-1}\\ \vdots\end{bmatrix}+\mathrm{diag}\begin{bmatrix}\vdots\\ j\mathbf{\Omega}\\ \mathbf{0}\\ -j\mathbf{\Omega}\\ \vdots\end{bmatrix}\cdot\begin{bmatrix}\vdots\\ \mathbf{F}_{1,1}\\ \mathbf{F}_{1,0}\\ \mathbf{F}_{1,-1}\\ \vdots\end{bmatrix}+\begin{bmatrix}\vdots\\ \mathbf{F}_{0,1}\\ \mathbf{F}_{0,0}\\ \mathbf{F}_{0,-1}\\ \vdots\end{bmatrix}=\begin{bmatrix}\vdots\\ 0\\ 0\\ 0\\ \vdots\end{bmatrix}$$

where $\Omega=\Omega_x\cdot\mathbf{E}=2\pi\cdot f_x\cdot\mathbf{E}$ and $\mathbf{E}$ is a unit matrix. Those equations can be written in a simple form

(8) $$-\left(\mathbf{\Omega}\right)^2\cdot\mathbf{F}_2\left(\mathbf{x}\right)+j\cdot\mathbf{\Omega}\cdot\mathbf{F}_1\left(\mathbf{x}\right)+\mathbf{F}_0\left(\mathbf{x}\right)=\mathbf{0}$$

using so called vector representations of the Fourier series [5].

Such infinite and nonlinear equation set can be solved numerically using Newton-Raphson algorithm when limiting its dimensions to the finite one [3][6]

(9) $$\mathbf{x}_{n+1}=\mathbf{x}_n-\mathbf{J}\left(\mathbf{x}_n\right)^{-1}\cdot\mathbf{F}\left(\mathbf{x}_n\right)$$

where

(10a) $$\mathbf{F}\left(\mathbf{x}\right)=-\left(\mathbf{\Omega}\right)^2\cdot\mathbf{F}_2\left(\mathbf{x}\right)+j\cdot\mathbf{\Omega}\cdot\mathbf{F}_1\left(\mathbf{x}\right)+\mathbf{F}_0\left(\mathbf{x}\right)$$

(10b) $$\mathbf{J}\left(\mathbf{x}\right)=\frac{\partial\mathbf{F}\left(\mathbf{x}\right)}{\partial\mathbf{x}}$$

The crucial point of this algorithm is the creation the Jacobi matrix (10b). This procedure is described in detail in [3][6].

Results of the numerical tests

The numerical algorithm has been implemented using the commercial MATLAB package and its convergence has been confirmed [6].

Calculations has been carried out for a motor with rated data P_N =1,25MW, U_N,=6kV, n_0 =750rpm and cosφ_N=0,9. It has been assumed that motor is supplied by a balanced three phase voltages and loaded by a torque given by the formula

(11) $$T_m(\varphi)=T_N+0,1\cdot T_N\cdot\sin(2\cdot\varphi+\pi/2)$$

Basing on the vector $\mathbf{x}=\left[i^+\quad i^-\quad i_f^{'}\quad i_D^{'}\quad i_Q^{'}\quad \Delta\varphi\right]^T$, the Fourier spectra of the following quantities have been calculated:

- the stator current of the phase 'a'

(12) $$i_a(t)=\sum_{k=-N}^{N}I_k\cdot\cos\left[2\pi\cdot f_s\cdot(1+2k/p)\cdot t+\alpha_k^+\right]$$

- rotor currents, i.e. the field winding current and the equivalent starting cage currents

(13a) $$i_f^{'}(t)=I_{f,0}+\sum_{k=1}^{N}I_{f,k}\cdot\cos\left[(2k/p)\cdot 2\pi\cdot f_s\cdot t+\beta_k\right]$$

(13b) $$i_D^{'}(t)=\sum_{k=1}^{N}I_{D,k}\cdot\cos\left[(2k/p)\cdot 2\pi\cdot f_s\cdot t+\chi_k\right]$$

