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Problems of modeling an electrical arc with variable geometric dimensions

Abstract. *Problems with representing power dissipation in simple, dynamic arc models with variable plasma column length have been described. Combined models (serial and parallel-hybrid) have also been presented that can be applied in wide range of currents. Then the analysis have been extended to generalized models with variable geometric sizes, in which energy dissipation is proportional to plasma area or volume. Problems with the choice of weight function and attenuation coefficient in hybrid models have been described. Results of processes simulation in circuit with controlled arc have been presented.*

Streszczenie. *Opisano problemy odwzorowania mocy dyssypacji w prostych modelach dynamicznych łuku o zmiennej długości kolumny plazmowej. Przedstawiono także modele kombinowane (szeregowy i równoległy-hybrydowy), mogące mieć zastosowanie w szerokim zakresie zmian prądu. Następnie rozszerzono tę analizę na uogólnione modele o zmiennych rozmiarach geometrycznych łuku, w których dyssypacja energii jest proporcjonalna do powierzchni lub objętości plazmy. Opisano problemy wyboru funkcji wagowej i współczynnika tłumienia w modelach hybrydowych. Zamieszczono wyniki symulacji procesów w obwodzie z łukiem sterowanym. (**Zagadnienia modelowania łuku elektrycznego o zmiennych rozmiarach geometrycznych**).*

Keywords: electrical arc, Berger model, Kulakov model, Voronin model, Sawicki model, Voronin-Sawicki hybrid model.
Słowa kluczowe: łuk elektryczny, model Bergera, model Kułakowa, model Woronina, model Sawickiego, model hybrydowy Woronina-Sawickiego.

Introduction

An arc column of variable size is a frequent physical phenomenon occurring in a number of electrotechnological devices. It is caused by various kinds of external interference or by the functioning of control systems [1]. Its influence on the electrical arc dynamics is often disregarded due to high inertia of mechanical systems as compared to relaxation times of plasma processes. In the case of modeling switching apparatus or devices with electromagnetic effect on the arc column, it is necessary to take into consideration the arc length and diameter variation in time. Besides, it is essential to know how the arc parameters and characteristics change even during slow controlling of the arc power. The modeling of such arcs has been described in a number of publications [2-9] and the results obtained in these studies will be restated with greater precision in order to reduce discrepancies between them and the results of experiments, especially in the range of very weak currents.

Representing the energy dissipation in variable-length arc models

Changing the arc length is the most popular way to influence the power and stability of the arc discharge. The effects obtained depend, among other things, on the power supply characteristics and other factors affecting the plasma channel diameter.

Popular models of the nonstationary electric arc, such as Mayr's model, or Cassie's model treat the plasma channel as cylindrical, from which energy is dissipated by conduction, or convection. In Cassie's model the power dissipated by convection is variable

$$P_{dis}(l) = u_C^2(l)\cdot g + p_v\left(\frac{dl}{dt}\right) \tag{1}$$

Since the voltage on the arc increases with the increase in the arc length l, the following formula has been proposed in [11] to account for the square voltage component in Cassie-Berger's model

$$u_C^2(l) = al \tag{2}$$

where the parameter a [V^2/m] is almost constant within the wide range of current i variation. The power $p_v(dl/dt)$ needed for producing an additional plasma volume is defined as

$$p_v\left(\frac{dl}{dt}\right) = \begin{cases} +b_1\dfrac{dl}{dt}, & \text{if } \dfrac{dl}{dt} > 0 \\ -b_2\dfrac{dl}{dt}, & \text{if } \dfrac{dl}{dt} \le 0 \end{cases} \tag{3}$$

where $b_2 \approx 0$ if $dl/dt < 0$ due to arc dissipativity. The arc equation modified by Berger with a variable Cassie voltage $U_C(t) = U_C\left(l, \frac{dl}{dt}\right)$ takes the conductance form [11]

$$\frac{1}{g}\frac{dg}{dt} = \frac{1}{\theta_C}\left(\frac{u_{col}^2}{u_C^2(l) + \frac{1}{g}p_v\left(\frac{dl}{dt}\right)} - 1\right) \tag{4}$$

where g – plasma column conductance; l – arc length; θ_C - time constant of the model; u_{col} – voltage drop at the arc column.

