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Mathematical analysis of transformer insulation state by means of composite indicator

***Abstract**. The paper is presenting mathematical model – composite indicator (CI), which was utilized on insulating state of distribution transformer to analyze and sensibility of individual measurements methods mutual comparison. We can uniquely determine importance of these measurements methods with this mathematical apparatus in these measurements methods in insulating state of transformers.*

***Streszczenie.** W artykule zaprezentowano matematyczny model wskaźnika syntetycznego, który został wykorzystany do oceny stanu izolacji transformatorów mocy. Opracowany model został zweryfikowany poprzez porównanie z innymi metodami. Wykazano zastosowanie statystycznych metod wieloatrybutowej analizy porównawczej dla oceny stanu izolacji transformatorów. (**Matematyczny model wskaźnika do oceny stanu izolacji transformatorów mocy**)*

Keywords: transformers, insulation, composite indicator, dendogram, cluster
Słowa kluczowe: transformator, izolacja, wskaźnik syntetyczny, dendogram, cluster

Introduction

With regard to the development of world and national economics, also control, maintenance and their analysis by mathematic calculations becomes an important sphere. This sphere also includes power transformers, where their proper function has a positive impact on the trouble-free supply of electricity and heat for industries and households. The importance of diagnostics at the premises of strategic significance, results not only from an economic viewpoint, but also because of safety and faultless operation, i.e. reliability of technical equipment in the power industry. It is therefore necessary, in the absence of scientific and research potential in a distribution organizations (e.g. power plants, heating plants), to achieve the objectives of the proposed activities, i.e. in-depth analysis of undesirable impacts on the state of devices, design of measurements and their verification, and design of new diagnostic procedures for improving reliability of power transformers.

Unless we want to determine real insulating state of transformer and then lifetime of insulation, is necessary to analyze some measurements in individual type of assays and then determine their exactness and reliability with mathematical model. We can exactly prove the importance of these assays by mathematical and statistical models in region of analyze of insulating state of transformers.

For mathematical analyzing these assays measurements we chose within the frame of comparison of degree of sensitivity in single methods of insulating state of distribution transformer 110/22kV:

- insulation resistance and absorption coefficient: R_{60}/R_{15},
- dissipation factor and capacity: $\tan\delta$ and C,
- relative change of short-circuit voltage $\mathrm{d}U_k$.

Description of chosen measurements

The oldest and easiest method of inspecting the state of insulators is by means of insulation resistance measuring. Main disadvantage of this method is that insulation resistance does not only depend on state of insulation but also on its type and dimensions. Therefore, insulation resistance method can be used to evaluate the state of insulation of electric device only on the basis of previous experience with the same insulation on the same device. Moreover, this method enables to identify even small insulation degradation, if it passes through insulation layer e.g. oil – paper, but it can not identify whether the degradation is on the side of oil or paper.

The method is based on the following principle: change in insulator state causes change in time dependence of a current flowing through the insulator by direct voltage [8]. Current flowing through insulator consists of time-decreasing absorption element and stabilized element. More water content is there in insulation, more apparent increase of stabilized element of a current is observed comparing to absorption element. Absorption element of a current has a low effect on characteristics of time dependency in relation to current as well as resistance, and flattens with increasing humidity (Fig.1).

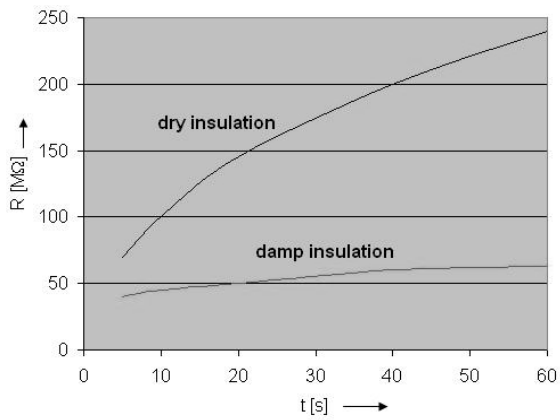

Fig.1. Time dependence of the insulation resistance

Utilizing this knowledge for evaluation of insulation state does not require determining full time dependence of a current. It is enough to determine value of a current (resistance) in two different moments from the time of connection to direct voltage. Ratio of these two values defines the state of insulation and is called polarizing index. Since it is a non-dimensional parameter, it does not depend on dimension of insulation. Polarizing index is measured after 1 and 10 minutes or after 15 and 60 seconds.