(13c) $$i_Q^{'}(t)=\sum_{k=1}^{N}I_{Q,k}\cdot\cos\left[(2k/p)\cdot 2\pi\cdot f_s\cdot t+\lambda_k\right]$$

- the rotor speed

(14) $$\omega(t)=\Omega_0+\sum_{k=1}^{N}\Omega_k\cdot\cos\left[(2k/p)\cdot 2\pi\cdot f_s\cdot t+\gamma_k\right]$$

- the power angle

(15) $$\vartheta(t)=\vartheta_0+\sum_{k=1}^{N}\vartheta_k\cdot\cos\left[(2k/p)\cdot 2\pi\cdot f_s\cdot t+\nu_k\right]$$

Figures 1 – 6 show the Fourier spectra of the variables listed above that characterize the steady-state. The magnitudes of the first greatest harmonics are only presented. The decibel calibration of vertical axes is used because of the disproportions between magnitudes of components. It should be mentioned that presented spectra contain harmonics with strictly determined frequencies, which do not depend on the accuracy of calculations. Those frequencies depend on properties of the loaded torque, and for the considered case they differ by $2 \cdot f_s / p$, i.e. by 25Hz. So, a qualitative analysis of the spectra is uniquely done.

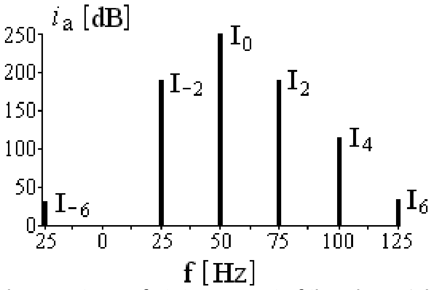

Fig. 1. Fourier spectrum of stator current of the phase 'a'

Fig.1 shows the Fourier spectrum of the stator current of the phase 'a'. It contains the fundamental component I_0 with supply frequency 50Hz and additional components shifted successively by 25Hz. According the formula given in Fig.1 those components have the frequencies of $(50 \pm k \cdot 25)\text{Hz}$, i.e. for $|k| \le 3$ they take the values

(16) $$(\cdots, -25, 0, 25, 50, 75, 100, 125, \cdots)\text{Hz}$$

The constant component cannot arise in the stator current due to the magnetic coupling and it disappears in calculated spectrum, confirming the correctness of the analysis. Negative frequencies in the spectrum indicate that a set of stator currents with such frequencies has oposite sequence. So, the stator currents have both positive and negative sequences for each frequency. It means that stator currents are asymmetrical and each one has different rms value. However, the magnitudes of additional components are rather small and decrease very fast with the frequency. The 75Hz component is approximately 60dB less than the 50Hz component. The currents of the opposite sequence for 50Hz, related to the component with frequency -50Hz, are very low. Then, the stator currents are practically symmetrical.

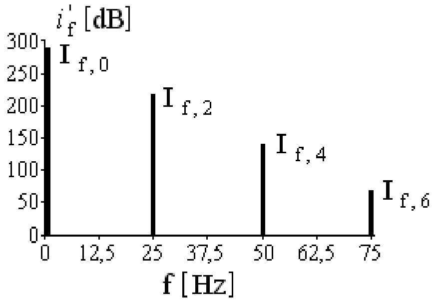

Fig. 2. Fourier spectrum of the field current

The Fourier spectrum of the field winding current is presented in Fig.2. It contains constant component related to the DC voltage supplying that winding and components generated by pulsating component of the loaded torque. Those components have frequencies of 25Hz, 50Hz 75Hz, etc. Their magnitudes decrease even faster than in the stator currents. Two successive harmonics differ approximately by 70dB.