When creating some dynamic arc models it is convenient to adopt the static characteristic $P_{stat}(I)$ [10], which is easy to obtain experimentally, when the following simple formula is employed

$$P_{stat}(I,l) = U_{stat}(I,l)\cdot I = E_{stat}(I)\cdot l\cdot I \tag{5}$$

In the classical version of Mayr's model it is assumed that $P_{dis}(t) = P_M = const$. For larger currents this condition does not hold, in which case Cassie's model is applied [7, 11]. Since the heat dissipation processes react slowly to external interference, it is possible to assume, that the power loss is mainly determined by the static chracteristics [7], $P_{dis}(t) \approx P_{stat}(i(t))$.

Kulakov suggested a modification of Mayr's model and Shellhase's model [10], employing the static characteristics by allowing for the arc length variation. The I-order model in the conductance form is [4]

$$\frac{1}{g}\frac{dg}{dt}=\frac{1}{\theta_{Ms}}\left[\frac{i}{g\cdot l\cdot E_{stat}(i)}-1\right]-\frac{1}{l}\frac{dl}{dt} \tag{6}$$

where i – arc current; $E_{stat}(i)$ – static characteristic of the electric field intensity; θ_{Ms} – time constant of the model.

Connecting in a series two non-linear conductances g_C and g_M, corresponding respectively to Cassie-Berger's model and to the Kulakov's model, makes it possible to obtain an arc model analogous to Habedank's one [3, 8]

$$\frac{1}{g_C}\frac{dg_C}{dt}=\frac{1}{\theta_C}\left[\frac{u_{col}^2}{u_C^2(l)+\frac{1}{g_C}p_v\left(\frac{dl}{dt}\right)}\left(\frac{g}{g_C}\right)^2-1\right] \tag{7}$$

$$\frac{1}{g_M}\frac{dg_M}{dt}=\frac{1}{\theta_{Ms}}\left[\frac{u_{col}}{l\cdot E_{Mstat}(i)}\frac{g}{g_M}-1\right]-\frac{1}{l}\frac{dl}{dt} \tag{8}$$

$$\frac{1}{g}=\frac{1}{g_C}+\frac{1}{g_M} \tag{9}$$

where $E_{Mstat}(i)$ – is a virtual static characteristic of the electric field intensity $\left(E_{Mstat}(i)=E_{stat}(i)\cdot g_c/(g_C+g_M)\right)$.

The hybrid model of the arc column allowing for its length variation combines models (4) and (6) by means of a weight function $\varepsilon(i)$ and becomes [3, 8]

$$\begin{aligned}\frac{1}{g}\frac{dg}{dt}=\frac{1}{\theta(i)}\Bigg\{&[1-\varepsilon(i)]\frac{u_{col}^2}{u_C^2(l)+\frac{1}{g}p_v\left(\frac{dl}{dt}\right)}+\\&+\varepsilon(i)\frac{i}{g\cdot l\cdot E_{stat}(i_M)}-1\Bigg\}-\varepsilon(i)\frac{1}{l}\frac{dl}{dt}\end{aligned} \tag{10}$$

where $i_M=u_{col}g\cdot\varepsilon(i)$.

It has to be noted that the first serial models were created in order to simulate long high-voltage free arcs in power devices. Such arcs are typically fed from regular voltage sources. The parallel models, on the other hand, are used for simulating processes in electrotechnological devices with shorter, low-voltage arcs, which are either free or stabilized. Such arcs are fed from current sources, which can be even perfect within a limited voltage range.