So as to better illustrate change in values of polarizing index, it needs to be expressed by both elements of current – absorption element i_a and stabilized element i_∞

$$p_i = \frac{R_{60}}{R_{15}} = \frac{i_{a15} + i_\infty}{i_{a60} + i_\infty} \qquad (1)$$

Humid and contaminated insulation is determined by i_∞, therefore numerator and denominator are very close values and their ration tends to 1. On the other hand, dry and clean insulation which is in good condition has stabilized current very low and time dependent element i_a is dominant. Thus, fraction value is noticeably higher than 1.

Absorption coefficient of new transformers before usage in operation should reach at least 1,3, usually value varies from 1,7 to 1,8.

The measurements of dissipation factor and capacities of transformer windings are used for additional determination of insulation quality as whole or only of some parts of transformer. The value of dissipation factor indicates presence of polar and ion compounds in oil and it also determinate the aging of oil. The degree of oil humidity can be measured by temperature dependence of tgδ [8].

Changes state of short-circuit voltage dU_k (impedance) express geometrical winding movement and their construction changes in transformer. This technical condiction depends on thermal and mechanical effects of short-circuit currents.

By means of meeasurement of short-circuit voltage we can identify the following damages of transformer:

- mechanical and insulating deformation of winding and its movement,
- disconnection of coils,
- interturn short-circuits.

Absolute value of short-circuit voltage usually are not sufficient to qualify condition of winding without knowledge of their evolution in time, so the analysis is based on comparision of values for specified time of operation of transformer (in this case is comparised between actual and factory new condition).

Composite indicator

A composite indicator (CI) is an mathematical aggregation of a set of individual indicators that measure multi-dimensional concept but usually no common units of measurement [1]. CI is used for performance measurement, benchmarking, monitoring, communication via providing an aggregated performance index in various fields such as sustainable energy index, Human Development Index, Road Safety Index [3, 4]. The mains steps in constructing CIs are weighting and aggregating of a set of given sub-indicators which directly affect the quality and reliability of calculated CI.

The graphical representation of CI construction is illustrated on Fig.2. There are *m* decision making units [DMU] which means comparised alternatives, each DMU consist *n* sub-indicators I_{ij}. Sub-indicators need to of the benefit type (the larger the better). For each DMU is evaluated CI. Sub-indicators usually have no common measurable units.

$$\begin{array}{c} DMU_1 \\ DMU_2 \\ \vdots \\ DMU_m \end{array} \begin{bmatrix} I_{11} & I_{12} & \cdots & I_{1n} \\ I_{21} & I_{12} & \cdots & I_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ I_{m1} & I_{m2} & \cdots & I_{mn} \end{bmatrix} \rightarrow \begin{bmatrix} CI_1 \\ CI_2 \\ \vdots \\ CI_m \end{bmatrix}$$

Fig.2. Construction of CI

Generally, the structure of CI can by expressed by equation:

$$CI = \sum_{i-1}^{n} w_i I_i \tag{2}$$

where w_i means weight assigned to indicator *i*.

A major problem in creating CI is the determination of the weights for the underlying sub-indicators. From the methodological point of view, there exist many weighting methods which can be used to derive weights for sub-indicators [1]. The most common used methods of CI construction are [3]:

- Factor analysis (FA),
- Analytic hierarchy process (AHP),
- Budget allocation (BA),
- Data envelopment analysis (DEA).

FA method is based on a reducing the dimensions of the problem, where the *n* dimensions are transformed into a smaller number of factor and weights of indicators are based on correlations. FA method requires some correlation between indicators, so can be used not for every types of DMUs. AHP and BA are based on the opinions of the experts, so values of CI could be subjective. Of the four method DEA is the most universal and the most objective.