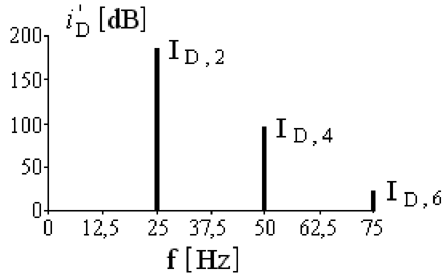

Fig. 3. Fourier spectrum of the 'd' equivalent damping cage current

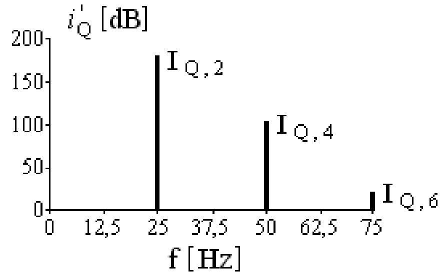

Fig. 4. Fourier spectrum of the 'q' equivalent damping cage

The algorithm presented in this paper solves all equations of a synchronous machine, therefore the unreal currents of the equivalent damping cages in 'd' and 'q' axes are calculated as well. Despite being not observable, they allow to determine power losses in the starting or damping cage of the machine. Fig.3 and Fig.4 show the Fourier spectra of those currents. The constant components do not appear in the spectra because at the synchronously running machine those currents are equal to zero. The damping cage currents have the same features as the additional components in the field winding. Their Fourier spectrum contain the frequencies of 25Hz, 50Hz 75Hz, etc., and the successive harmonics differ approximately only by 70dB.

One of the variables calculated by the Newton-Raphson algorithm is the angle deviation $\Delta\varphi(t)$, which is periodic. It allows determining the power angle changes at considered steady-state. In Fig.5 the Fourier spectrum of the power angle is presented. The constant value follows from the mean value of the loaded torque. The deviations have harmonics with frequencies of 25Hz, 50Hz 75Hz, etc. Their magnitudes decrease also very fast, and the two successive harmonics differ approximately by 70dB. It should be noted that even if the torque has only one harmonic, the machine responds with many harmonics due to the nonlinearity of full set of equations.

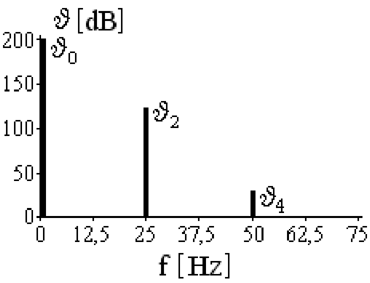

Fig. 5. Fourier spectrum of the power angle

The alternating component in the torque generates perturbations of the angular velocity, which can be calculated as the derivatives of the deviations of is the angle deviation $\Delta\varphi(t)$. Fig. 6 shows the Fourier spectrum of

the angular rotor speed. The constant value has been added because perturbations affect the steady-state at the synchronous speed, what has been taken into account by introducing the angle deviation as the difference between the real value of the rotational angle and linearly growing part due to the synchronous speed $\Delta\varphi(t) = \varphi(t) - (\Omega_S / \mathrm{p}) \cdot t$.
The additional components in the Fourier spectrum of rotor speed have the same frequencies as the power angle deviation, i.e. 25Hz, 50Hz 75Hz, etc. That spectrum is more sensitive compared to the spectrum of the power angle, and the successive harmonics differ approximately only by 60dB.

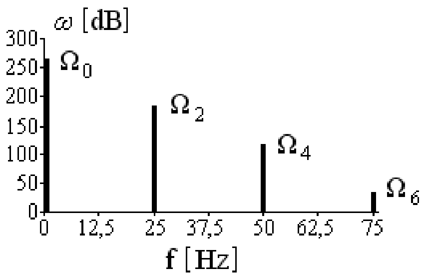

Fig. 6. Fourier spectrum of the rotor speed

Conclusions

This paper describes the algorithm which allows to directly determine a steady-state of a synchronous machine loaded by an angle dependent torque. Application of that algorithm is illustrated by a case study for synchronously running 4-pole motor when the constant torque is disturbed by a component repeated twice per revolution. It has been confirmed that the Fourier spectra of all important quantities characterizing the steady-state of synchronous motor can be both qualitatively predicted and quantitatively determined by solving the set of nonlinear algebraic equations.

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