Representing the power dissipation in generalized variable-size arc models

Voronin's arc model takes into account the external influence on the cylindrical arc length and diameter. It is necessary to make a number of assumptions [5, 12] to create the model, which is based on the simplified equation of the arc heat balance. It is therefore assumed that the dissipated power is proportional to the arc lateral surface

$$P_{disS}(l,S)=p_S l\sqrt{4\pi S} \tag{11}$$

The arc model obtained is one with variable geometrical parameters $S(t)$ and $l(t)$ of the arc. It takes the general conductance form [12]

$$\begin{aligned}\frac{1}{g}\frac{dg}{dt}=&\frac{1}{\theta_S(S)}\left(\frac{u_{col}i}{P_{disS}(l,S)}-1\right)+\\&-\frac{1}{l}\frac{dl}{dt}\left(1+\ln\frac{g\,l}{K_gS}\right)+\frac{1}{S}\frac{dS}{dt}\left(1-\ln\frac{g\,l}{K_gS}\right)\end{aligned} \tag{12}$$

where the attenuation coefficient is

$$\theta_S(S)=\frac{Q_0Sl}{P_{disS}(l,S)}=\frac{Q_0}{p_S}\sqrt{\frac{S}{4\pi}} \tag{13}$$

and Q_0 – reference coefficient, J/m^3; K_g – coefficient of approximation of unitary conductance, S/m; l – arc length, m; p_S – density of power dissipated through the lateral surface of the arc column, W/m^2; S – arc cross-section area, m^2. All the three parameters Q_0, K_g, p_S are obtained experimentally and assumed to be constant.

For a free, or partly free arc

$$\begin{aligned}P_{disS}(l,S(i))&=p_S l\sqrt{4\pi\cdot S(i)}=\\&=\pi p_S l\cdot d(i)=P_{disS}(l,d(i))\end{aligned} \tag{14}$$

and the attenuation coefficient is

$$\theta_S(S(i))=\frac{Q_0}{p_S}\frac{d(i)}{4}=\theta_S(d(i)) \tag{15}$$

As can be seen, in this model the dissipated power coefficient and the attenuation coefficient are proportional to the arc column diameter, which in turn depends on the current. Equation (12), then, becomes

$$\begin{aligned}\frac{1}{g}\frac{dg}{dt}=&\frac{1}{\theta_S(d(i))}\left(\frac{u_{col}i}{P_{disS}(l,d(i))}-1\right)+\\&-\frac{1}{l}\frac{dl}{dt}\left(1+\ln\frac{4g\,l}{\pi K_g d^2(i)}\right)+\\&+\frac{2}{d(i)}\frac{dd(i)}{dt}\left(1-\ln\frac{4g\,l}{\pi K_g d^2(i)}\right)\end{aligned} \tag{16}$$

In publication [2] static characteristics of that model are analysed and it is demonstrated that by means of the experimentally obtained function $d(i)$ it is possible to arrive at curves corresponding to the conditions conducive to heat transfer from the arc to the cool external region, occurring mainly by means of surface radiation. Such curves and conditions are encountered in welding and in initial stages of charge melting in arc furnaces. The graphs $u(i)$ are also similar to the characteristics of arcs operating in the range of low currents.

In Sawicki's model [2, 8] it is assumed that the dissipated power is proportional to the plasma volume

$$P_{disV}(l,S)=p_V lS \tag{17}$$

and the model obtained is one with variable geometrical parameters $S(t)$ and $l(t)$. It takes the general conductance form

$$\begin{gathered}\frac{1}{g}\frac{dg}{dt}=\frac{1}{\theta_V}\left(\frac{u_{col}i}{P_{disV}(l,S)}-1\right)+\\ -\frac{1}{l}\frac{dl}{dt}\left(1+\ln\frac{g\,l}{K_g S}\right)+\frac{1}{S}\frac{dS}{dt}\left(1-\ln\frac{g\,l}{K_g S}\right)\end{gathered} \tag{18}$$

where the attenuation coefficient is

$$\theta_V(S)=\frac{Q_0 Sl}{P_{disV}(l,S)}=\frac{Q_0}{p_V}=const \tag{19}$$

For a free, or partly free arc

$$P_{disV}(l,S(i))=p_V l\cdot S(i)=\frac{\pi}{4}p_V l\cdot d^2(i) \tag{20}$$

As evident above, the value of the dissipated power in this model is proportional to the area of the arc cross-section, which in turn depends on the current, and the attenuation coefficient is constant. Then, Equation (18) becomes