DEA is a performance measurement technique that can be used for evaluating the relative efficiency of DMUs. For each DMU the efficiency is defined as a ratio of the weighted sum of outputs to the weighted sum of inputs [4]. An application of DEA was proposed in [1] to construct CI, where two sets of weights are calculated by using two slightly different DEA models. The final CI values are results of combination of these two DEA models.

In the first DEA model for a particular DMU$_j$ in the data set (*j* = 1, 2, …, *m*) a gI_i value is determined using a set of the best indicator weights w_{ij} (*i* = 1, 2, …, *n*) which gI_i value of the DMU$_j$ and satisfies the restrictions.

$$gI_i = \max \sum_{j=1}^{n} w_{i,j}^{g} I_{i,j} \tag{3}$$

$$s.t. \sum_{j=1}^{n} w_{i,j}^{g} I_{i,j} \le 1, \quad k = 1,2,...,m \tag{4}$$

$$w_{i,j}^{g} \ge 0, \quad j = 1,2,...,n \tag{5}$$

In the second DEA model for a each DMU$_j$ a bI_i value is determined using a set of the worst indicator weights w_{ij} (*i* = 1, 2, …, *n*), with the similarly restrictions:

$$bI_i = \min \sum_{j=1}^{n} w_{i,j}^{b} I_{i,j} \tag{6}$$

$$s.t. \sum_{j=1}^{n} w_{i,j}^{g} I_{i,j} \ge 1, \quad k = 1,2,...,m \tag{7}$$

$$w_{i,j}^{b} \ge 0, \quad j = 1,2,...,n \tag{8}$$

In different words, DEA is a linear programming model, where each entity selects a set of weights which are most favorable for itself to give a standardized efficiency score (between zero and one). The first DEA model (3-5) can help each entity select the "best" set of weights for use, the second DEA model measures how close the evaluated entity is from the worst case under the worst possible weights.

Models (3-5) and (6-8) are giving as a results indexes based on weights w_{ij} that are most favorable and less favorable for each entity and these two indexes are combined to CI form in the following way:

$$CI_i = k \frac{gI_i - gI^-}{gI^* - gI^-} + (1-k) \frac{bI_i - bI^-}{bI^* - bI^-} \tag{9}$$

where:

$$gI^* = \{\max gI_i, \ i = 1,2,...,m\}$$

$$gI^- = \{\min gI_i, \ i = 1,2,...,m\}$$

$$bI^* = \{\max bI_i, \ i = 1,2,...,m\}$$

$$bI^{-} = \{\min bI_i,\ \ i = 1,2,...,m\}$$
$$0 \le k \le 1$$

Coefficient k is an adjusting parameter which is determined by decision maker. Usually has a value 0,5.

Hierarchical cluster analysis

Cluster is an multivariate technique which informs about similarity in the data set. Clustering is task of assigning objects into groups – cluster. The objects in the same cluster are more similar to each other than to those in other clusters. The classification aims to reduce the dimensionality of a data set by finding similarities between classes [2].

Classification criterion is the Euclidean distance D between objects x and y, defined as:

$$D(x,y) = \frac{\sum_{i=1}^{N} (x'_i - y'_i)^{\frac{1}{2}}}{N} \tag{10}$$

where N – number of dimension of objects, x_i', y_i' – standardized values of the object x and y.

Standardization is implemented according formula:

$$x'_i = \frac{x_i - x_{av}}{\sigma_x} \tag{11}$$

where x_{av} – average value of entity x, σ – deviation of entity x.

Similarly as above, clusters are grouping into upper-clusters, which express similarity between groups. This operation is repeated until every object is grouped into one cluster. The result is a dendogram that illustrates the relationships between objects.

Principal components analysis (PCA) is a mathematical method which performs a reduction of data dimensionality and allows the visualization of underlying structure in experimental data [3, 6, 7]. Indicators are grouped in a limited number of components and still are able to explain majority of the variance in data. The idea of PCA can be expressed by equitation:

$$V = WX \tag{12}$$

where x – matrix of the input data, V – matrix of the principal components, W – matrix of eigenvectors of the covariance matrix X. The dimensionality of input data can be reduce by using only the first n singular vectors of W:

$$V' = W_n X \tag{13}$$

The factor n depends on the required share of variance explained by the selected number of components.