$$\begin{gathered}\frac{1}{g}\frac{dg}{dt}=\frac{1}{\theta_V}\left(\frac{u_{col}i}{P_{disV}(l,d(i))}-1\right)+\\ -\frac{1}{l}\frac{dl}{dt}\left(1+\ln\frac{4g\,l}{\pi K_g d^2(i)}\right)+\\ +\frac{2}{d(i)}\frac{dd(i)}{dt}\left(1-\ln\frac{4g\,l}{\pi K_g d^2(i)}\right)\end{gathered} \tag{21}$$

In publication [2] the static characteristics of this model are analysed and it is demonstrated that when the experimentally obtained function $d(i)$ is applied it is possible to arrive at curves representing the adverse conditions of heat transfer from the arc to the hot surrounding region, mostly by radiation and convection. Such conditions and curves are attested in the final stages of charge melting in arc furnaces. The graphs $u(i)$ are also close to characteristics of arcs in high current ranges.

The experimental studies on the free arc during the quasi-static current alteration show that the following holds

$$d(i)=b|i|^q\text{, cm} \tag{22}$$

If the arc is in air, then b = 0,0025 cm·A^{-q}. If the arc is submersed in a longitudinal gas stream, q = 0,6÷0,7 [13]. The value of (22) is close to the theoretically obtained one, in which q = 2/3, but the coefficient b given in [14] is almost three times larger (b = 0,0062). This proves high discrepancy between different sets of experimental data. Due to inertia of heat processes in AC arcs, especially those of higher frequency or with steep curves passing the zero value of current, during the momentary lack of current there still exists a conducting plasma channel, which facilitates the next discharge. The current drop is accompanied by a very quick weakening of the effect of plasma being shrunk by its own magnetic field, since the pressure increase in the radial direction is $\Delta p_r \propto i^2$. Additionally, on some electrodes the current drop can result in a change in the structure of the cathode spot, a change in the cathode emission state, restructuring of the conical part of the plasma column, etc. Owing to these factors, the arc diameter function can have its minimum at the point of relatively small, non-zero value of current. This is confirmed by experimental studies on the time constant [15], which, according to (15) is proportional to the arc diameter. To approximate the dependence of the arc diameter on the arc current it is possible to employ one of the symmetrical functions with an appropriate derivative

$$\left.\begin{aligned}&d(i_*)=a\exp\left(-pi_*^2\right)+b(i_*+\delta)^q\\ &\frac{dd(i_*)}{di_*}=-2api_*\exp\left(-pi_*^2\right)+bq\cdot(i_*+\delta)^{q-1}\end{aligned}\right\} \tag{23}$$

$$\left.\begin{aligned}&d(i_*)=a\exp\left(-pi_*^2\right)+b\cdot\operatorname{ar\,sinh}(qi_*)\\ &\frac{dd(i_*)}{di_*}=-2api_*\exp\left(-pi_*^2\right)+bq\frac{1}{\sqrt{1+(qi_*)^2}}\end{aligned}\right\} \tag{24}$$

$$\left.\begin{aligned}&d(i_*)=a\exp\left(-pi_*^2\right)+b\ln\left(1+qi_*^2\right)\\ &\frac{dd(i_*)}{di_*}=-2api_*\exp\left(-pi_*^2\right)+2bq\frac{i_*}{1+qi_*^2}\end{aligned}\right\} \tag{25}$$

where $i_*(t)=|i(t)|$, δ - very small constant (eg. δ = 1·10^{-2}), and $d(i)=d(|i|)\rightarrow\frac{dd(i)}{di}=\frac{dd(|i|)}{d|i|}\operatorname{sgn}(i)$.

Approximation (22) and its associate (23) suggested in the literature [13, 14] are in fact inconvenient in numerical calculations due to very high values of the derivative $dd(i)/di$ for very low currents $i\approx 0$. Functions (24) and (25) offered here are much more manageable in this respect.

In the hybrid TWV model of the arc [16] it is assumed that there are various heat dissipating channels (Mayr's model - conduction, Cassie's model – convection), depending on the value of the current. It can also be assumed here that Voronin's model is operative for weak currents, whereas Sawicki's model for strong currents . In this way, Voronin-Sawicki's hybrid model (VS) is obtained

$$\begin{gathered}\frac{1}{g}\frac{dg}{dt}=\frac{1}{\theta(d(i))}\left\{[1-\varepsilon(i)]\frac{u_{col}i}{P_{disV}(l,d(i))}+\right.\\ \left.+\varepsilon(i)\frac{u_{col}i}{P_{disS}(l,d(i))}-1\right\}-\frac{1}{l}\frac{dl}{dt}\left(1+\ln\frac{4g\,l}{\pi K_g d^2(i)}\right)\\ +\frac{2}{d(i)}\frac{dd(i)}{dt}\left(1-\ln\frac{4g\,l}{\pi K_g d^2(i)}\right)\end{gathered} \tag{26}$$

where $\theta(d(i))$ – the generalized attenuation function, which can be described by means of a formula analogous to (23)-(25). In can be approximately assumed that in this model

$$\theta(i) = \theta_S(d(i)) \approx \approx \begin{cases} \frac{p_V}{4p_S}\theta_V d(i) = const \cdot d(i), & \text{if } |i| \text{ small} \\ \frac{p_V}{4p_S}\theta_V d_{\max} = const, & \text{if } |i| \text{ big} \end{cases} \tag{27}$$

where $d_{\max}$ – corresponds to the current amplitude ($d_{\max} = d(I_{\max})$) .

Problems of selecting the weight function in hybrid arc models

In the original hybrid arc model TWV [16] the simplest form of the weight function was suggested (a Gaussian function)

$$\varepsilon_0(i) = \exp\left(-\frac{i^2}{I_0^2}\right) \tag{28}$$

where I_0 – is the boundary current of switching between Cassie's and Mayr's models – the inflection point (I_0, e^{-1}).
A number of simulations were performed and the results were compared to experiments in which high power AC plasma torches were used [17]. On the basis of that a modified weight function was put forward, with strong asymmetry with respect to the inflection point

$$\varepsilon_1(i) = \exp\left(-\frac{|i|^\alpha}{I_0^\alpha}\right) \tag{29}$$

The advantage of formula (29) is that it is simple, on the other hand, it is also characterised by limited flexibility in curve shape alterations. In the search for a new weight function a modified form of formula [9] can be employed

$$\varepsilon_2(i) = \exp\left[-\left(\frac{|i|}{I_0}\right)^{\alpha/(\delta+|i|)}\right] \tag{30}$$

where δ is a very small quantity ($0 < \delta << 1$). Here $\delta = 1*10^{-3}$.
If the departure point is a unipolar sigmoid function, another weight function is obtained [9, 18]

$$\varepsilon_3(i) = \frac{1}{1+\exp(\beta\cdot(|i|-I_0))} \tag{31}$$

where $\beta >> 1$. Increase in the coefficient β causes that the curves become steeper near the boundary point (I_0, 0,5). The weight function can be easily rendered asymmetrical, analogously to (29) and (30), by modifying the formula

$$\varepsilon_4(i) = \frac{1}{1+[\exp(\beta\cdot(|i|-I_0))]^{\alpha/(\delta+|i|)}} \tag{32}$$

The choice of a particular weight function depends on the required accuracy of approximating the experimental results and the shape of dynamic VC arc characteristics. The latter, in turn, depend on the chemical composition, pressure and thermal state of the gas, shape, material, dimensions and position of the electrodes, parameters (amplitude, frequency) of the periodical forced current, external interference, etc. The effectiveness of applying the weight functions described above in hybrid arc models is discussed in [9].