Results of the calculations

The values of sub-indicators were normalized to scale 0-1. In case of coefficient R_{60}/R_{15} and C, which are of "benefit type" was used formula:

$$x'_i = \frac{x_i - \min_i\{x_i\}}{\max_i\{x_i\} - \min_i\{x_i\}} \tag{14}$$

"Cost type" sub-indicators – dU_k and tanδ – are normalized in the following way:

$$x'_i = \frac{\max_i\{x_i\} - x_i}{\max_i\{x_i\} - \min_i\{x_i\}} \tag{15}$$

Table 1 presents the results of the calculations as well as the data for four sub-indicators for randomly of chosen 13 transformers 110/22 kV.

On Fig.3 is shown dendogram obtained by clustering. For hierarchical clustering, the distances at which clusters are combined can be used as criteria. Applying k-means clustering [9] the transformers T1,…, Tn were divided into the following four classes:

- T1, T2, T4, T8, T9, T10, T11, T12, T13 (cluster 1),
- T3, T6 (cluster 2),
- T5 (cluster 3),
- T7 (cluster 4).

The strengths and weaknesses of each cluster can be deduced from Fig.4, which presents the median values of sub-indicators for each clusters.

PCA analysis was applied to the data set of transformers after standardization. The sample score of PCA is presented on Fig.5. To analysis were chosen the first two components (PC1 and PC2) of matrix W which explain 73,1% of total variance: PC1 explain 48,32%, PC2 – 24,78%.

Table 1. Four sub-indicators and CI values for 13 transformers

Tr	Sub-indicators				Indicators		
	$\frac{R_{60}}{R_{15}}$	dU_k [‰]	tanδ	C [pF]	*gI*	*bI*	CI
T1	1,36	0,47	0,0217	2881,4	0,873	1,053	0,362
T2	1,37	8	0,0186	4746,7	1	1,442	0,643
T3	1,58	4,5	0,0123	2996,5	1	1,348	0,613
T4	1,44	7,9	0,0075	2731,5	0,974	1,346	0,580
T5	1,31	42,7	0,0046	1957,5	0,591	1	0,001
T6	1,55	32,4	0,0135	3815,5	1	1,004	0,501
T7	1,25	0	0,0424	3940,0	0,839	1	0,303
T8	1,31	21,4	0,0177	4882,0	0,954	1,225	0,517
T9	1,30	8,95	0,0160	4235,3	0,972	1,487	0,623
T10	1,39	2,41	0,0122	2236,0	0,936	1	0,422
T11	1,38	8,93	0,0153	3825,2	0,950	1,508	0,603
T12	1,32	20,4	0,0126	4030,7	0,882	1,310	0,457
T13	1,31	3,64	0,0187	3775,0	0,953	1,542	0,618