Representing the variable attenuation coefficient in arc models

The amount of internal energy accumulated in an arc depends on a number of physical factors: plasma volume (radius, length, shape of the column), temperature distribution in the arc column, gas pressure, degree of plasma ionization, etc. The arc internal energy cannot change in a discrete manner during commutation, and consequently, other parameters, such as dissipated power and conductance cannot change discretely either. Since the arc is a non-linear element of the electric circuit, the curves of transient processes are not exactly exponential functions. Despite this, a constant arc attenuation coefficient is introduced and assumed to be such during the whole process [19].

The arc time constant is shorter by 2÷3 orders of magnitude ($10^{-6}\div10^{-7}$ s) in arc torches as compared to free arcs. The greater the velocity of gas flow around the arc column or the velocity of arc in gas, the smaller the time constant. For high gas velocities the arc time constant does not depend on gas composition or electrode types. As the current increases up to the magnitude of hundreds and thousands of amperes, the values of time constants in various gases converge and approach the value of about 10^{-4} s [19].

The methods of obtaining the time constant can be generally divided into experimental and theoretical ones. The dynamic properties of the arc can be best observed during current modulations. Due to the fact that the distributions of current intensity, temperature, plasma flow, etc. are highly non-homogeneous, there are differences in relaxation times between processes occurring in the core and on the surface of the arc column. The analysis of the shape of the function $\ln(u(t))$ representing the voltage response to a discrete current fluctuation makes it possible to find the time constant of heat processes in the core θ_f and on the surface θ_S, where $\theta_f < \theta_S$. Typically, however, an equivalent time constant is introduced, based on generalized relaxation times not only in the arc column, but also on electrode spots. This kind of a characteristic is shown in Fig. 1. As is seen, the decrease in current results in a local minimum around the value of 18 A. Further decrease in current corresponds to significant increase in the value of the time constant. The difference becomes even more dramatic when the gas flow around the arc is intensified [15].

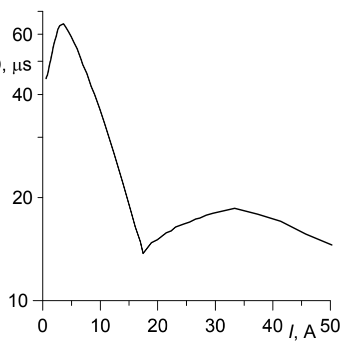

Fig. 1. Time constant characteristics as a function of arc discharge current in the constrictor channel of plasma torch with nitrogen [15]

The arc time constant obtained experimentally by introducing small fluctuations during the quasistatic alteration of direct current can be different from the AC arc time constant, especially in the range of very weak currents where the arc becomes unstable. Due to that it is more reliable to determine the time constant by measuring the time from the moment of the current passing zero to the moment of arc ignition or extinction. Alternatively, the constant can be determined on the basis of the analysis of arc voltage harmonics.

Since the simplified mathematical models are approximations of the electrical arc characteristics within narrow ranges of current, the choice of the form of the dissipated power function is correlated to the time constant. In some constant-length arc models, however, such as Schwarz-Avdonin's model, the arc time constant is approximated by a power function $\theta(g)=\theta_0 g^{\alpha}\cong\theta_0' S^{\alpha}$ In hybrid models, encompassing wide ranges of currents, the choice of the attenuation coefficient function often requires taking into account a more complex function $\theta(i)$. In [16] the general behaviour of the equivalent attenuation coefficient in a hybrid TWV model is described by the formula

$$\theta=\theta_0+\theta_1\exp\left(-\alpha|i|\right)\approx\begin{cases}\theta_1\,, & \text{if } |i| \text{ small}\\ \theta_0, & \text{if } |i| \text{ big}\end{cases} \tag{33}$$

where $\alpha > 0$ i $\theta_1 >> \theta_0$. On the basis of Equation (15) and Fig.1 it is possible to offer more precise approximations of the attenuation coefficient, applying the proportion $\theta(i) \propto d(i)$. In order to do this, one can use one of the formulas (23)-(25).

Simulation of processes in models with variable-length free arcs

Assume that arc length changes are relatively slow ($dl/dt \approx 0$). Then (16) becomes

$$\begin{aligned}\frac{1}{g}\frac{dg}{dt}&=\frac{1}{\theta_S\left(d(i)\right)}\left(\frac{u_{col}i}{P_{disS}\left(l,d(i)\right)}-1\right)+\\&+\frac{2}{d(i)}\frac{dd(i)}{dt}\left(1-\ln\frac{4g\,l}{\pi K_g d^2(i)}\right)\end{aligned} \tag{34}$$

With new symbols, it becomes

$$\begin{aligned}\frac{1}{g}\frac{dg}{dt}&=\frac{1}{K_t d(i)}\left(\frac{u_{col}i}{K_S l d(i)}-1\right)+\\&+\frac{2}{d(i)}\frac{dd(|i|)}{d|i|}\operatorname{sgn}(i)\frac{di}{dt}\left(1-\ln\frac{K_G g\,l}{d^2(i)}\right)\end{aligned} \tag{35}$$

where $P_{disS}\left(l,d(i)\right)=K_S l\cdot d(i)$, [W]; $K_S=\pi p_S$, [W·m^{-2}]; $K_t=0.25Q_0/p_S$, [s·m^{-1}]; $K_G=4/\left(\pi K_g\right)$, [m·S^{-1}].
Static characteristics of the arc determines the relationship

$$u_{col}(i)=K_S l\,\frac{d(i)}{i} \tag{36}$$

which in the case of approximation (22) takes the form $u_{col}(i)=K_S b l|i|^{q-1}\operatorname{sgn}(i)$.

On the same assumption ($dl/dt \approx 0$) Equation (21) also becomes simpler

$$\begin{aligned}\frac{1}{g}\frac{dg}{dt}&=\frac{1}{\theta_V}\left(\frac{u_{col}i}{P_{disV}\left(l,d(i)\right)}-1\right)+\\&+\frac{2}{d(i)}\frac{dd(i)}{dt}\left(1-\ln\frac{4g\,l}{\pi K_g d^2(i)}\right)\end{aligned} \tag{37}$$

and with new symbols

$$\begin{aligned}\frac{1}{g}\frac{dg}{dt}&=\frac{1}{\theta_V}\left(\frac{u_{col}i}{K_V l d^2(i)}-1\right)+\\&+\frac{2}{d(i)}\frac{dd(|i|)}{d|i|}\operatorname{sgn}(i)\frac{di}{dt}\left(1-\ln\frac{K_G g\,l}{d^2(i)}\right)\end{aligned} \tag{38}$$

where $K_V=0.25\pi p_V$, [W·m^{-3}]. Static characteristics of the arc determines the relationship

$$u_{col}(i)=K_V l\,\frac{d^2(i)}{i} \tag{39}$$

which in the case of approximation (22) takes the form $u_{col}(i)=K_V b^2 l|i|^{2q-1}\operatorname{sgn}(i)$.
Simplified Voronin-Sawicki's model (26) can be represented as

$$\begin{aligned}\frac{1}{g}\frac{dg}{dt}&=\frac{1}{\theta\left(d(i)\right)}\left\{\left[1-\varepsilon(i)\right]\frac{u_{col}i}{P_{disV}\left(l,d(i)\right)}+\right.\\&\left.+\varepsilon(i)\frac{u_{col}i}{P_{disS}\left(l,d(i)\right)}-1\right\}+\\&+\frac{2}{d(i)}\frac{dd(i)}{dt}\left(1-\ln\frac{4g\,l}{\pi K_g d^2(i)}\right)\end{aligned} \tag{40}$$

and with the newly introduced symbols it becomes

$$\begin{aligned}\frac{1}{g}\frac{dg}{dt}&=\frac{1}{K_t d(i)}\left\{\left[1-\varepsilon(i)\right]\frac{u_{col}i}{K_V l d^2(i)}+\right.\\&\left.+\varepsilon(i)\frac{u_{col}i}{K_S l d(i)}-1\right\}+\\&+\frac{2}{d(i)}\frac{dd(|i|)}{d|i|}\operatorname{sgn}(i)\frac{di}{dt}\left(1-\ln\frac{K_G g\,l}{d^2(i)}\right)\end{aligned} \tag{41}$$