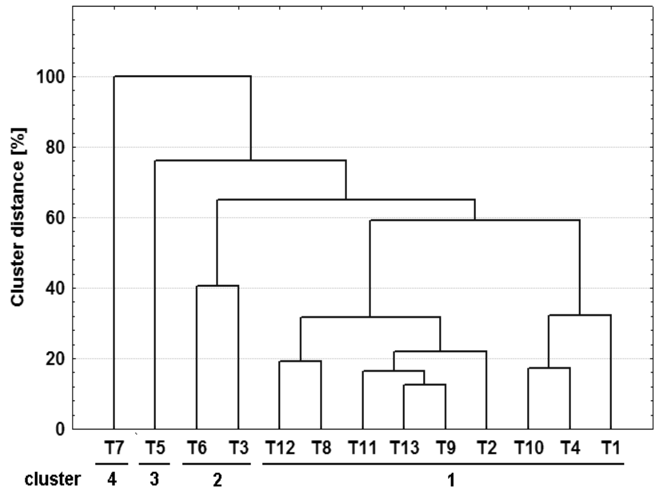

Fig.3. Dendogram based on hierarchical clustering

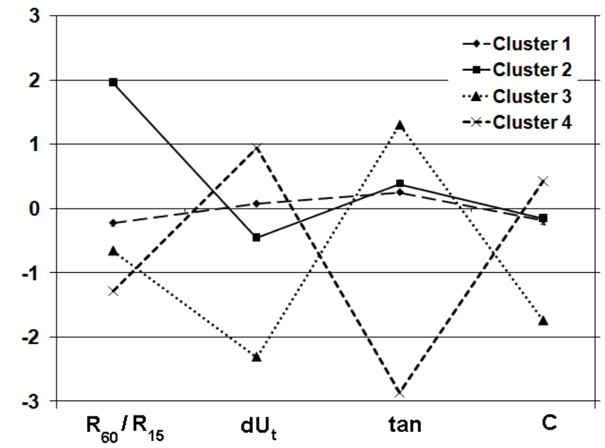

Fig.4. Cluster centers per sub-indicators

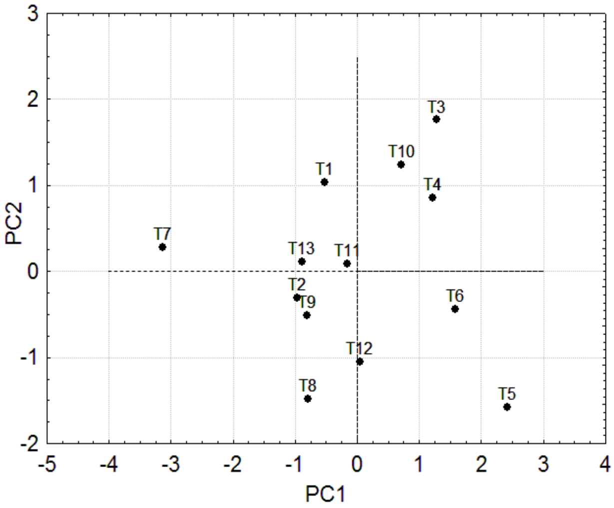

Fig.5. Principal component analysis (PCA) plots transformers

Discussion

On the basis of the performed cluster analysis, principal component analysis and created composed indicator can be found and explained relationships between technical parameters of the transformers.

Firstly, could be analyzed the average value of CI in each cluster, which equal:

- cluster 1: 0,538,
- cluster 2: 0,557,
- cluster 3: 0,001,
- cluster 4: 0,303.

It is noticeable, that values of CI for clusters 1 and 2 are similar and they are higher than values for clusters 3 and 4. This suggest the diametrically worst technical condition of transformers T5 (cluster 3) and T7 (cluster 4) in comparison to the other transformers. This relationship can be confirmed by cluster centers per sub-indicators analysis (Fig.3), where the locations of cluster centers for clusters 3 and 4 are significantly different from the locations for clusters 1 and 2. The average value of CI of cluster 2 is slightly bigger than value of cluster 1. It may be explained by higher location of the center of cluster 2 for parameter R_{60}/R_{15}.

The sample score plot for PC1 and PC2 is shown in Fig. 5 and a number of observations may be made. Firstly, the T5 and T7 are located furthest. This is to be expected because transformer T5 and T7 have the worst values of CI. In contrast the transformers T2, T9, T11, T13, which have the best values of CI (bigger than 0,6), are located in the centre of the diagram. Interestingly, despite the fact that transformer T3 also got big value of CI (0,613), is located relatively quite far from the centre of PCA diagram (Fig.5). It may be explained by relatively small value of insulation capacitance *C* of this transformer, even though the other parameters of T3 are good. It should be noticed, that transformers T1, T4 and T10, which form "under-cluster" in Fig.4 are also closely located on Fig.5.

Conclusion

On the basis of summary results of the mathematical CI model, there can be set optimized modern techniques for the diagnosis of insulation state chosen oil transformers, thereby a higher quality of trouble-free distribution of heat and electricity will be achieved.

Composed indicators have been accepted as a useful tool in many non-technical areas, such economy, society, and environment. In this paper is presented application of CI in field of technical sciences. CI describing the technical condition of high power transformer was developed. By using statistical methods CPA and cluster analysis was proofed, that proposed approach is reliable and can be applicable for using in industry. Moreover, it was showed that other method of statistical analysis – PCA and cluster analysis – can be used as e useful tool for analyze of condition of transformers.

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