Since the approximations of the function $d(i)$ cover the full current range, then the attenuation coefficient $\theta(d(i)) = K_t d(i)$ can be applied in the hybrid model. Static characteristics of the arc determines the relationship

$$u_{col}(i)=\frac{l}{i\cdot\left[\dfrac{1-\varepsilon(i)}{K_V d^2(i)}+\dfrac{\varepsilon(i)}{K_S d(i)}\right]} \tag{42}$$

which in the case of approximation (22) takes the form

$$u_{col}(i)=\frac{l}{\dfrac{1-\varepsilon(i)}{K_V b^2}|i|^{1-2q}+\dfrac{\varepsilon(i)}{K_S b}|i|^{1-q}}\operatorname{sgn}(i) \tag{43}$$

Practical applications of the mathematical models of the variable-length arc for simulating the working conditions of welding and electrothermal devices can be largely facilitated by implementing their macromodels in the popular software MATLAB-Simulink. Effects of such a simulation in a serial circuit with a current source ($I_{\max}$ =200 A, f = 50 Hz), reactive elements *RL* (R = 0.01Ω, L = 1 mH) and electric arc are shown in Fig.2 where the appropriateness of the arc models developed is confirmed. It was assumed the resultant voltage drop near the electrodes 18 V. Uses the function approximation (24) with coefficients: a=0.054, b=0.64, p=0.0059, q=0.0072.

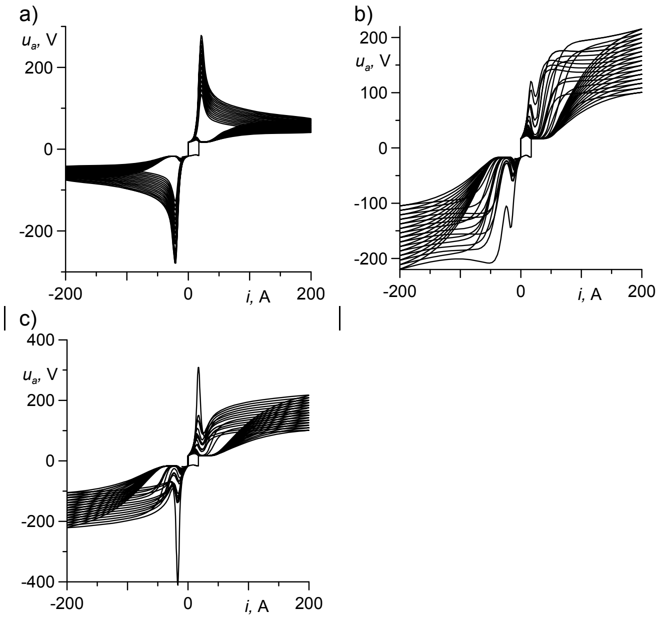

Fig 2. Dynamic hysteresis loops of an extending arc: a) Voronin's model (35), (K_S=30·10^5 W·m^{-2}, K_t=0.02 s·m^{-1}, K_G=0.05 m·S^{-1}); b) Sawicki's model (38), (K_V=15·10^7 Wm^{-3}, θ_V=6·10^{-4} s, K_G=0.2 m·S^{-1}); c) VS hybrid model (41) (K_S=30·10^5 W·m^{-2}, K_V=15·10^7 Wm^{-3}, K_t=0.016 s·m^{-1}, K_G=0.2 m·S^{-1}, I_0=5 A)

Conclusions

1. Combined models of the non-stationary, variable-length electric arc offer new possibilities of simulating processes in circuits with various electric currents and various moments when fluctuations occur or control is applied.
2. Tying the dissipation power and attenuation coefficient with the geometrical dimensions of the arc column makes it possible to precisely represent the dynamic arc characteristics in various operating conditions and stages of electrotechnological devices.

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